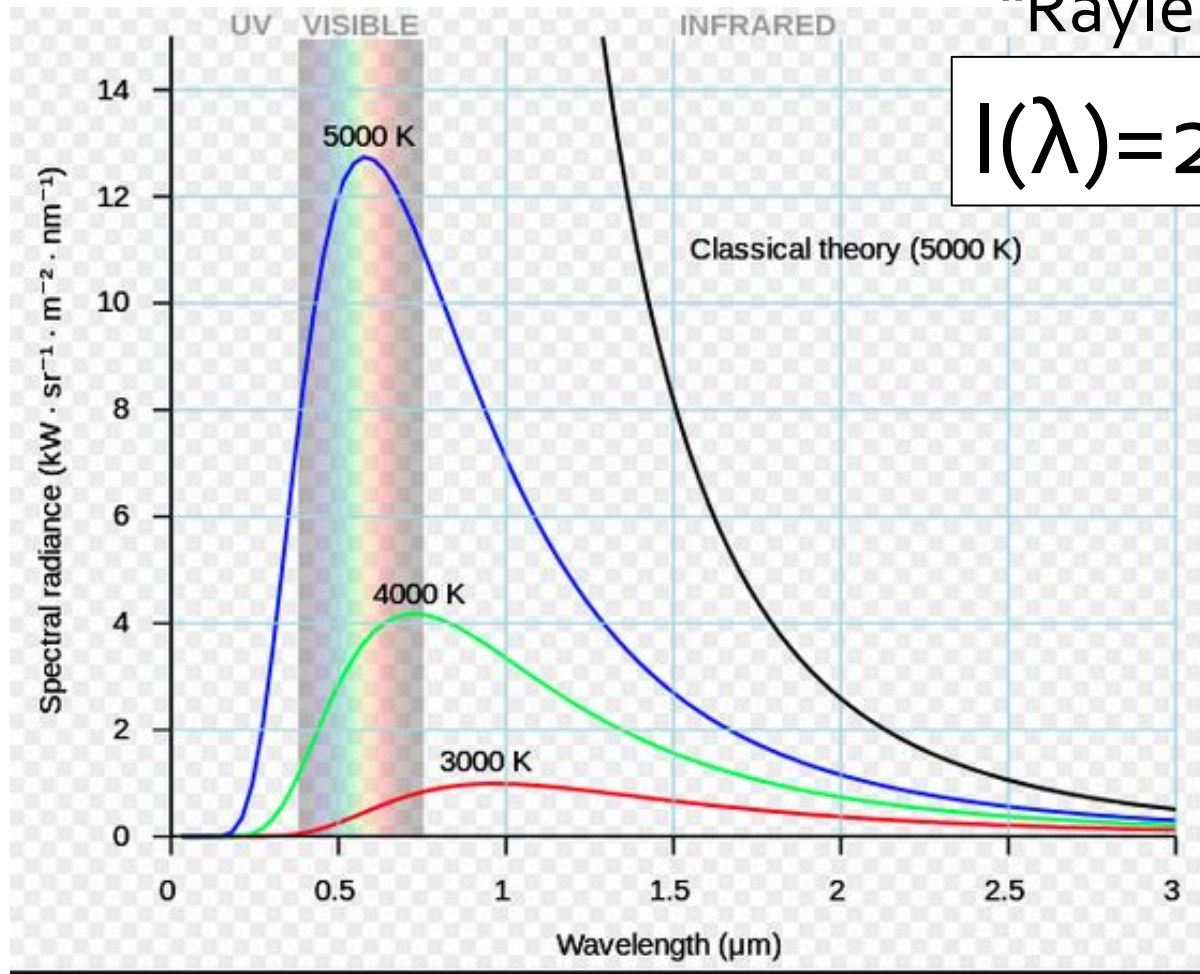


Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Ultraviolet Catastrophe



“Rayleigh-Jeans Law”

$$I(\lambda) = 2\pi c / \lambda^4 * kT$$

Maxwell-Boltzmann Statistics

With increasing energy E , it is progressively less likely that any given particle will attain that energy, so more particles will be found with lower energies. It is assumed that an unlimited number of particles can occupy any energy state.

The probability that a particle will have energy E

$$f(E) = \frac{1}{Ae^{E/kT}} = Be^{-E/kT}$$

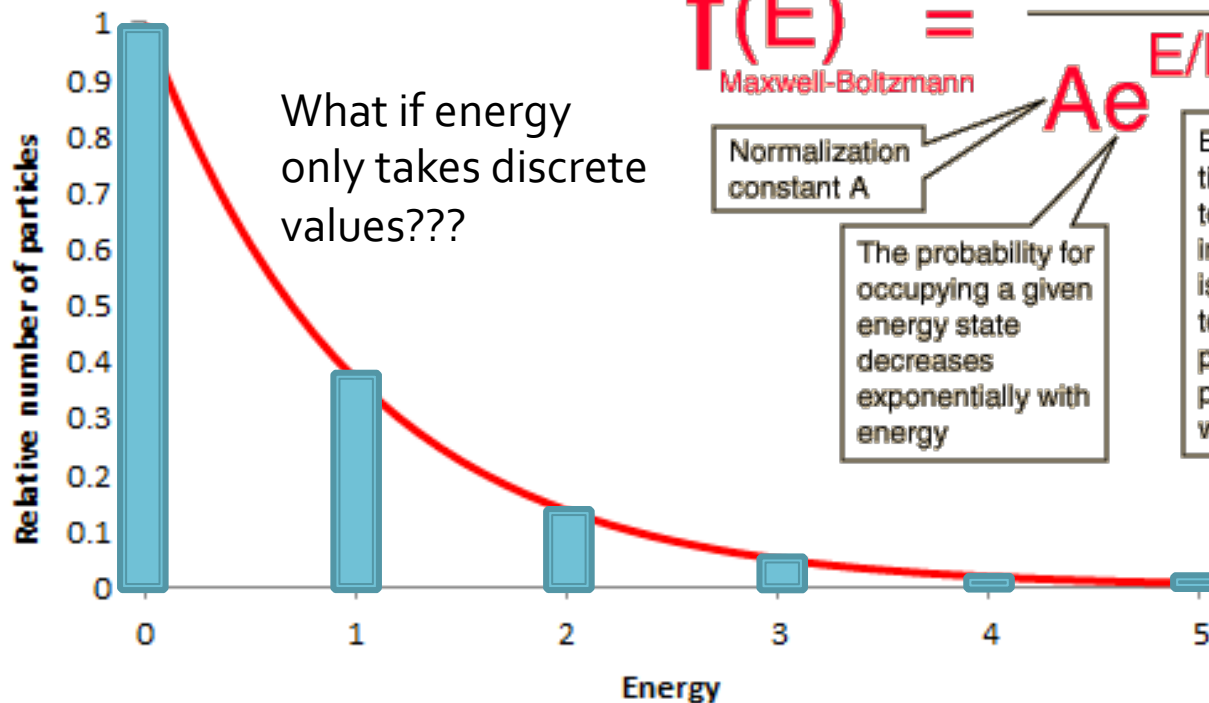
Maxwell-Boltzmann

Normalization constant A

The probability for occupying a given energy state decreases exponentially with energy

Boltzmann's constant k times the absolute temperature T . The implication of this term is that for a higher temperature, it is more probable that a given particle can be found with energy E .

What if energy only takes discrete values???



Planck Theory

- $E_{\text{oscillator}} = nh\nu = n\varepsilon$
 - In other words, energy comes only in discrete multiples of a fundamental unit $\varepsilon = h\nu$ (should look familiar!)
- $N_n = Be^{-n\varepsilon/kT} = Be^{-nh\nu/kT}$
- $\sum N_n = N$
- $\langle E \rangle = 1/N * \sum N_n E_n$

Concept Check

- The sum over all n of $Be^{-n\varepsilon/kT}$ should equal N .
What is B ?
- A. N
- B. $N*(1-\varepsilon/kT)$
- C. $N/(1-e^{-\varepsilon/kT})$
- D. $N*(1-e^{-\varepsilon/kT})$

Note: The sum of x^n
from $n=0$ to $n=\infty$ is $1/(1-x)$

Concept Check

- The sum over all n of $Be^{-n\epsilon/kT}$ should equal N .
What is B ?

A. N

B. $N*(1-\epsilon/kT)$

C. $N/(1-e^{-\epsilon/kT})$

D. $N*(1-e^{-\epsilon/kT})$

Maxwell-Boltzmann (discrete)

$$\sum_{n=0}^{N-1} x^n = 1 + x + x^2 + \dots + x^{N-1}$$

$$\sum_{n=0}^{N-1} x^{n+1} = x + x^2 + x^3 + \dots + x^N$$

$$\sum_{n=0}^{N-1} (1-x)x^n = 1 - x^N$$

$$\sum_{n=0}^{N-1} x^n = \frac{1 - x^N}{1 - x}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad (x < 1)$$

$$\Rightarrow \sum_{n=0}^{\infty} e^{-n\epsilon/kT} = \sum_{n=0}^{\infty} \left(e^{-\epsilon/kT} \right)^n$$

$$= \frac{1}{1 - e^{-\epsilon/kT}}$$

$$\Rightarrow N_n = N \left(1 - e^{-\epsilon/kT} \right) e^{-n\epsilon/kT}$$

$$\langle E \rangle = \frac{1}{N} \sum_{n=0}^{\infty} n\epsilon N_n$$

$$= \sum_{n=0}^{\infty} n\epsilon \left(1 - e^{-\epsilon/kT} \right) e^{-n\epsilon/kT}$$

$$\text{Trick } \sum_0^{\infty} x^n = \frac{1}{1-x}$$

$$\sum_0^{\infty} (ax)^n = \frac{1}{1-ax}$$

$$\begin{aligned} \frac{d}{d\alpha} \sum_0^{\infty} (ax)^n &= \sum_0^{\infty} \frac{d}{d\alpha} (ax)^n \\ &= \sum_0^{\infty} an (ax)^{n-1} \\ &= \frac{1}{ax} \sum_0^{\infty} an (ax)^n \end{aligned}$$

$$= \frac{d}{d\alpha} \left(\frac{1}{1-ax} \right) = \frac{a}{(1-ax)^2}$$

$$\Rightarrow \sum_0^{\infty} n (ax)^{n-1} = \frac{x}{(1-ax)^2}$$

$$\Rightarrow \langle E \rangle = \epsilon (1 - e^{-\epsilon/kT}) \cdot \frac{e^{-\epsilon/kT}}{(1 - e^{-\epsilon/kT})^2}$$

$$= \frac{\epsilon e^{-\epsilon/kT}}{1 - e^{-\epsilon/kT}}$$

$$= \boxed{\frac{\epsilon}{e^{\epsilon/kT} - 1}}$$

Planck Blackbody Formula

$$\langle E \rangle = kT$$

$$\rightarrow \langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}$$

$$= \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}$$

$$\Rightarrow I(\lambda) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

— For $h\nu \ll kT$

$$e^{hc/\lambda kT} \sim 1 + \frac{hc}{\lambda kT}$$

$$\Rightarrow I(\lambda) \sim \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{hc/\lambda kT}$$

$$= \frac{2\pi c kT}{\lambda^4}$$

same as
Rayleigh-Jeans

— For $h\nu \gg kT$

$$e^{hc/\lambda kT} \rightarrow \infty$$

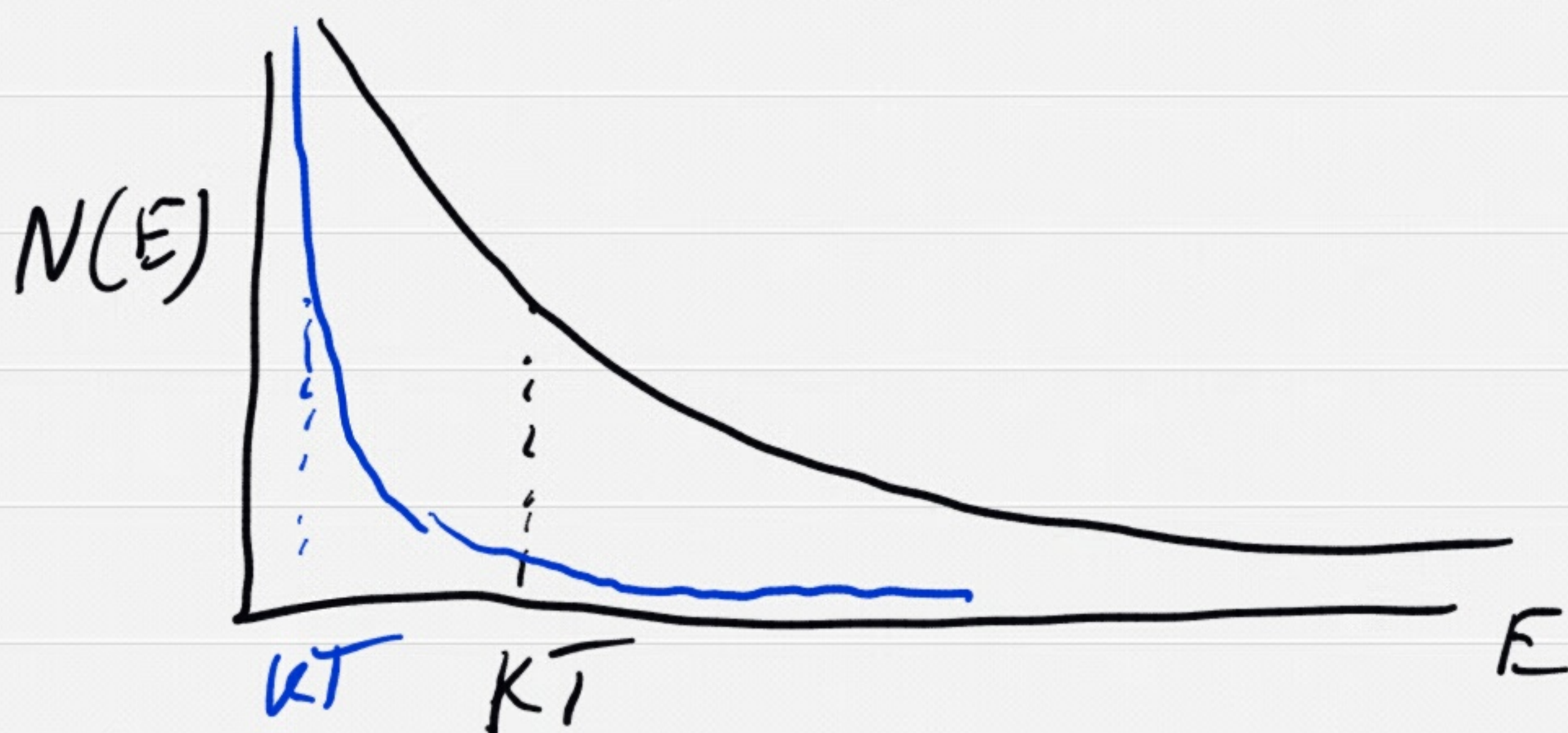
$$I(\lambda) \rightarrow 0$$

solves ultraviolet
catastrophe!

- Why does it work?
 classically

$$N(E) \propto e^{-E/kT}$$

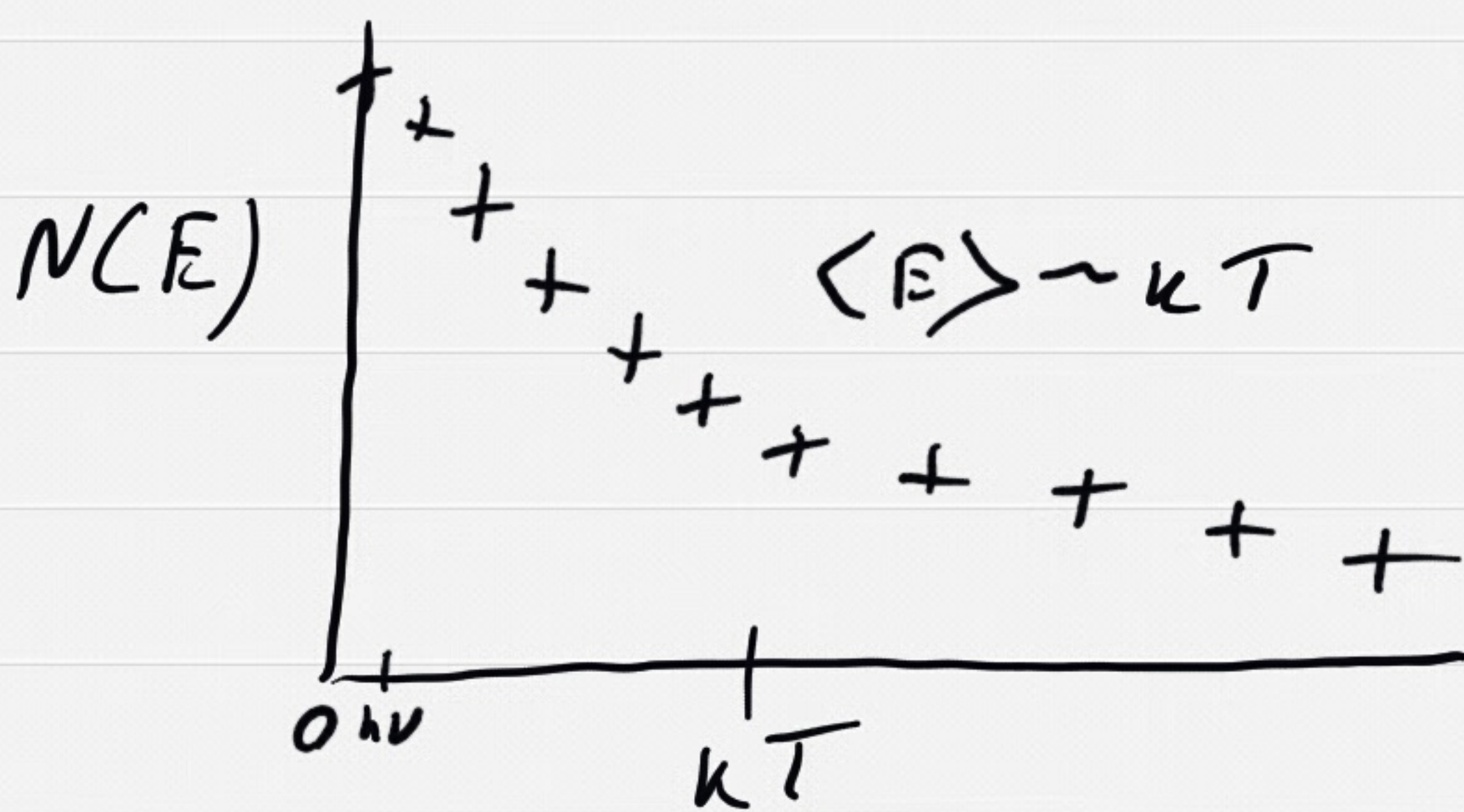
$\langle E \rangle$ always kT



quantum

$$N_n = e^{-nE/kT}$$

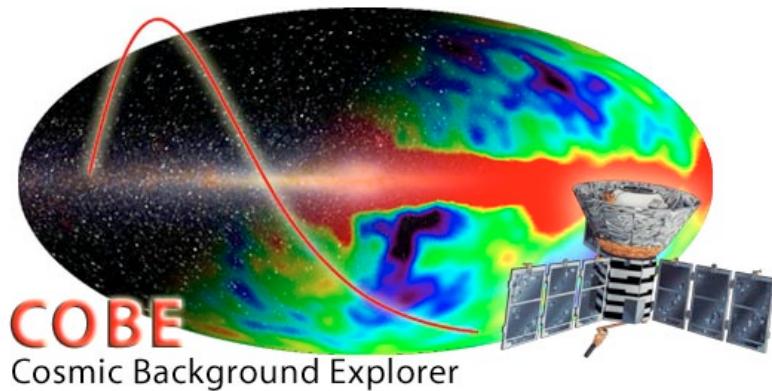
If $E \ll kT$ (low frequency)



If $E \gg kT$ (high frequency)

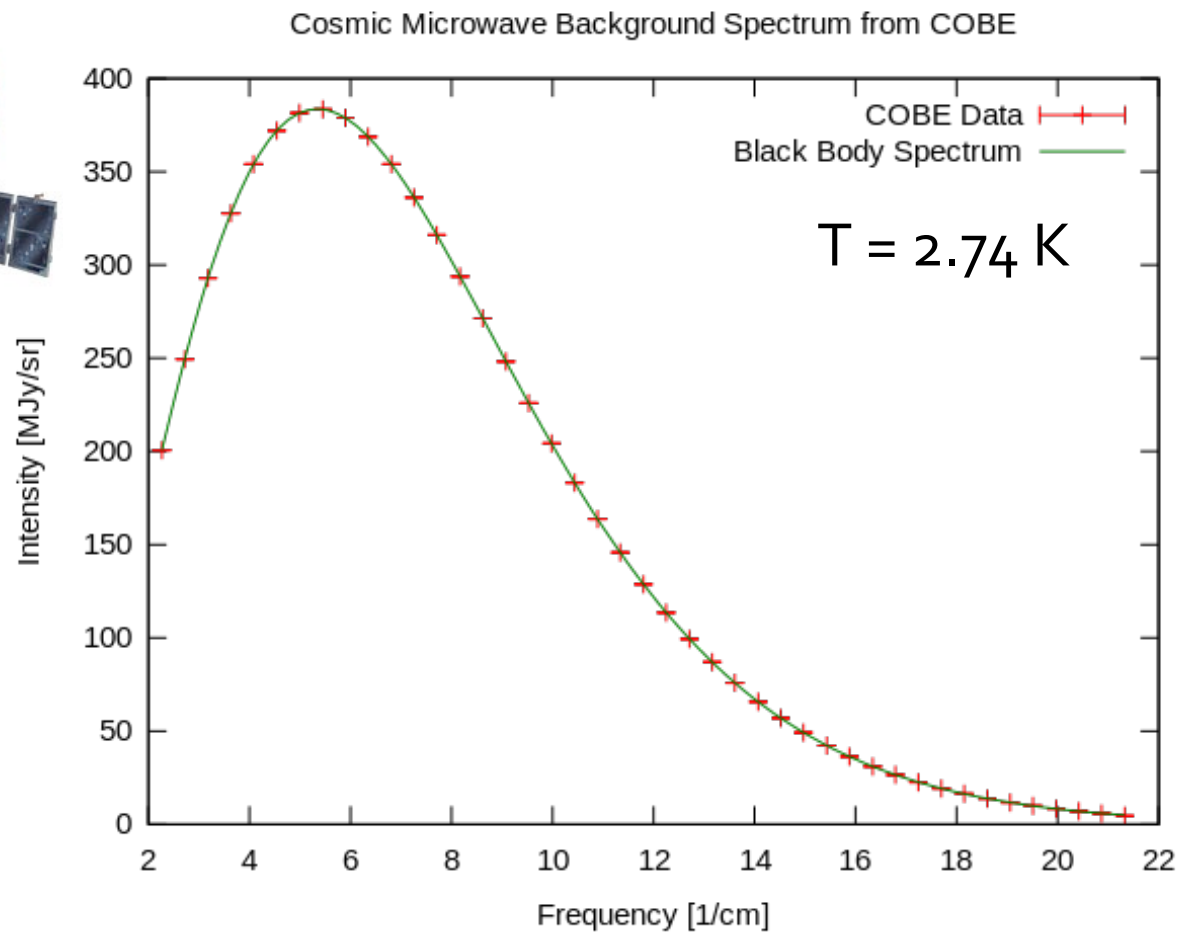


Black Body Radiation



The Planck Radiation Law

$$I_{\lambda} = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kt} - 1)}$$



Photons, Magical Photons

- Light is an electromagnetic wave
- Light is also a particle!
 - Photons have zero rest mass ($m = 0$)
 - Photons have energy $E = h\nu$
 - Photons have momentum $p = h\nu/c = h/\lambda$

Compton Scattering

