# Physics IV [2704] Midterm 1 Wednesday, February 21, 2018 


#### Abstract

Directions: This exam is closed book. You are allowed one 8.5 " $\times 11$ " sheet with equations etc., which should be turned in with your test. Read all the questions carefully and answer every part of each question. Please show your work on all problems partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect. Please use a calculator only to check arithmetic - all steps of calculations should be explicitly shown. Unless otherwise instructed, you may express your answers in terms of fundamental constants like $k, h, c$ rather than calculating numerical values. If your answer includes a $\sqrt{ } \mathrm{N}$ you can leave it that way unless the number $N$ is a perfect square. If the question asks for an explanation, please write at least a full sentence explaining your reasoning. Please ask if you have any questions, including clarification about any of the instructions, during the exam.


## Good luck!

A few useful numbers:

| $\mathrm{v} / \mathrm{C}=0.5$ | $\gamma=1.15$ | I | $\mathrm{v} / \mathrm{c}=0.8$ | $\gamma=5 / 3$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{v} / \mathrm{c}=0.6$ | $\gamma=1.25$ | I | $\mathrm{v} / \mathrm{c}=(\sqrt{ } 3) / 2=0.866$ | $\gamma=2$ |
| $\mathrm{v} / \mathrm{c}=1 / \sqrt{ } 2$ | $\gamma=\sqrt{ } 2$ | I | $\mathrm{v} / \mathrm{c}=0.98$ | $\gamma=5$ |
| $\mathrm{v} / \mathrm{c}=0.75$ | $\gamma=1.51$ | I | $\mathrm{v} / \mathrm{c}=0.999$ | $\gamma=22$ |

Speed of light $\mathrm{c}=3 \times 10^{8} \mathrm{~m} / \mathrm{s}$
Planck's constant $h=6.6 \times 10^{-34} \mathrm{Js}=4 \times 10^{-15} \mathrm{eV} \mathrm{s} \quad \hbar=\mathrm{h} /(2 \pi)$
Compton wavelength $\mathrm{h} /\left(\mathrm{m}_{\mathrm{e}} \mathrm{c}\right)=2.4 \times 10^{-12} \mathrm{~m}=0.0024 \mathrm{~nm}$
Photon energy $=\mathrm{h} v=\mathrm{hc} / \lambda \sim 1240 \mathrm{eV}-\mathrm{nm} /(\lambda$ in nm$)$

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit ( 0 points) for this test, and disciplinary action may result.

## Sign Your Name

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Question 1 (10 points): A particle with rest mass $M$ and no initial velocity ( $v_{0}=0$ ) decays into two smaller particles, each with rest mass $m$. The two new particles move away from the decay site, each with velocity $v_{f}=0.866 c$, in opposite directions (so that momentum is conserved). Is the total combined rest mass of the two new particles ( $2 m$ ) greater than, less than, or equal to M? EXPLAIN YOUR ANSWER.

Question 2 (24 points): Spaceman Spiff measures his spaceship and finds that it has a proper length of 10 meters. He then flies at a velocity $\mathrm{v}_{\text {ship }}=0.8 \mathrm{c}$ past Hobbes, who is at rest on a space station. Hobbes and Spiff each start their clocks $\left(t=0, t^{\prime}=0\right)$ as the front of the spaceship passes the station, and each defines the origin of their
 coordinates ( $x=0, x^{\prime}=0$ ) to be at this point in their frame, with their $x$-axes pointing along the spaceship velocity as shown.

2a (8 points): What are the space-time coordinates $\left(x_{2}, c t_{2}\right)$ at which the back of the spaceship passes Hobbes, according to Hobbes?

2b (8 points): What are the space-time coordinates ( $x_{2}$ ', $c t_{2}$ ) at which the back of the spaceship passes Hobbes, according to Spaceman Spiff?

2c (8 points): What are the space-time intervals $\Delta s^{2}=(c \Delta t)^{2}-\Delta x^{2}$ between the front and back of the spaceship passing the station, according to Hobbes and Spiff? Do they agree? Why or why not?

Question 3 (16 points): Two photons, each with energy $E_{Y}$, collide to produce a new particle with rest mass $M$. Assuming that the photons approach from opposite sides of the $x$-axis in the $x$-y plane, with their velocities at $\pm 60^{\circ}$ from the $x$-axis as shown, what are the final relativistic total energy $E_{f}$, the final relativistic vector momentum x and y components $p_{f x}$ and $p_{f y}$, and the rest mass $M$ of the new particle, in terms of $E_{\gamma}$ ?


Question 4 (10 points): A beam of electrons of speed $v$ is incident on a very small double slit and then strikes a screen where it is made visible. If the speed of the electrons is increased, does the spacing between the bright regions on the screen increase, decrease, or remain the same? EXPLAIN YOUR ANSWER.

Question 5 ( 20 points): In a Compton scattering experiment, a photon with an initial wavelength of 0.0024 nm scatters from an electron. In the process, the electron gains $250,000 \mathrm{eV}$ of kinetic energy. As an approximation to make the calculations easy, assume $h c / \lambda=1200 \mathrm{eV}-\mathrm{nm} /(\lambda$ in nm$)$.

5a (5 points): What is the change in frequency (not wavelength!) of the photon? You can leave the answer written in terms of constants like $e$ and $h$ rather than computing a numerical value.

5b (15 points): At what angle (relative to its incoming direction) did the photon scatter?

Question 6 ( 10 points): The uncertainty principle allows for the possibility of a particleantiparticle pair to be created from nothing (a so-called "quantum fluctuation"), as long as the two particles don't exist for very long. Imagine an electron-positron pair is created from vacuum. The electron and positron each have a rest energy of $\sim 500,000 \mathrm{eV}$. Approximately how long (in seconds) does the uncertainty principle allow this pair of particles to exist for? An answer correct to one order of magnitude is adequate.

Question 7 (10 points): Draw a wave packet (as a function of spatial coordinates). Now, assume that we affect this wave packet in such a way that we double the uncertainty in its momentum. Assuming that the product of the uncertainty in momentum and the uncertainty in position is approximately constant (as expected from the uncertainty principle), draw the new wave packet, on the same spatial scale.

