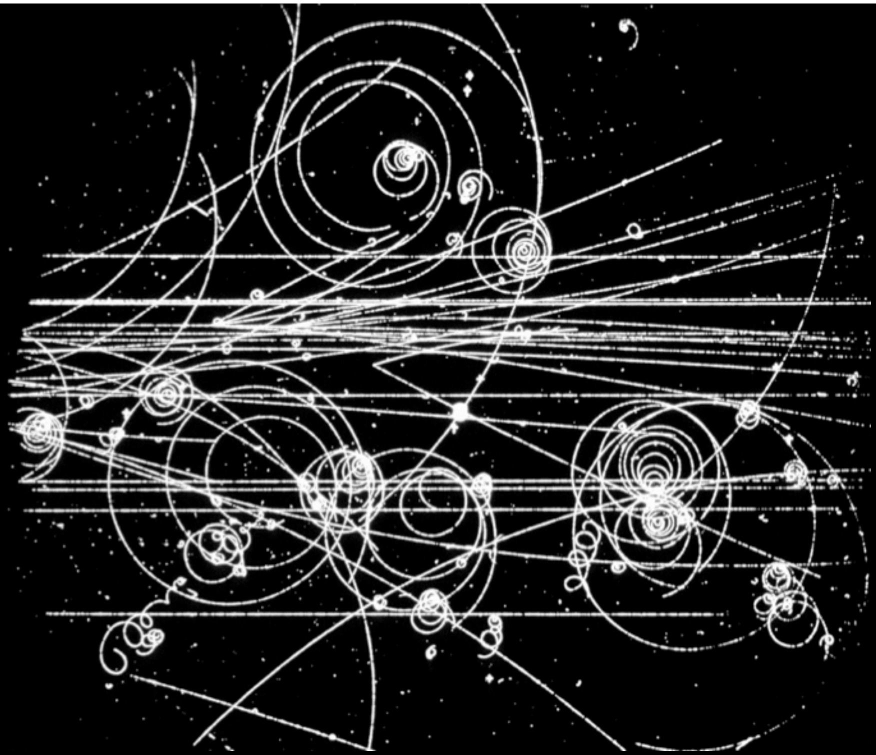
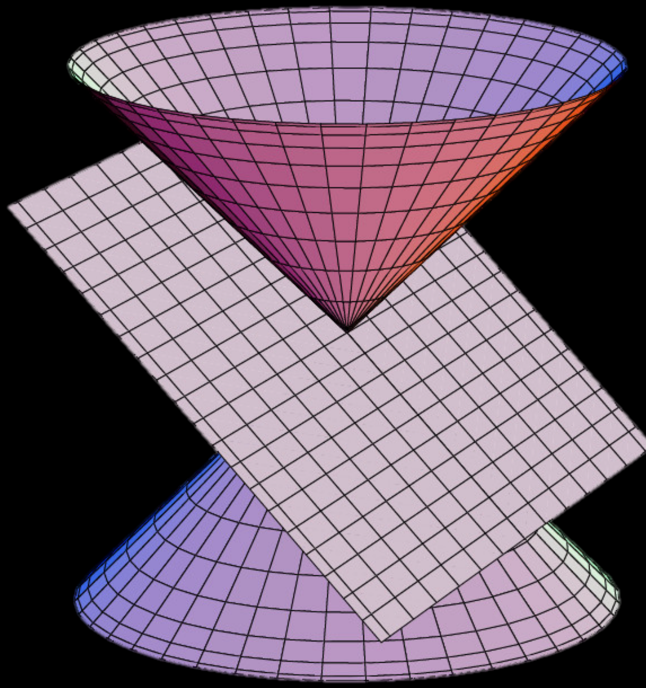




Modern Physics (Phys. IV): 2704

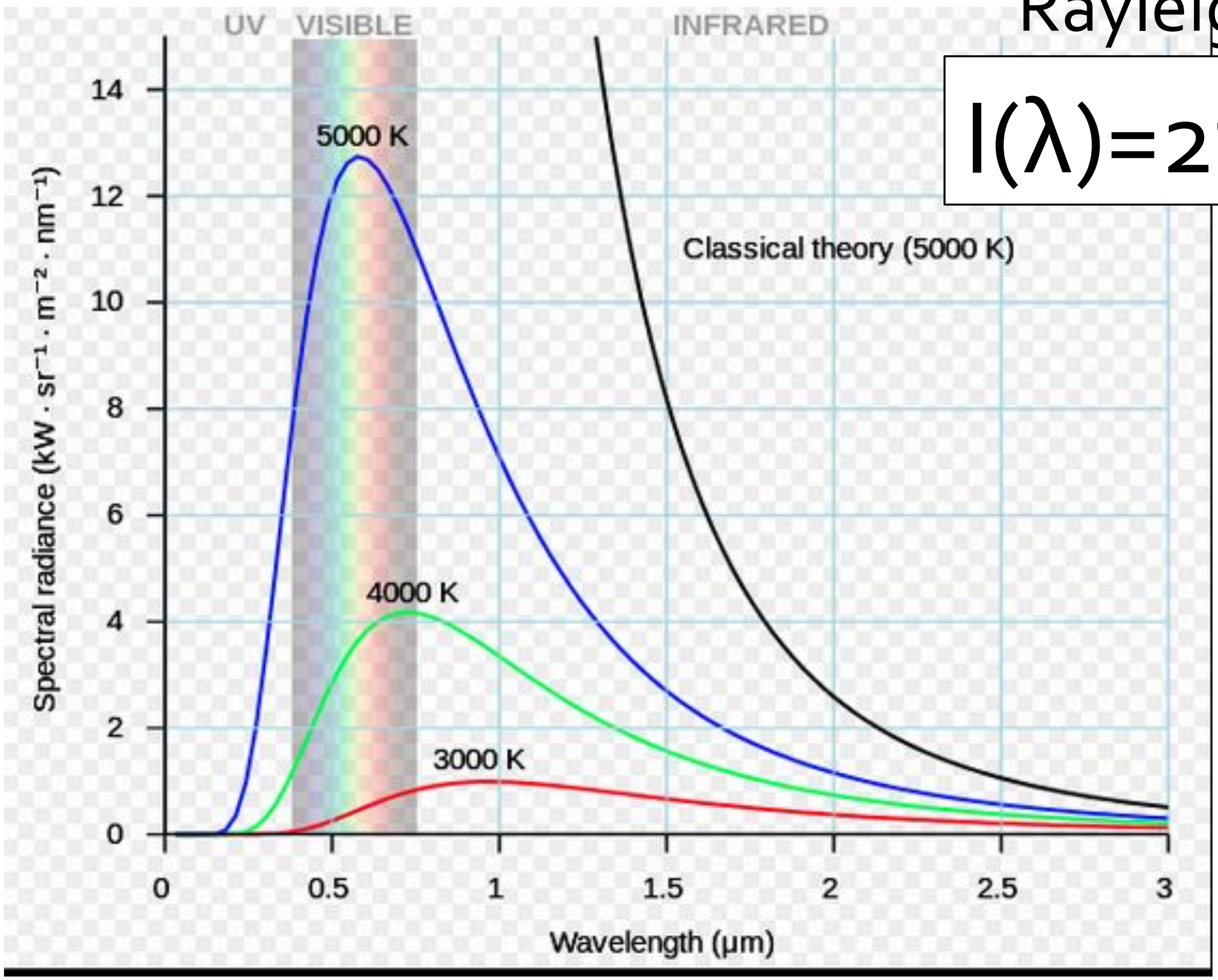
Professor Jasper Halekas

Van Allen 70
MWF 12:30-1:20 Lecture

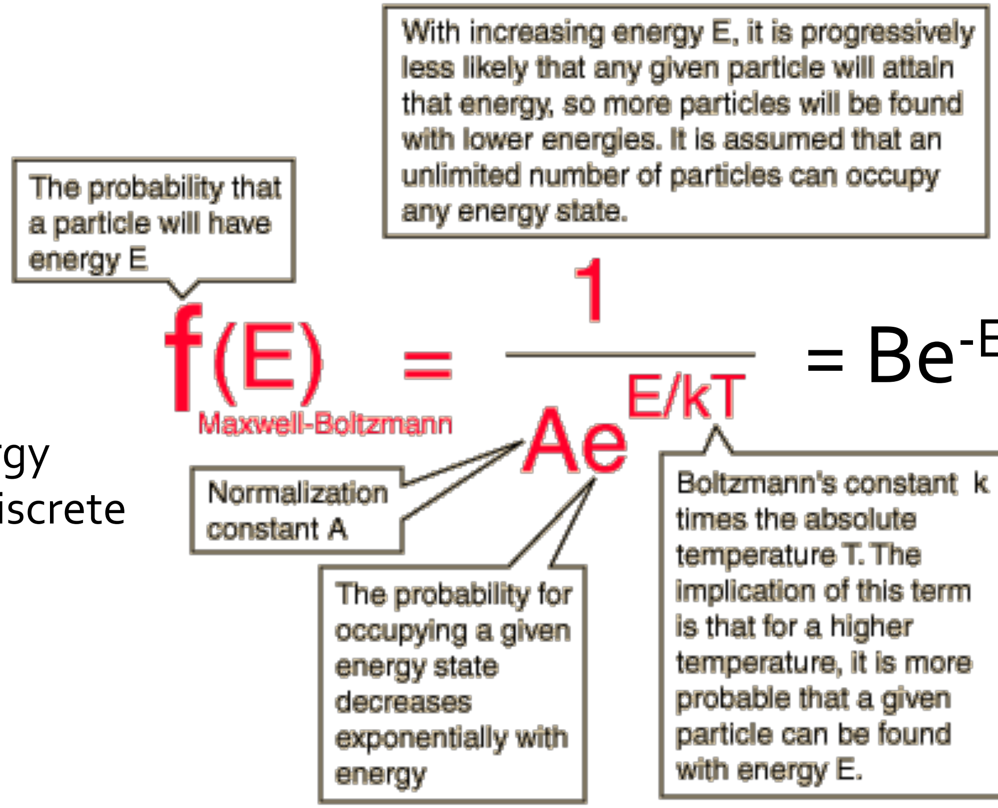
Ultraviolet Catastrophe

“Rayleigh-Jeans Law”

$$I(\lambda)=2\pi c/\lambda^4 * kT$$

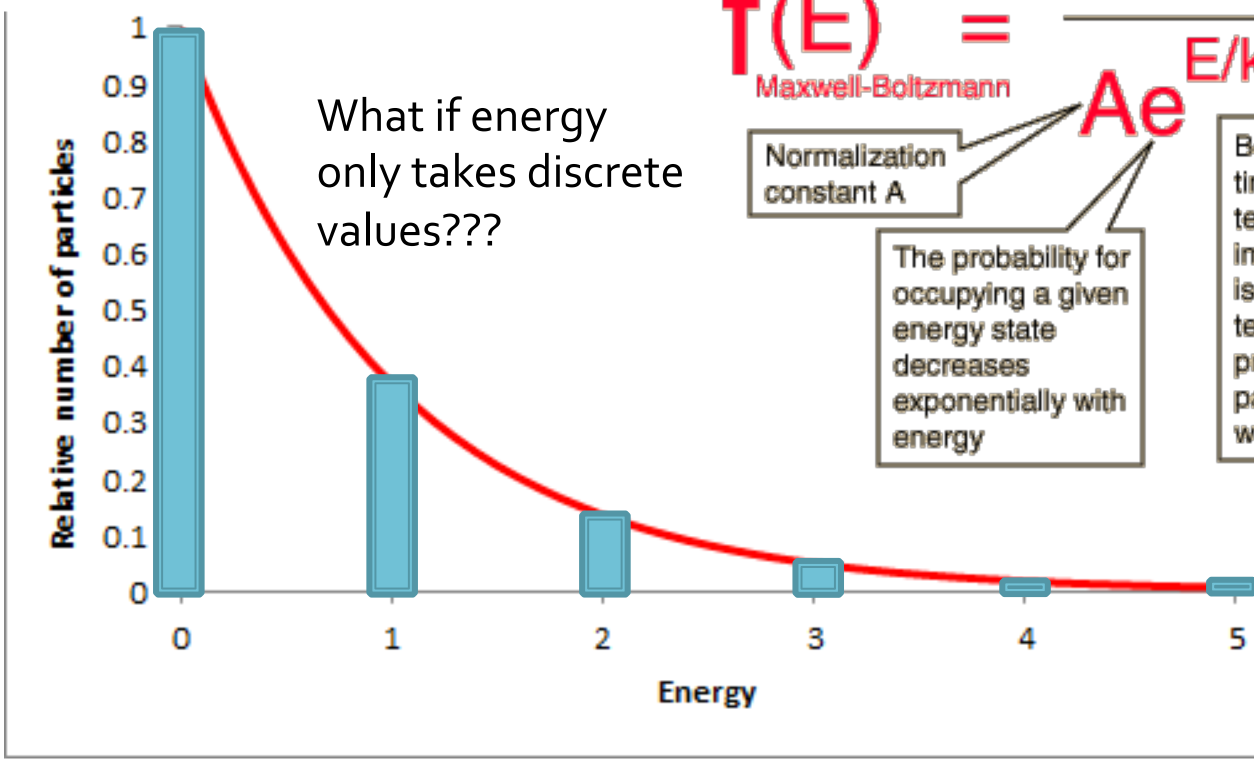


Maxwell-Boltzmann Statistics



$$f(E) = \frac{1}{Ae^{E/kT}} = Be^{-E/kT}$$

What if energy only takes discrete values???



Planck Theory

- $E_{oscillator} = nh\nu = n\varepsilon$
  - In other words, energy comes only in discrete multiples of a fundamental unit $\varepsilon = h\nu$ (should look familiar!)

- $N_n = Be^{-n\varepsilon/kT} = Be^{-nh\nu/kT}$

- $\Sigma N_n = N$

- $\langle E \rangle = 1/N * \Sigma N_n E_n$

Concept Check

- The sum over all n of $Be^{-n\varepsilon/kT}$ should equal N. What is B?

A. N
B. $N*(1-\varepsilon/kT)$
C. $N/(1-e^{-\varepsilon/kT})$
D. $N*(1-e^{-\varepsilon/kT})$

Note: The sum of x^n
from $n = 0$ to $n = \infty$ is $1/(1-x)$

Concept Check

- The sum over all n of $Be^{-n\varepsilon/kT}$ should equal N. What is B?

A. N
B. $N*(1-\varepsilon/kT)$
C. $N/(1-e^{-\varepsilon/kT})$
D. $N*(1-e^{-\varepsilon/kT})$

Maxwell-Boltzmann (discrete)

$$\sum_{n=0}^{N-1} x^n = 1 + x + x^2 + \cdots + x^{N-1}$$

$$\sum_{n=0}^{N-1} x^{n+1} = x + x^2 + x^3 + \cdots x^N$$

$$\sum_{n=0}^{N-1} (1-x)\, x^n = 1 - x^N$$

$$\sum_{n=0}^{N-1} x^n = \frac{1-x^N}{1-x}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad (x < 0)$$

$$\Rightarrow \sum_{n=0}^{\infty} e^{-n\varepsilon/kT} = \sum_{n=0}^{\infty} \left(e^{-\varepsilon/kT}\right)^n$$

$$= \frac{1}{1-e^{-\varepsilon/kT}}$$

$$\Rightarrow N_n = N\left(1-e^{-\varepsilon/kT}\right) e^{-n\varepsilon/kT}$$

$$\langle E \rangle = \frac{1}{N} \sum_{n=0}^{\infty} n\varepsilon\, N_n$$

$$= \sum_{n=0}^{\infty} n\varepsilon \left(1-e^{-\varepsilon/kT}\right) e^{-n\varepsilon/kT}$$

Trick $\sum_0^\infty x^n = \frac{1}{1-x}$

$$\sum_0^\infty (\alpha x)^n = \frac{1}{1-\alpha x}$$

$$d/d\alpha \sum_0^\infty (\alpha x)^n = \sum_0^\infty d/d\alpha (\alpha x)^n$$
$$= \sum_0^\infty \alpha n (\alpha x)^{n-1}$$
$$= \frac{1}{\alpha x} \sum_0^\infty \alpha n (\alpha x)^n$$

$$= d/d\alpha \left(\frac{1}{1-\alpha x} \right) = \frac{\alpha}{(1-\alpha x)^2}$$

$$\Rightarrow \sum_0^\infty n (\alpha x)^{n-1} = \frac{x}{(1-\alpha x)^2}$$

$$\Rightarrow \langle E \rangle = \varepsilon (1-e^{-\varepsilon/kT}) \cdot \frac{e^{-\varepsilon/kT}}{(1-e^{-\varepsilon/kT})^2}$$

$$= \frac{\varepsilon e^{-\varepsilon/kT}}{1-e^{-\varepsilon/kT}}$$

$$= \boxed{\varepsilon / (e^{\varepsilon/kT} - 1)}$$

Planck Blackbody Formula

$\langle E \rangle = kT$

$\longrightarrow \langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}$

$= \frac{hc/\lambda}{e^{hc/\lambda kT} - 1}$

$\Rightarrow \boxed{I(\lambda) = \frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}}$

- For $h\nu \ll kT$

$e^{hc/\lambda kT} \sim 1 + hc/\lambda kT$

$\Rightarrow I(\lambda) \sim \frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{hc/\lambda kT}$

$= \frac{2\pi c kT}{\lambda^4}$ same as Rayleigh-Jeans

- For $h\nu \gg kT$

$e^{hc/\lambda kT} \to \infty$

$I(\lambda) \to 0$

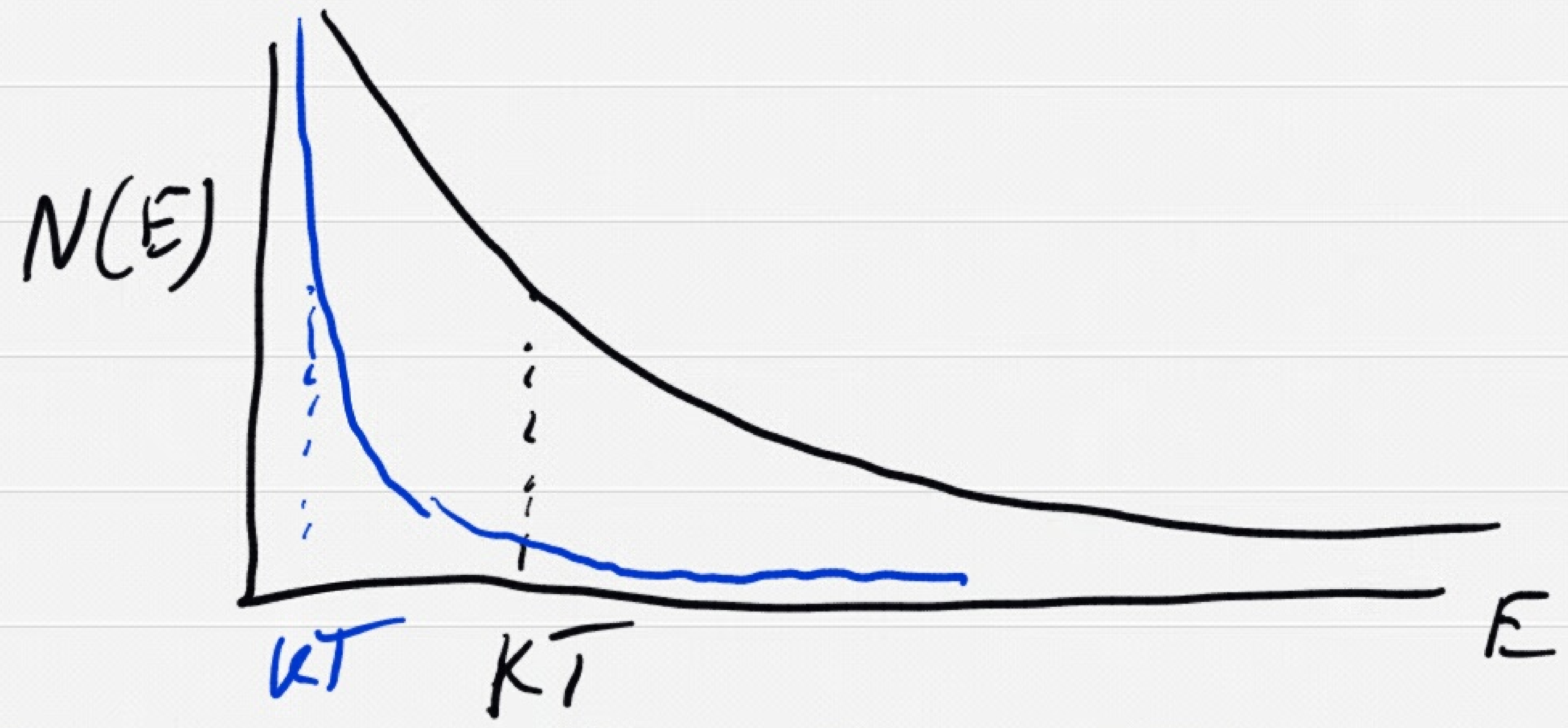
solves ultraviolet catastrophe!

– Why does it work?

classically

$$N(E) \propto e^{-E/kT}$$

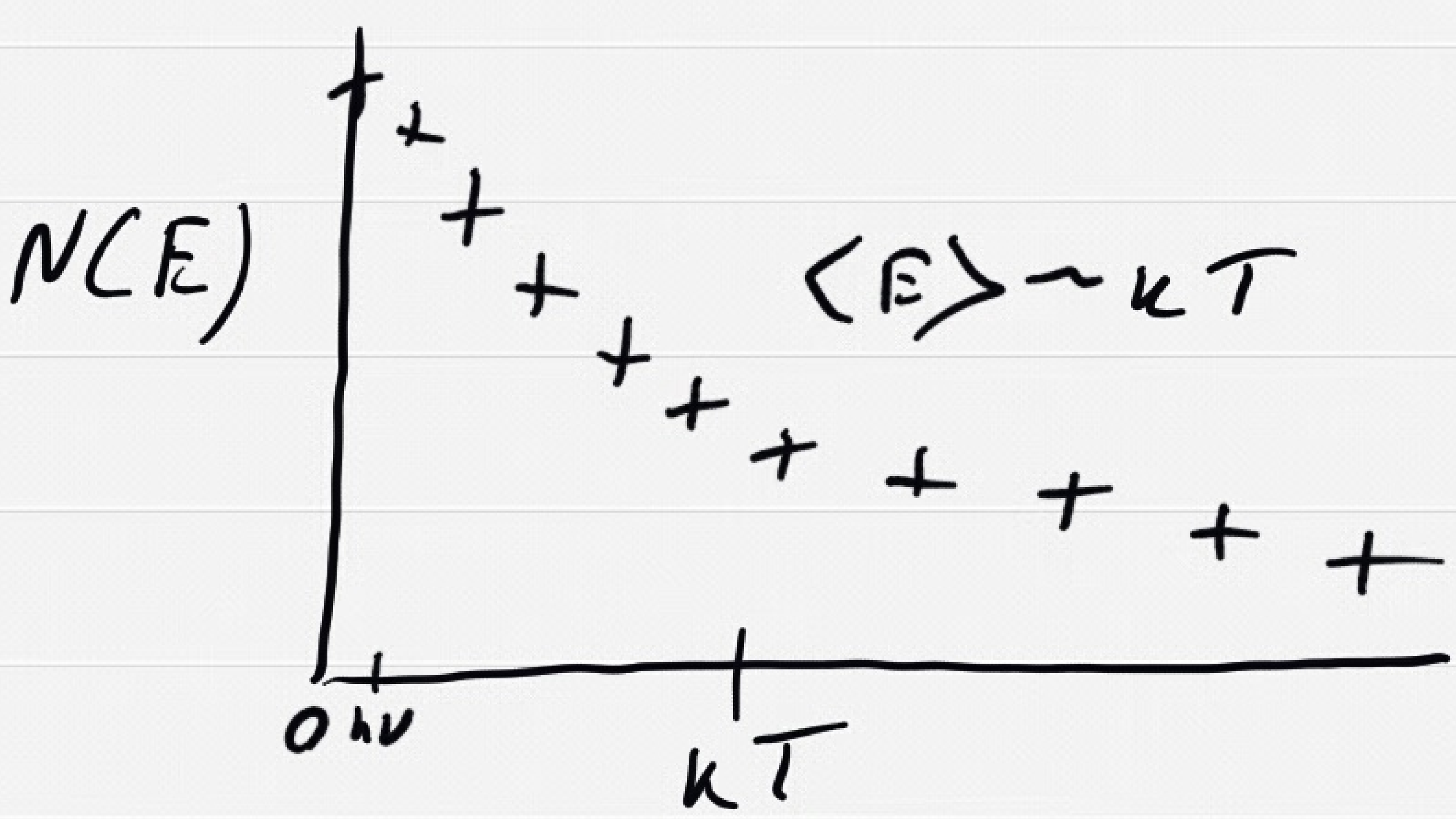
$\langle E \rangle$ always kT



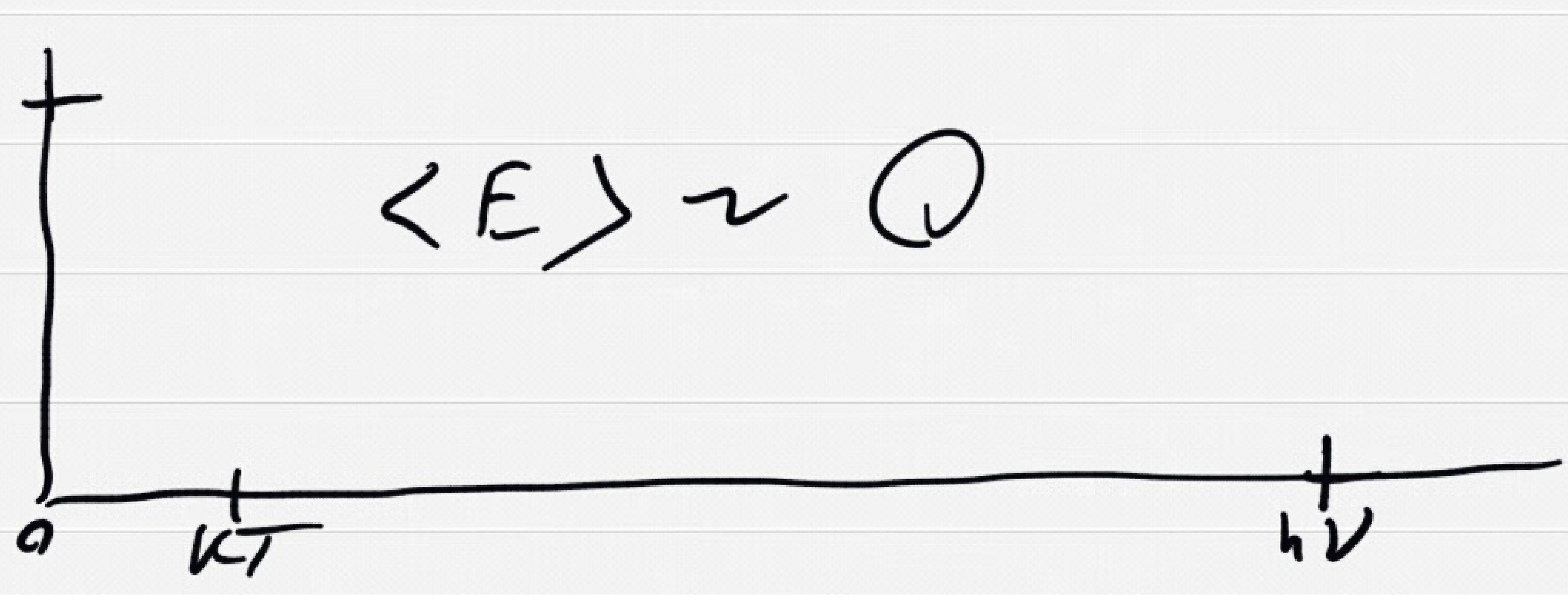
quantum

$$N_n = e^{-n\varepsilon/kT}$$

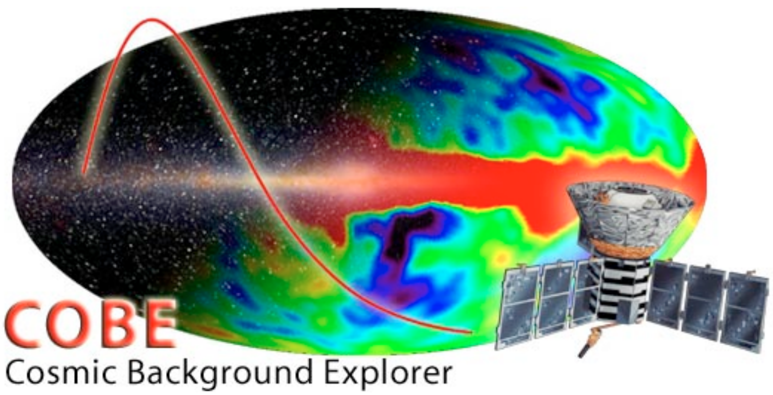
If $\varepsilon \ll kT$ (low frequency)



If $\varepsilon \gg kT$ (high frequency)

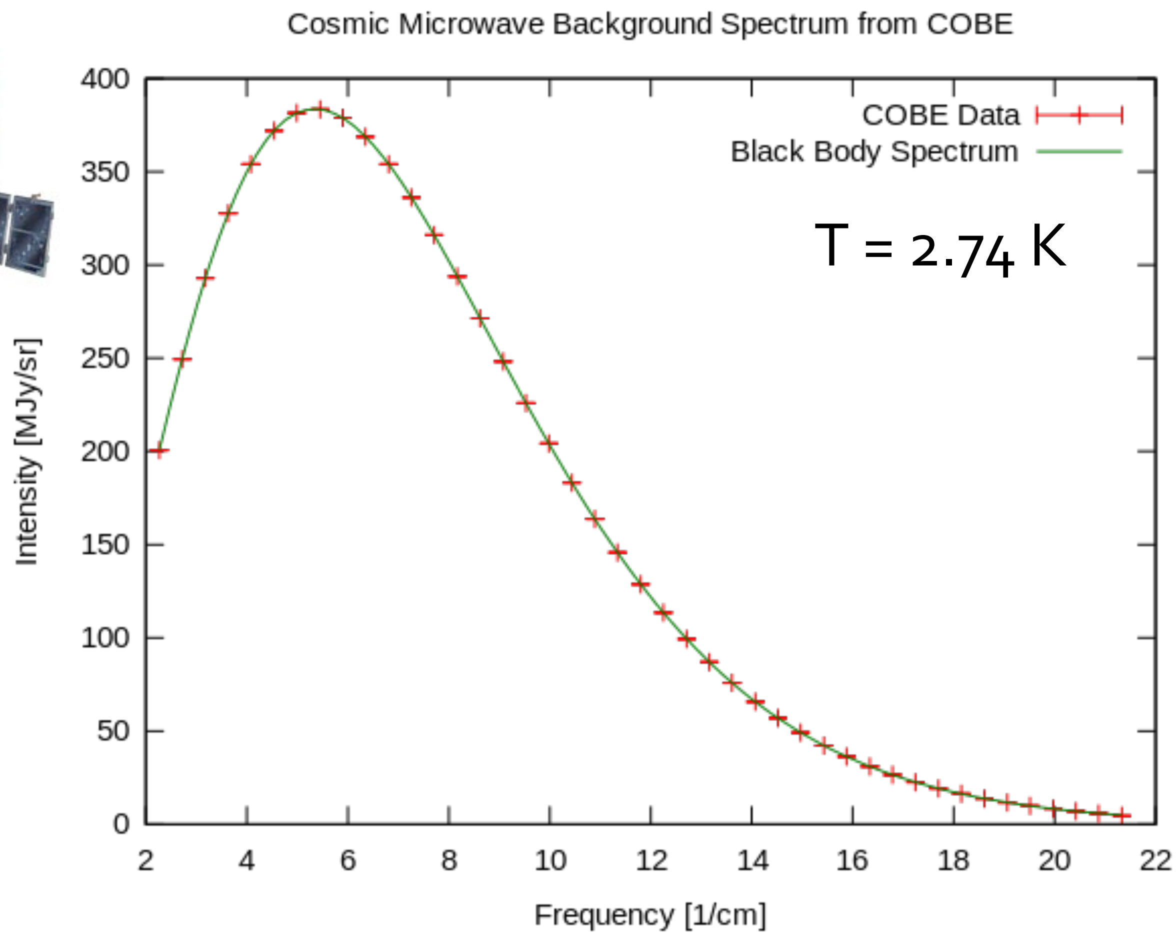


Black Body Radiation



The Planck Radiation Law

$$I_\lambda = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kt} - 1)}$$



Photons, Magical Photons

- Light is an electromagnetic wave
- Light is also a particle!
  - Photons have zero rest mass ($m = 0$)
  - Photons have energy $E = h\nu$
  - Photons have momentum $p = h\nu/c = h/\lambda$

Compton Scattering