Physics IV [2704] Practice Midterm 2 – Version 2

Directions: This exam is closed book. You are allowed one 8.5”x11” sheet with equations etc., which should be turned in with your test. Read all the questions carefully and answer every part of each question. Please show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Please use a calculator only to check arithmetic – all steps of calculations should be explicitly shown. Unless otherwise instructed, you can express your answers in terms of fundamental constants like $k, h, \hbar, c, \varepsilon_0$ rather than calculating numerical values. If the question asks for an explanation, please write at least a full sentence explaining your reasoning. Please ask if you have any questions, including clarification about any of the instructions, during the exam.

This test is designed to be gender and race neutral.

Good luck!

A few useful numbers:

Speed of light $c = 3\text{x}10^{8}$ m/s
Planck’s constant $h = 6.6\text{x}10^{-34}$ J s $= 4\text{x}10^{-15}$ eV s
Compton wavelength $h/(m_e c) = 2.4\text{x}10^{-12}$ m
Photon energy $= h\nu = hc/\lambda \sim 1240$ eV-nm /(λ in nm)
Bohr energies $E_n = -me^4/(32\pi^2\varepsilon_0^2\hbar^2) * Z^2/n^2 = -13.6\text{eV} * Z^2/n^2$
Bohr radius $a_0 = 4\pi\varepsilon_0\hbar^2/(me^2) = 0.0529$ nm

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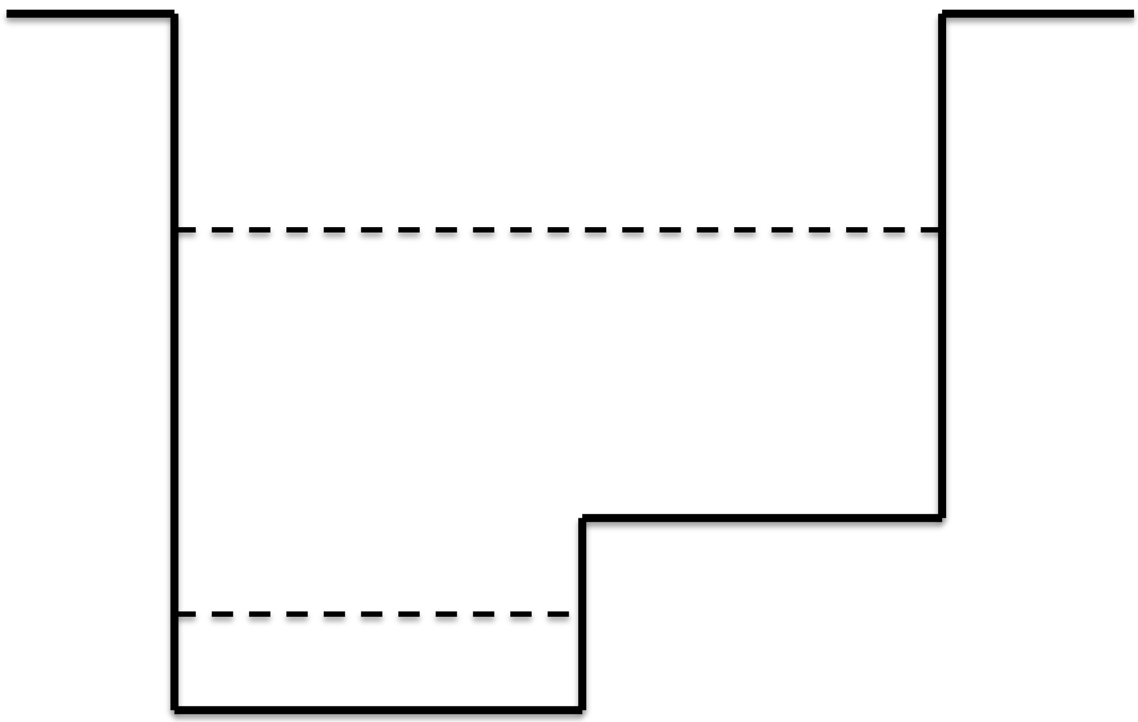
Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else’s test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name______________________________________

Print Your Name______________________________________

Question 1 (20 points): Draw the real component of two wave functions that might occur in an asymmetric potential like that shown below, for electrons at the two energy levels shown. Make sure your wave functions include at least three wavelengths, and are qualitatively accurate in how both their wavelength and amplitude vary across the drawing.



Question 2 (30 points): Consider the following one-dimensional potential energy step:

$U(x) = 0$ for $x < 0$
$U(x) = U_0$ for $x > 0$ (U_0 is a finite positive constant)

Particles of mass m are incident on this barrier from the negative x direction, with an energy $E < U_0$.

2a (10 points): Write down the general time-independent wave function ψ_1 valid for $x < 0$, explicitly show that each term of this wave function is a solution to the appropriate time-independent Schrödinger equation, and relate the wave number k_1 that appears in these terms to the energy E.

2b (10 points): Write down the general time-independent wave function ψ_2 valid for $x > 0$, explicitly show that it is a solution to the appropriate time-independent Schrödinger equation, and relate the wave number k_2 that appears in the wave function to E and U_0.

2c (10 points): Derive (do not try to solve) two equations that would allow you to relate the normalization constants in your two wave functions to each other and to k_1 and k_2.

Question 3 (15 points): In a Rutherford scattering experiment, the closest approach of an alpha particle (z=2) with a kinetic energy of 5 MeV and an impact parameter b =0 to a gold nucleus (Z = 79) is 50 femtometers. What would be the closest approach of a proton (z=1) with a kinetic energy of 10 MeV, also on a head-on trajectory with b=0? Either show the math or describe a qualitative argument supporting your answer.

Question 4 (20 points): The 3d (*n* =*3*, *l* = 2) radial wave function of an electron in atomic hydrogen is $R(r) = A\frac{r^2}{{a_o}^{3.5}}e^{-r/(3a_o)}$ where *A* is a constant.

4a (10 points): Find the most probable value of *r* (i.e. the most likely radius of the electron around the nucleus) from the radial probability density.

4b (10 points): The angular part of the 3d wave function for $m_l = +2$ is proportional to $sin^2\theta\, e^{2i\phi}$. Sketch the angular probability density as a function of theta (θ), and separately as a function of phi (φ). Discuss what this probability distribution means in terms of the electron's orbit and the direction of its angular momentum.

Question 5 (15 points): Describe qualitatively the physical principle that allows an electron to tunnel through a potential barrier. For what types of barrier is this tunneling more likely, and how does that relate to the physical principle you described?