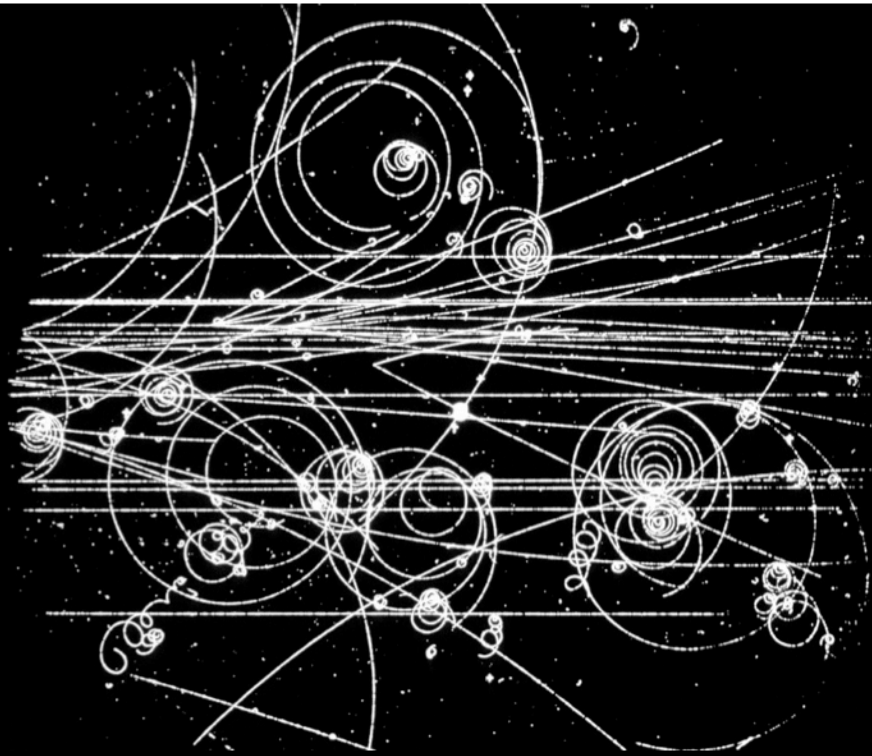
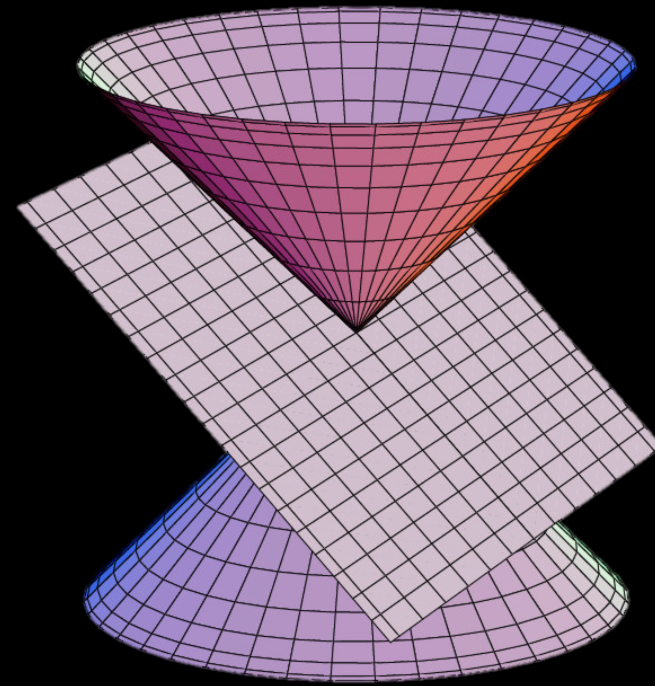
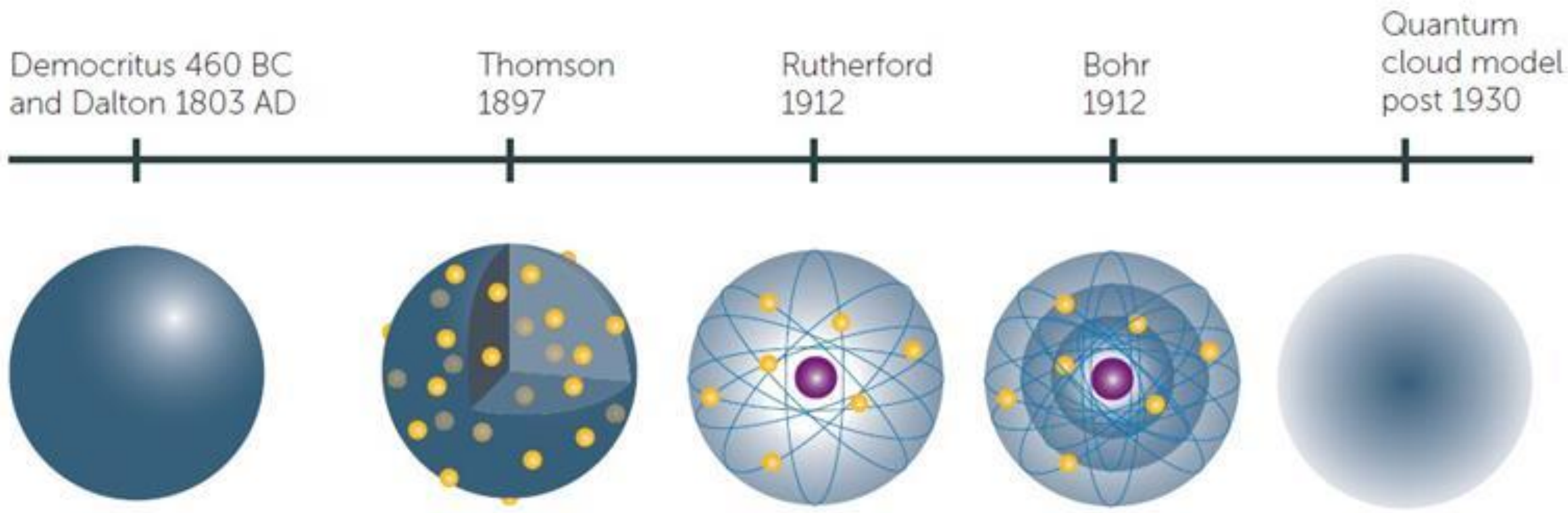




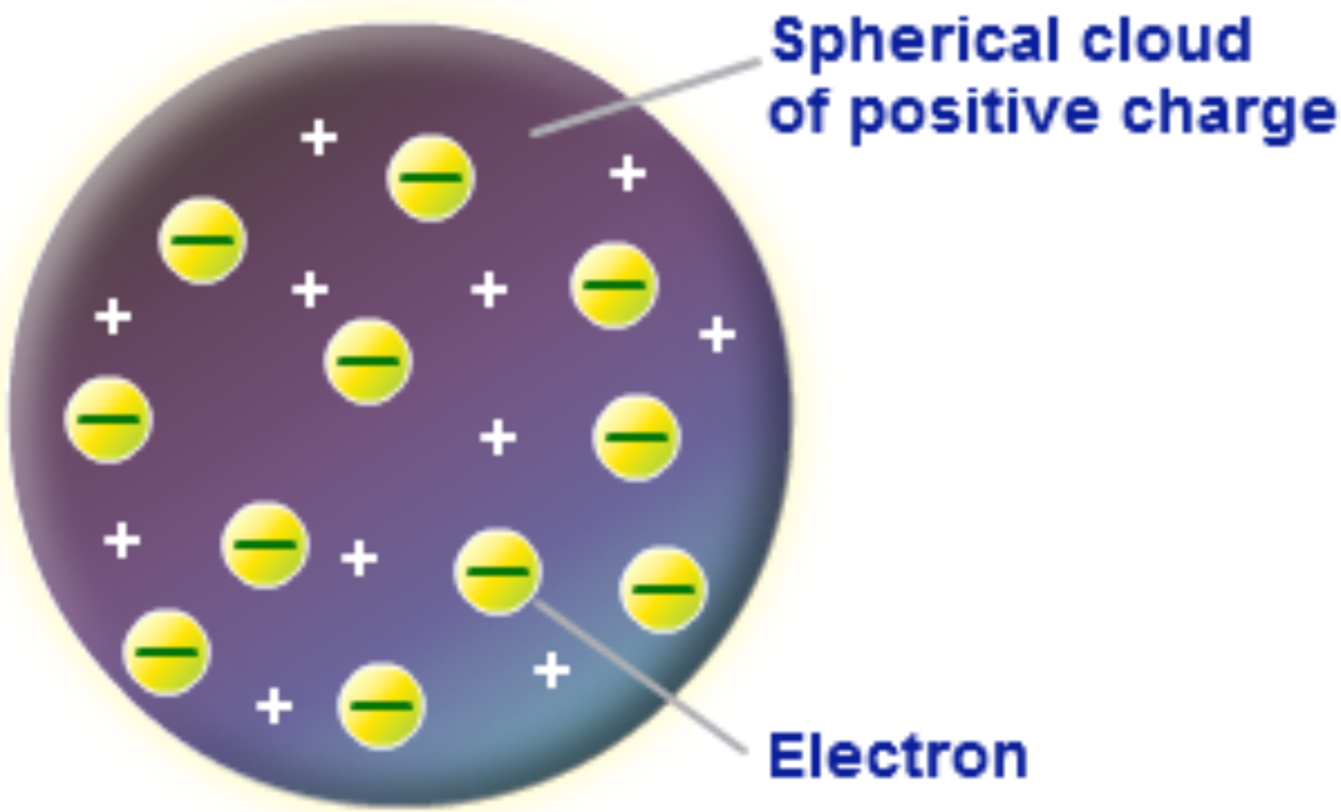
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Models of the Atom

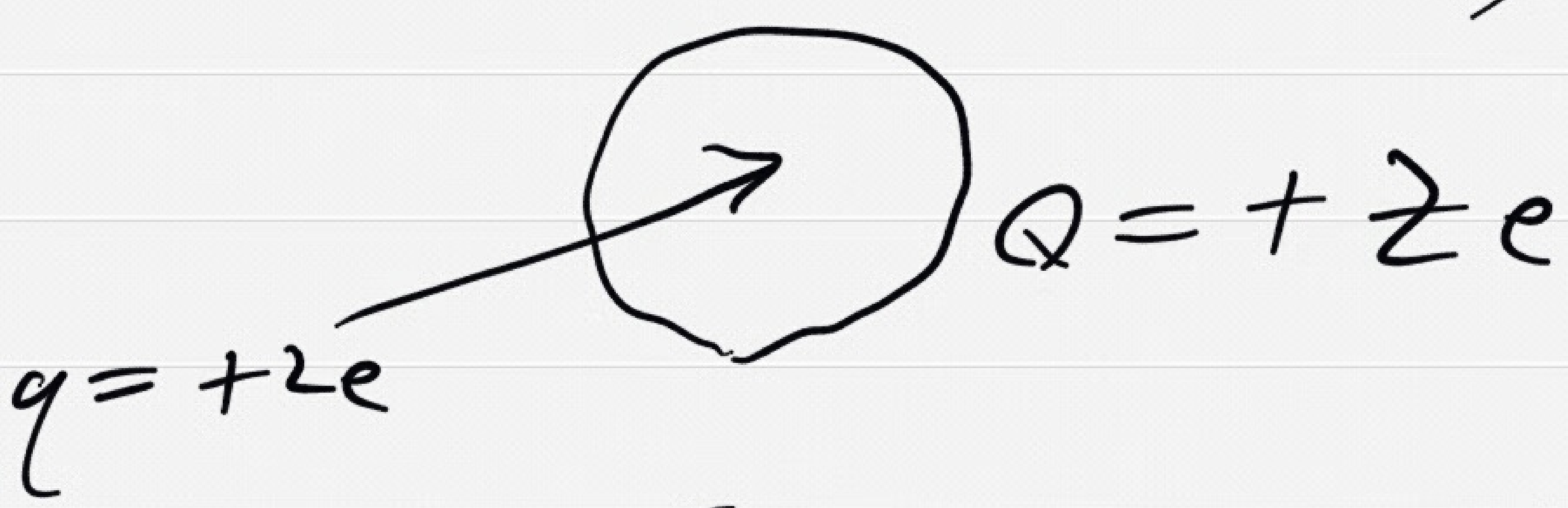


Thomson Atom



Thomson's Plum pudding model

Thompson Scattering



E in uniform sphere

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}$$

$$F = qE = \frac{2Ze^2 r}{4\pi\varepsilon_0 R^3}$$

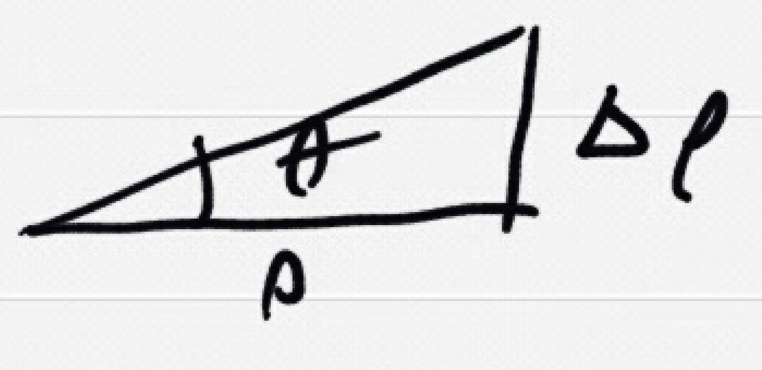
$$\langle F \rangle \sim \frac{2Ze^2 R/2}{4\pi\varepsilon_0 R^3} = \frac{Ze^2}{4\pi\varepsilon_0 R^2}$$

Impulse – Momentum

$$\Delta p = \langle F \rangle \Delta t$$

$$\Delta t \sim R/v$$

$$\Delta p \sim \frac{Ze^2}{4\pi\varepsilon_0 R v}$$



$$\theta \sim \Delta p / p$$

$$= \frac{Ze^2}{4\pi\varepsilon_0 R v} \cdot \frac{1}{m_\alpha v}$$

$$= \frac{Ze^2}{8\pi\varepsilon_0 R \cdot K_\alpha}$$

R of gold atom ~ 0.18 nm
Z of gold atom = 79
Kinetic energy of α ~ 5 MeV

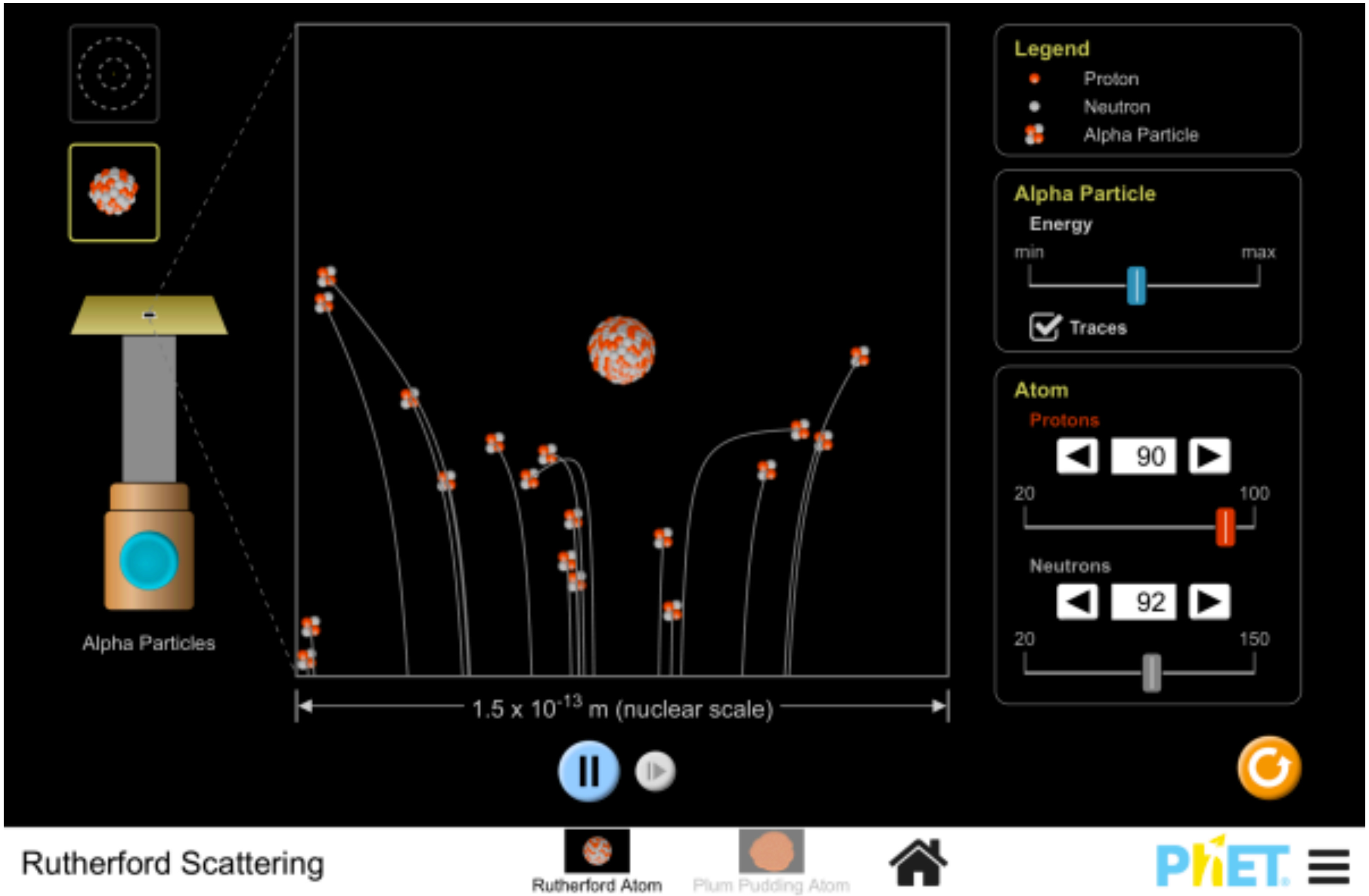
$$\theta \sim \frac{79 \cdot (1.6\times10^{-19})^2}{8\cdot\pi\cdot 8.85\times10^{-12}\cdot 2\times10^{-10}\cdot 5\times10^{6}\cdot 1.6\times10^{-19}}$$

$\sim 7\times10^{-5}$ radians

– Each scatter is very small!!

Thomson Scattering Simulation

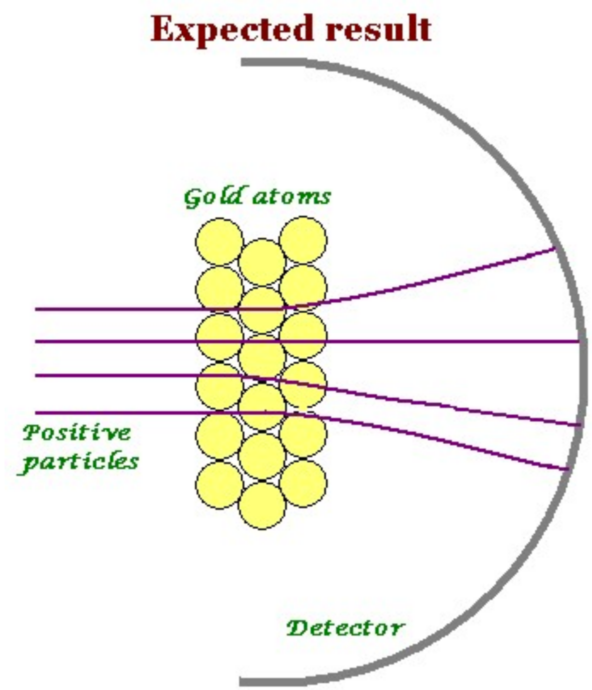
https://phet.colorado.edu/en/simulation/rutherford-scattering



Concept Check

- For each interaction, alpha particles scatter by ~10^{-4} radians. If each alpha interacts with 10^4 gold atoms while passing through a foil, what is the chance of an alpha particle scattering by a total of one radian?

A. 1
B. $1/2^{10}$
C. $1/2^{100}$
D. $1/2^{10000}$



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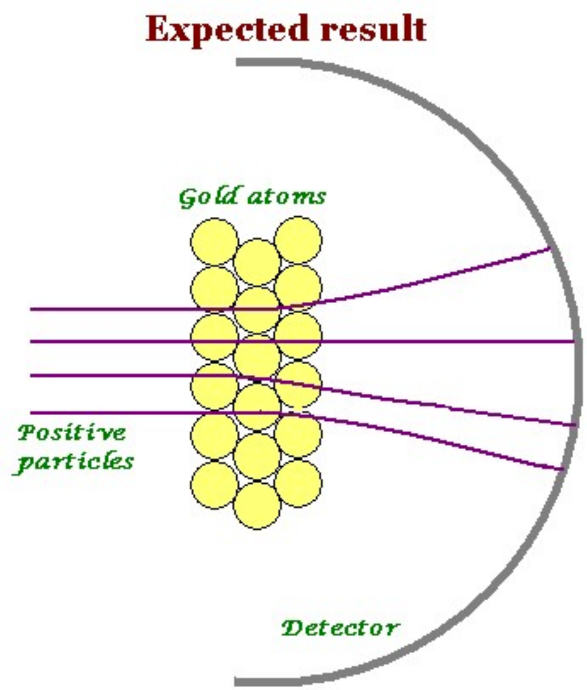

A. 1
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C. $1/2^{100}$
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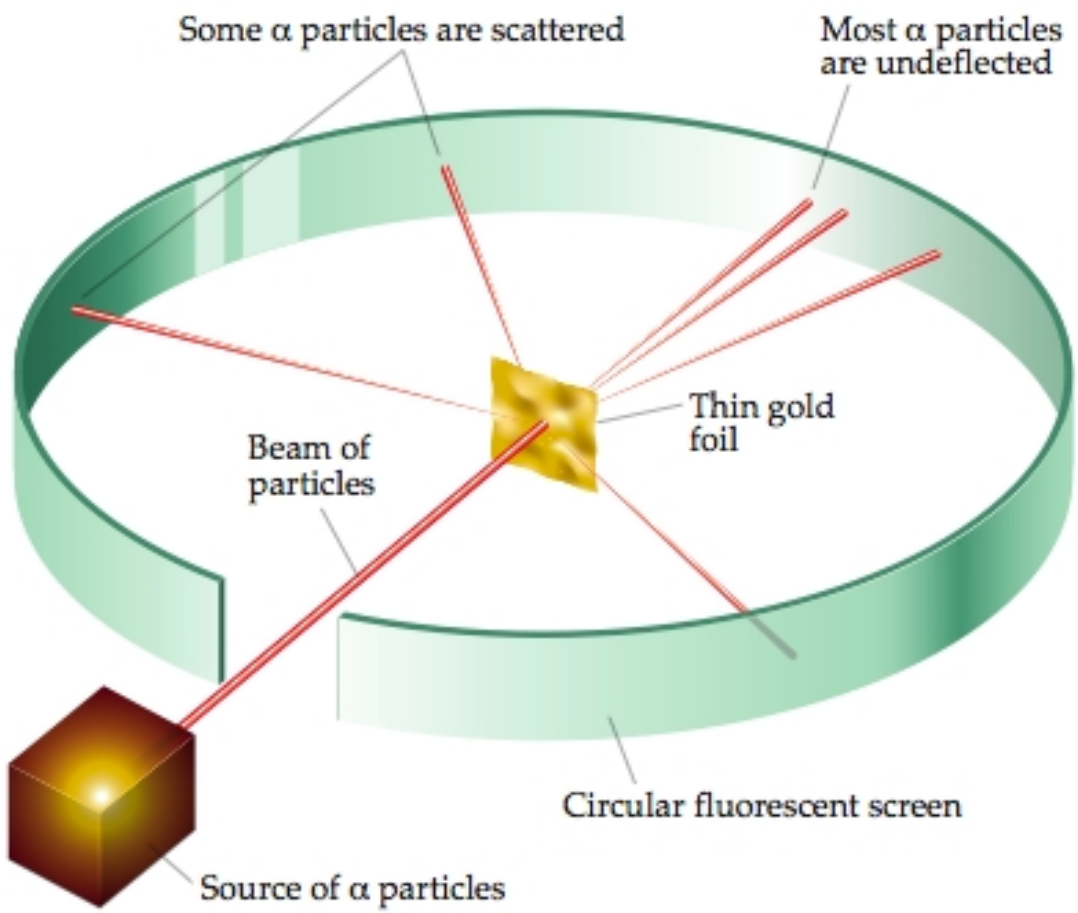
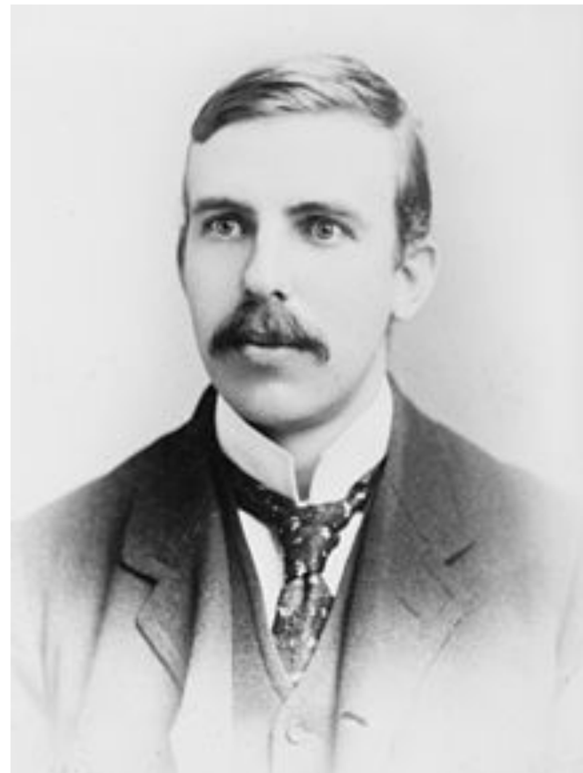
Alpha Particle Scattering Results

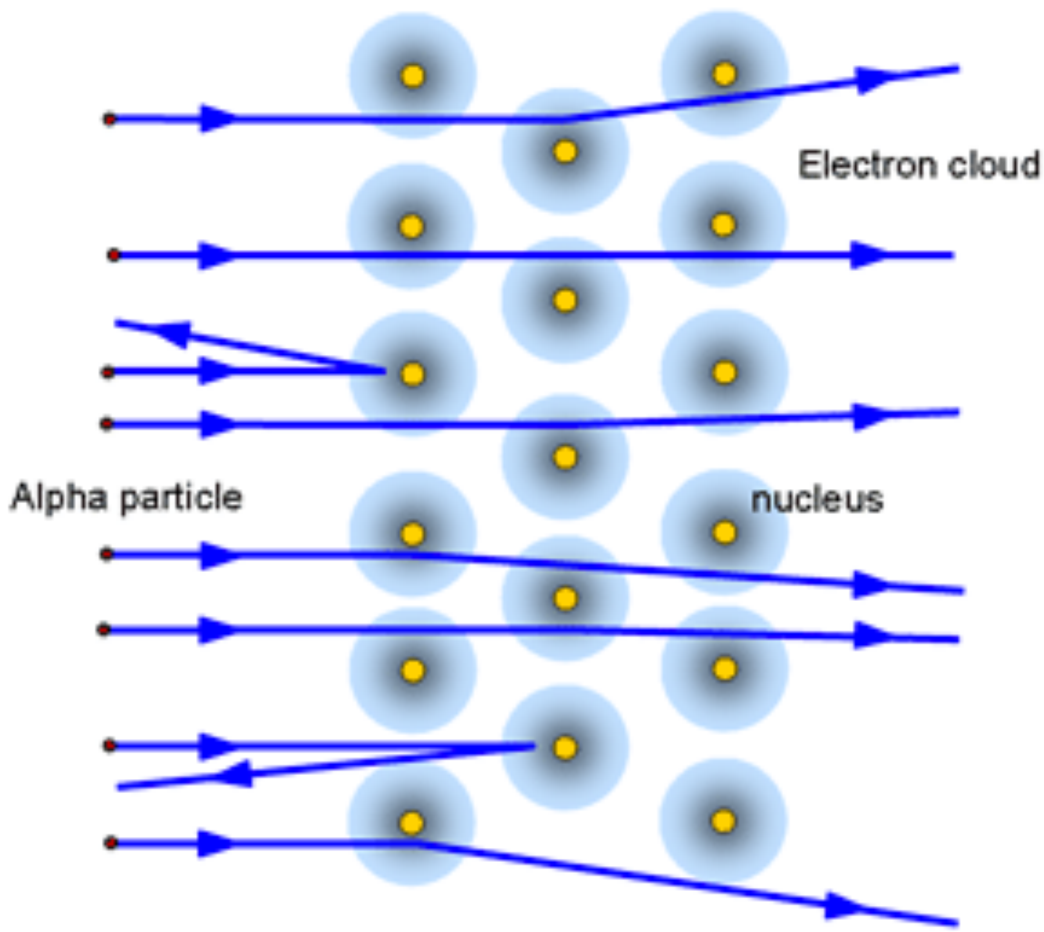


Alpha Particle Scattering

- *"It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you"* – Ernest Rutherford

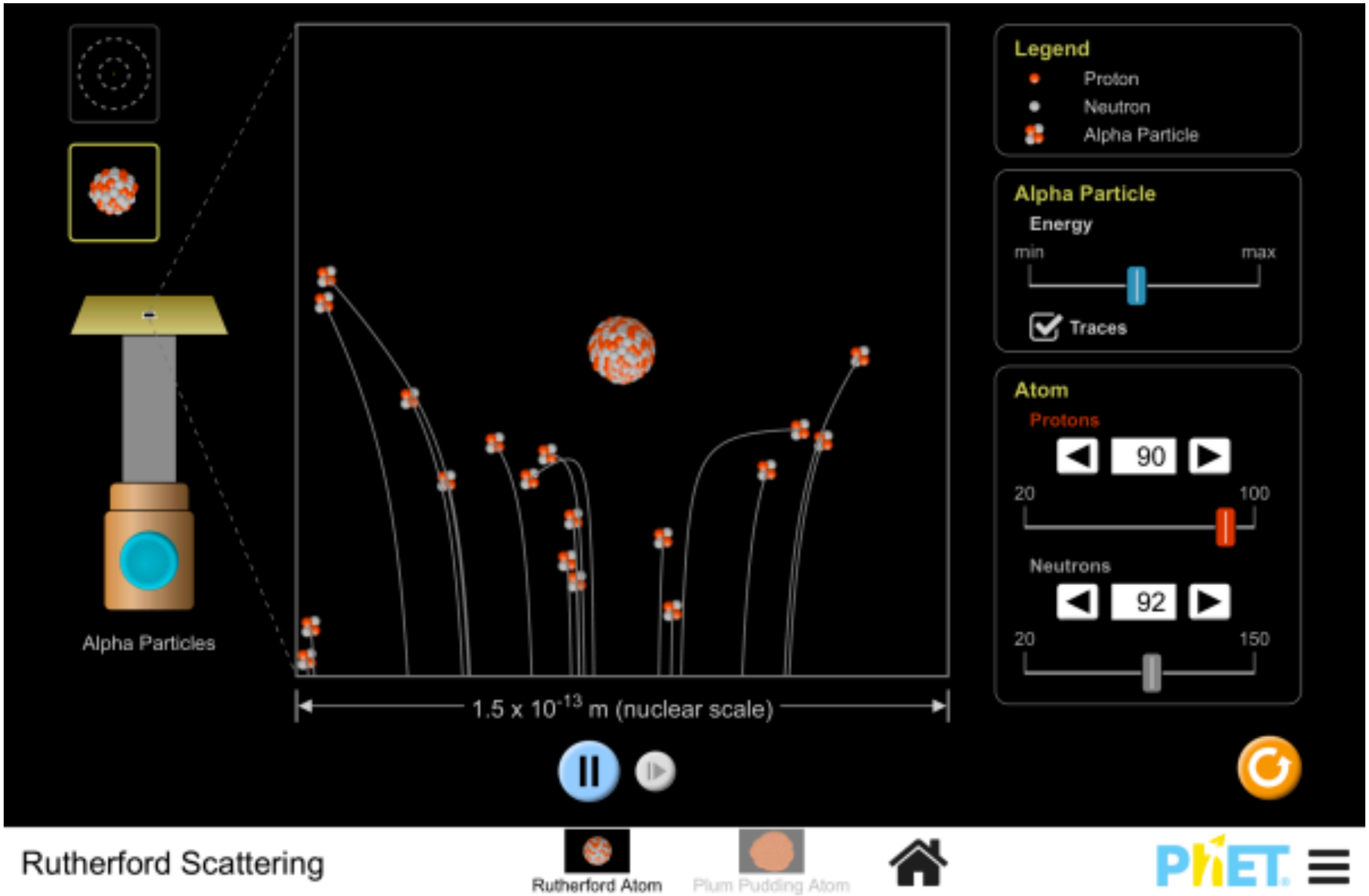


Alpha Particle Scattering Interpretation

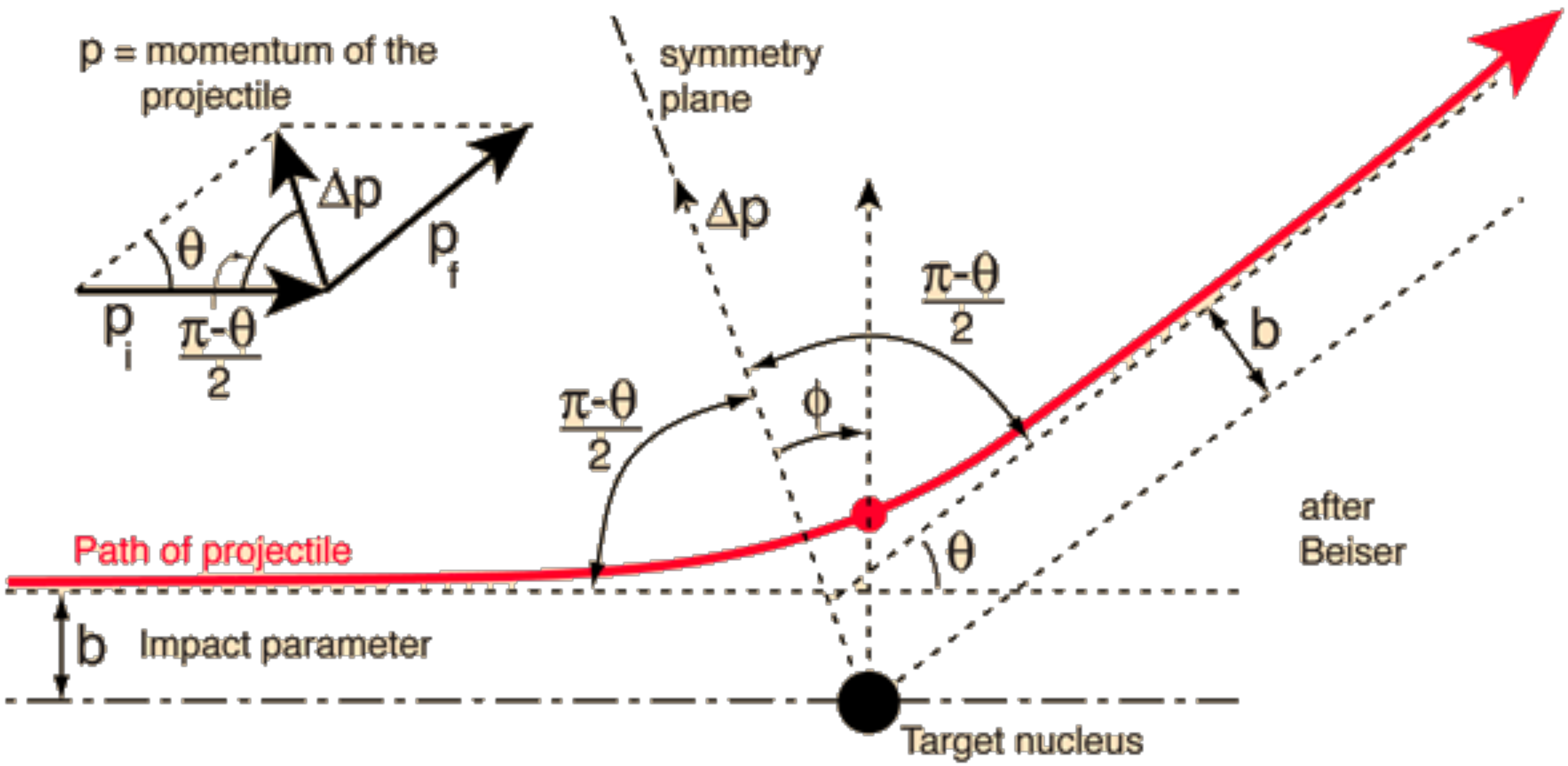


Rutherford Scattering Simulation

https://phet.colorado.edu/en/simulation/rutherford-scattering



Rutherford Scattering



Rutherford scattering

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} = \alpha/r$$

$$E = \tfrac{1}{2}mv_r^2 + \tfrac{1}{2}mv_\varphi^2 + U(r) = \text{const.}$$

$L = m v_\varphi r$ = angular momentum

$E(\infty) = K$ = initial kinetic energy

$$K = \tfrac{1}{2}mv_r^2 + \frac{L^2}{2mr^2} + \alpha/r$$

$$= \tfrac{1}{2}m\dot{r}^2 + L^2/2mr^2 + \alpha/r$$

$$\dot{r} = dr/dt = dr/d\varphi \; d\varphi/dt$$

$$= dr/d\varphi \cdot L/mr^2$$

$$u = 1/r \rightarrow du = -1/r^2 \, dr$$

$$\Rightarrow \dot{r} = -du/d\varphi \cdot L/m$$

$$K = \frac{L^2}{2m}\left(du/d\varphi\right)^2 + \frac{L^2}{2m}u^2 + \alpha u$$

Take $d/d\varphi$

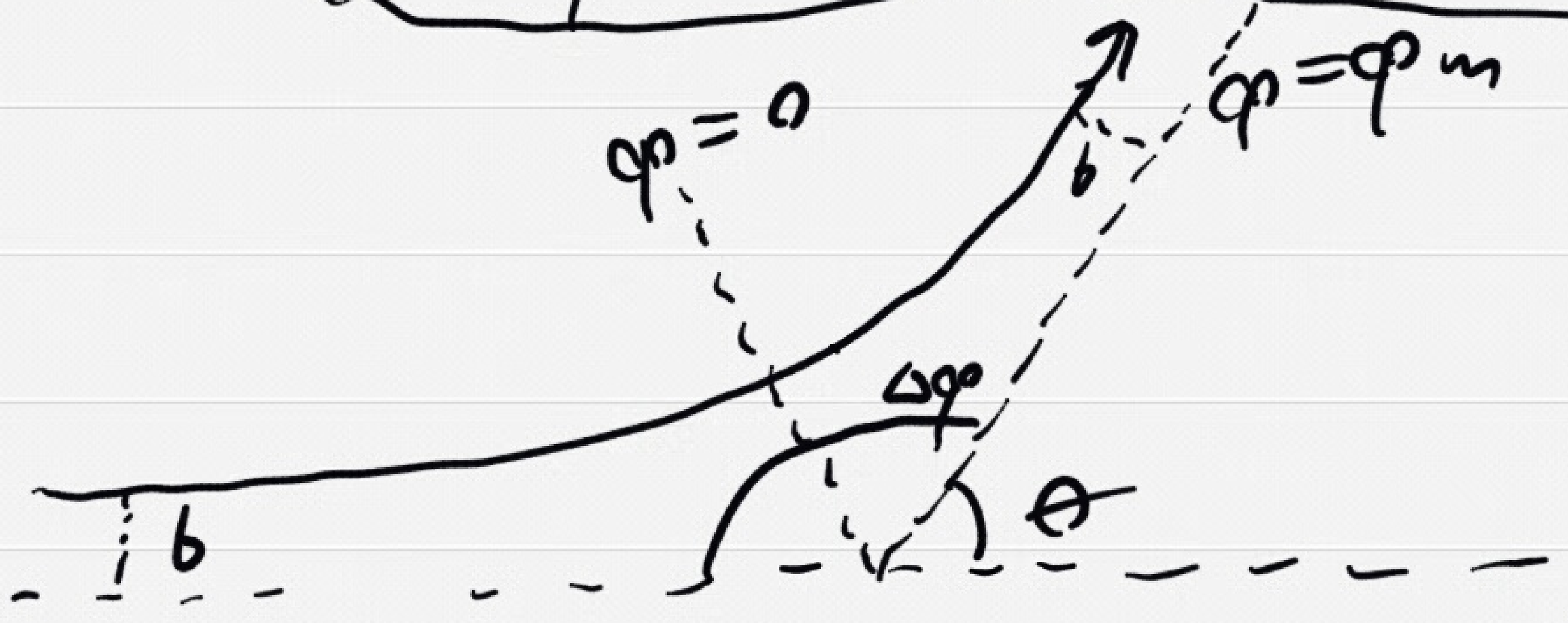
$$\Rightarrow 0 = \frac{L^2}{m}\, du/d\varphi \cdot d^2u/d\varphi^2 + \frac{L^2}{m}u \, du/d\varphi + \alpha \, du/d\varphi$$

$$\Rightarrow L^2/m \; d^2u/d\varphi^2 + \frac{L^2}{m}u + \alpha = 0$$

$$\Rightarrow \quad \frac{d^2u}{d\varphi^2} + u = -\frac{\alpha m}{L^2}$$

solution:

$$u(\varphi) = -\frac{\alpha m}{L^2}\left[1 - \varepsilon\cos\varphi\right]$$



Want relation between b and θ (impact parameter & scattering angle)

$$\theta = \pi - \Delta\varphi$$
$$= \pi - 2\varphi_m$$

$$K = \frac{L^2}{2m}\left(\frac{du}{d\varphi}\right)^2 + \frac{L^2}{2m}u^2 + \alpha u$$

$$\frac{du}{d\varphi} = -\frac{\alpha m}{L^2}\varepsilon\sin\varphi$$

$$\Rightarrow K = \frac{L^2}{2m}\left(\frac{\alpha m\varepsilon}{L^2}\right)^2\sin^2\varphi + \frac{L^2}{2m}\left(\frac{\alpha m}{L^2}\right)^2(1-\varepsilon\cos\varphi)^2 - \frac{\alpha^2 m}{L^2}(1-\varepsilon\cos\varphi)$$

$$= \frac{\alpha^2 m}{L^2}\left[\frac{1}{2}\varepsilon^2\sin^2\varphi + \frac{1}{2} - \varepsilon\cos\varphi + \frac{1}{2}\varepsilon^2\cos^2\varphi - 1 + \varepsilon\cos\varphi\right]$$

$$= \frac{\alpha^2 m}{L^2}\left[\frac{1}{2}\varepsilon^2 - \frac{1}{2}\right]$$

$$\Rightarrow \varepsilon = \sqrt{1 + \frac{2KL^2}{\alpha^2 m}}$$

$$L = mv_0 b = \sqrt{2mK}\, b$$

$$\Rightarrow \varepsilon = \sqrt{1 + 4K^2b^2/\alpha^2}$$

So $u(\varphi) = \frac{\alpha}{2\kappa b^2}\left[\varepsilon \cos\varphi - 1\right]$

w/ $\varepsilon = \sqrt{1 + 4\kappa^2 b^2/\alpha^2}$

$$\Rightarrow \quad 1 + \frac{2\kappa b^2 u}{\alpha} = \varepsilon \cos\varphi$$

$$\Rightarrow \quad \cos\varphi = \frac{1 + \frac{2\kappa b^2 u}{\alpha}}{\sqrt{1 + 4\kappa^2 b^2/\alpha^2}}$$

Solve for $\varphi_m = \frac{\pi - \theta}{2}$ $\left[\begin{array}{c} u \to 0 \\ r \to \infty \end{array}\right]$

$$\cos\varphi_m = 1/\sqrt{1 + 4\kappa^2 b^2/\alpha^2}$$

$$\cos\left(\frac{\pi}{2} - \theta/2\right) = \sin(\theta/2)$$

$$\Rightarrow \quad \sin(\theta/2) = 1/\sqrt{1 + 4\kappa^2 b^2/\alpha^2}$$

$$\Rightarrow \quad \frac{4\kappa^2 b^2}{\alpha^2} = \frac{1}{\sin^2(\theta/2)} - 1$$
$$= \cot^2(\theta/2)$$

$$\Rightarrow \quad b = \frac{\alpha}{2\kappa} \cot(\theta/2)$$

$$\Rightarrow \quad \boxed{b = \frac{z Z e^2}{8\pi \varepsilon_0 K} \cot(\theta/2)}$$

Rutherford Scattering