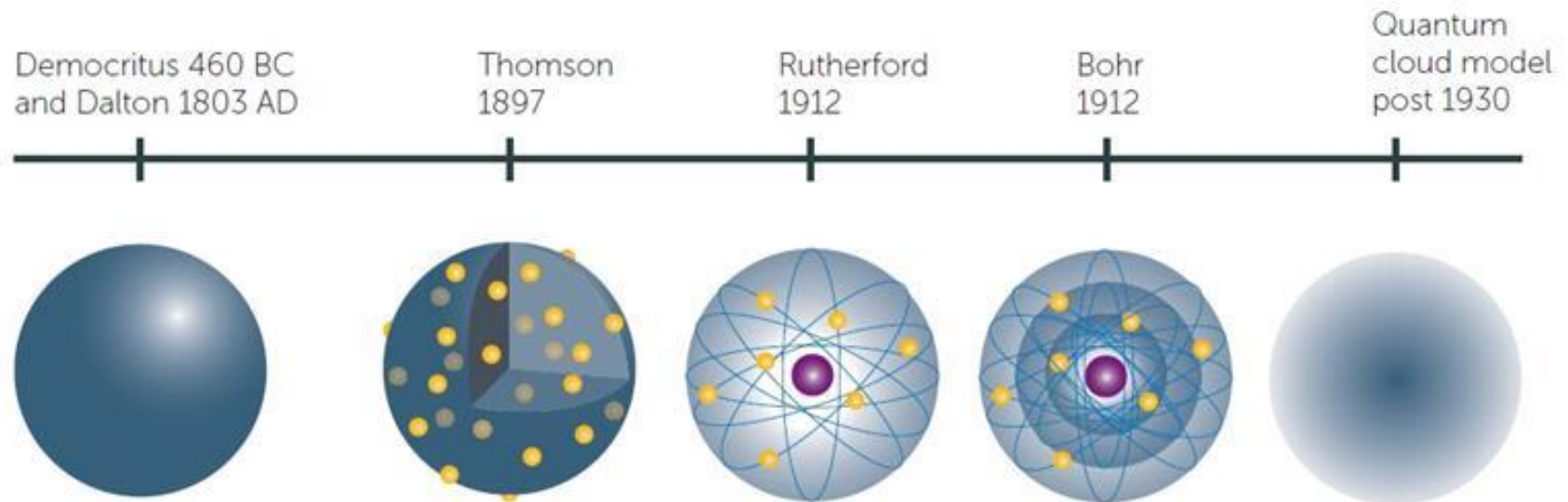


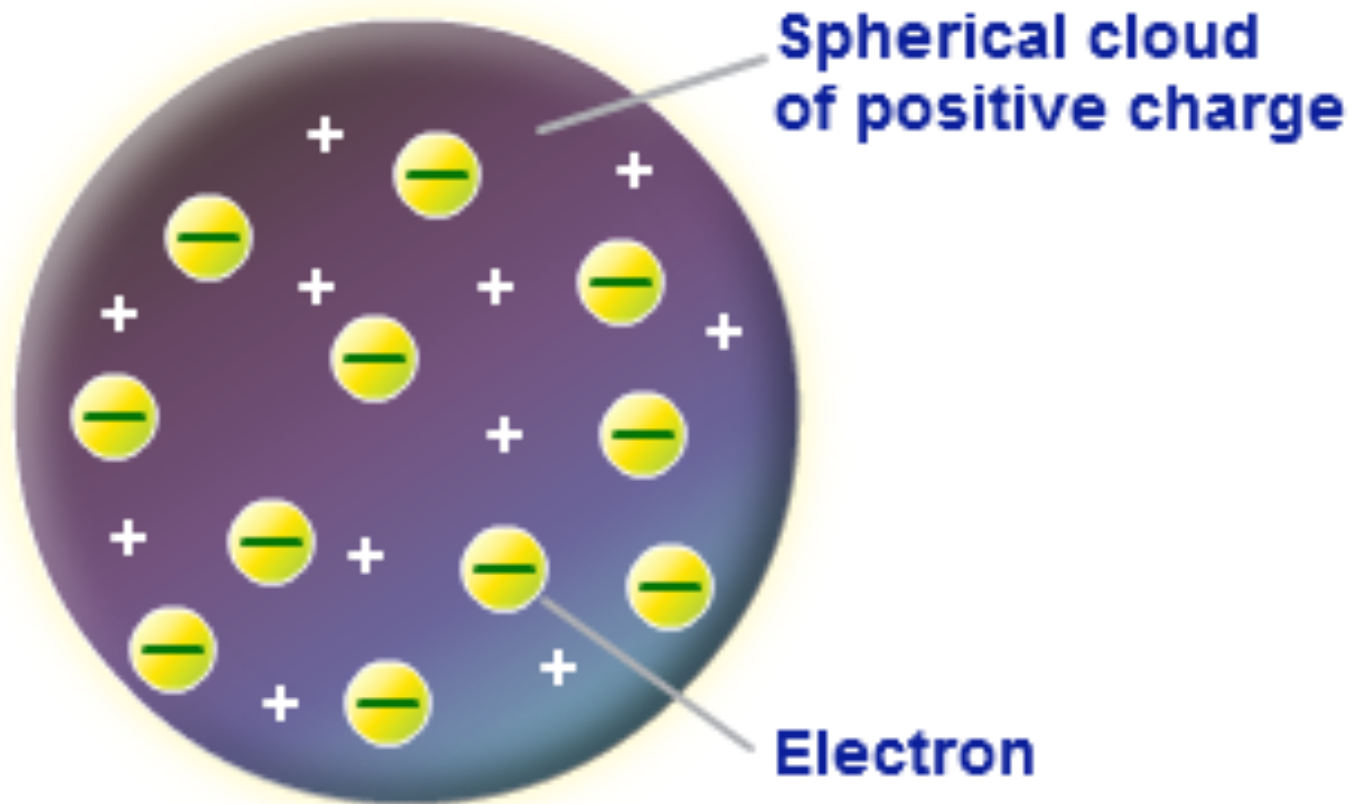
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Models of the Atom

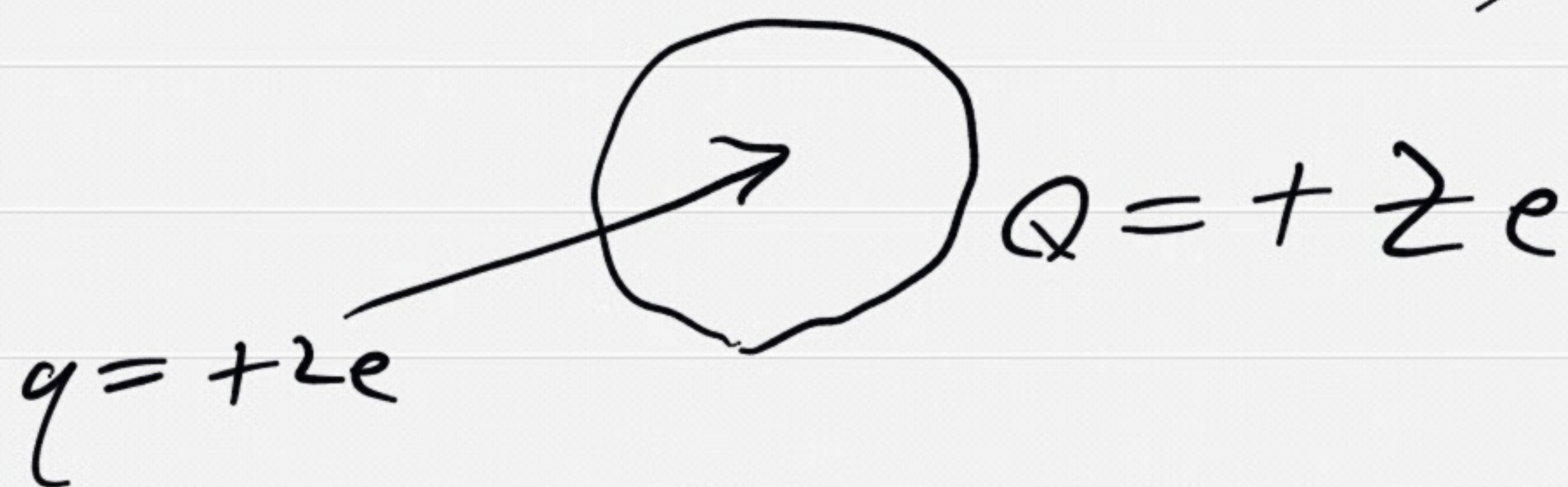


Thomson Atom



Thomson's Plum pudding model

Thompson Scattering



E in uniform sphere

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}$$

$$F = qE = \frac{2ze^2 r}{4\pi\epsilon_0 R^3}$$

$$\langle F \rangle \sim \frac{2ze^2 R/2}{4\pi\epsilon_0 R^3} = \frac{ze^2}{4\pi\epsilon_0 R^2}$$

Impulse - Momentum

$$\Delta p = \langle F \rangle \Delta t$$

$$\Delta t \sim R/v$$

$$\Delta p \sim \frac{ze^2}{4\pi\epsilon_0 R v}$$



$$\theta \sim \frac{\Delta p}{p}$$

$$= \frac{ze^2}{4\pi\epsilon_0 R v} \cdot \frac{1}{m\alpha v}$$

$$= \frac{ze^2}{8\pi\epsilon_0 R \cdot K\alpha}$$

R of gold atom $\sim 0.18 \text{ nm}$

Z of gold atom $= 79$

Kinetic energy of $\alpha \sim 5 \text{ MeV}$

$$\theta \sim \frac{79 \cdot (1.6 \times 10^{-19})^2}{8 \cdot \pi \cdot 8.85 \times 10^{-12} \cdot 2 \times 10^{-10} \cdot 5 \times 10^6 \cdot 1.6 \times 10^{-19}}$$

$\sim 7 \times 10^{-5}$ radians

- Each scatter is
very small!!

Thomson Scattering Simulation

<https://phet.colorado.edu/en/simulation/rutherford-scattering>

Alpha Particles

1.5 x 10⁻¹³ m (nuclear scale)

Legend

- Proton
- Neutron
- Alpha Particle

Alpha Particle

Energy

min max

Traces

Atom

Protons

20 90 100

Neutrons

20 92 150

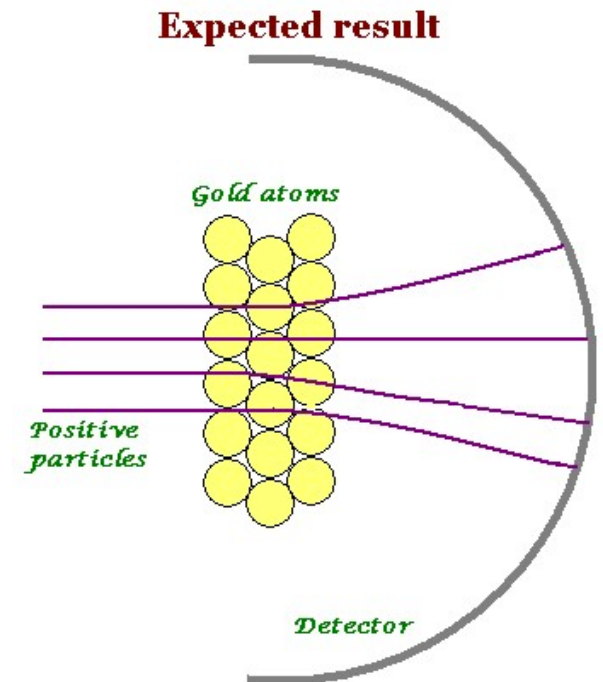
Rutherford Scattering

Rutherford Atom Plum Pudding Atom

PhET

Concept Check

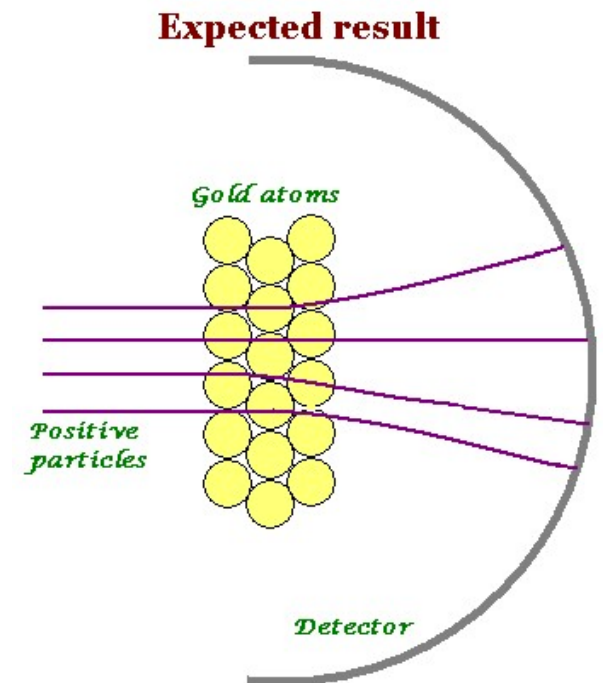
- For each interaction, alpha particles scatter by $\sim 10^{-4}$ radians. If each alpha interacts with 10^4 gold atoms while passing through a foil, what is the chance of an alpha particle scattering by a total of one radian?
- A. 1
B. $1/2^{10}$
C. $1/2^{100}$
D. $1/2^{10000}$



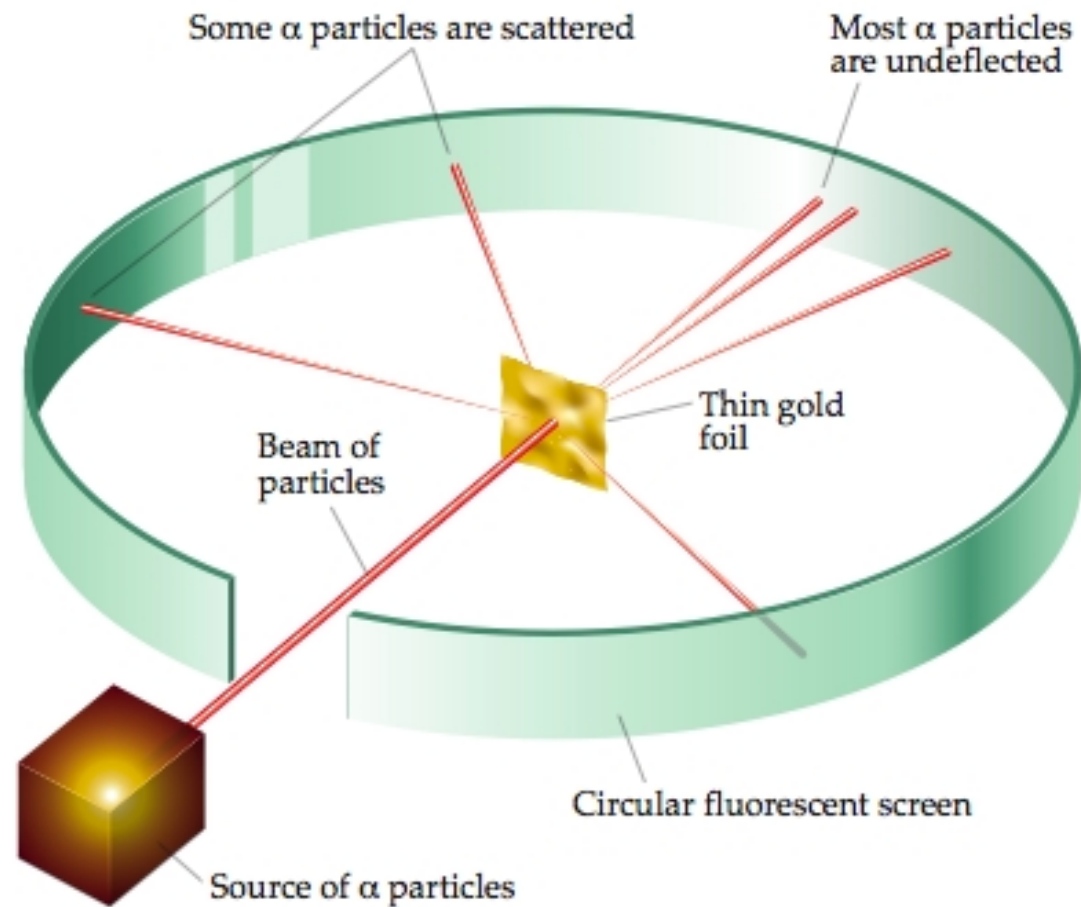
Concept Check

- For each interaction, alpha particles scatter by $\sim 10^{-4}$ radians. If each alpha interacts with 10^4 gold atoms while passing through a foil, what is the chance of an alpha particle scattering by a total of one radian?

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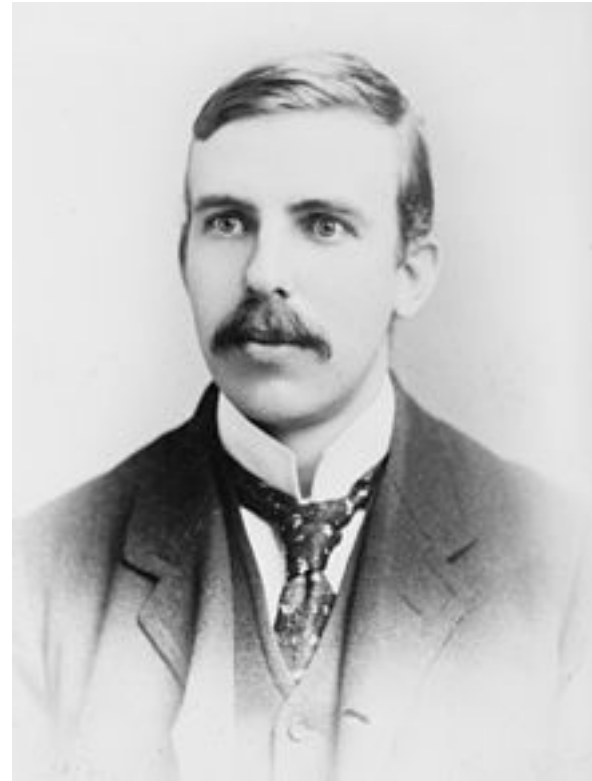


Alpha Particle Scattering Results

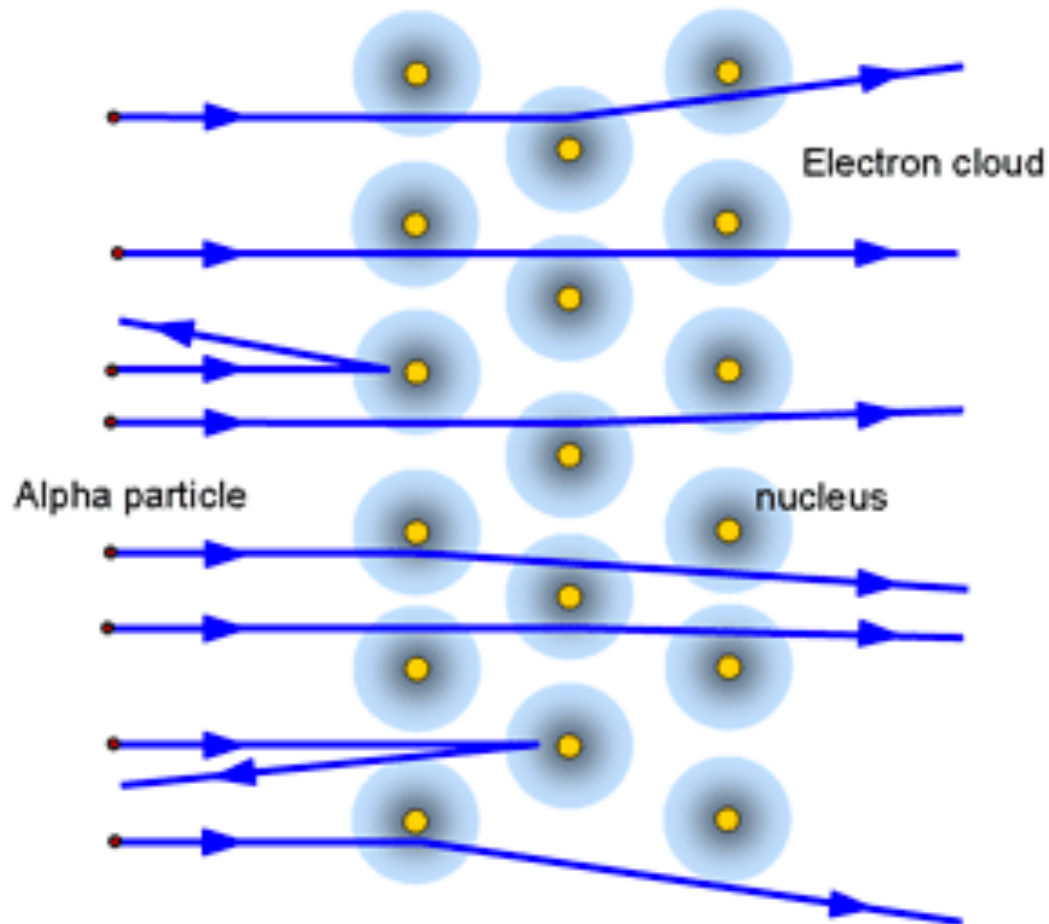


Alpha Particle Scattering

- *"It was as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you"*
– Ernest Rutherford



Alpha Particle Scattering Interpretation



Rutherford Scattering Simulation

<https://phet.colorado.edu/en/simulation/rutherford-scattering>

The simulation interface includes the following components:

- Legend:**
 - Proton (red dot)
 - Neutron (grey dot)
 - Alpha Particle (orange square)
- Alpha Particle Controls:**
 - Energy slider: min to max
 - Traces
- Atom Controls:**
 - Protons: 20 to 100, current value 90
 - Neutrons: 20 to 150, current value 92
- Scale:** 1.5×10^{-13} m (nuclear scale)
- Navigation:** Rutherford Atom, Plum Pudding Atom, Home, PhET logo

Rutherford Scattering

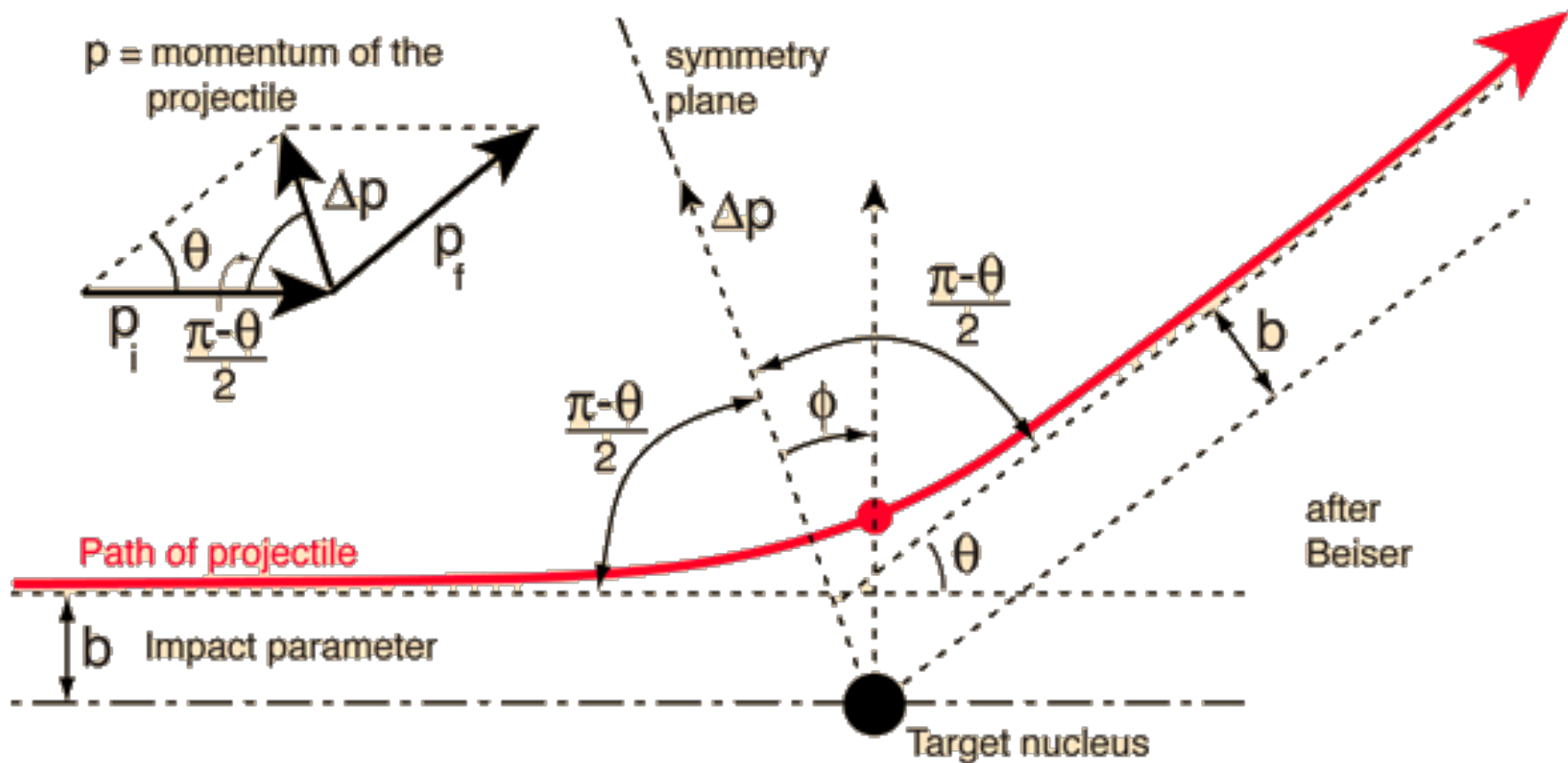
Rutherford Atom

Plum Pudding Atom



PhET

Rutherford Scattering



Rutherford Scattering

$$U(r) = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} = \frac{a}{r}$$

$$E = \frac{1}{2} m v_r^2 + \frac{1}{2} m v_\phi^2 + U(r) = \text{const.}$$

$$L = m v_\phi r = \text{angular momentum}$$

$$E(\infty) = K = \text{initial kinetic energy}$$

$$\begin{aligned} K &= \frac{1}{2} m v_r^2 + \frac{L^2}{2mr^2} + \frac{a}{r} \\ &= \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + \frac{a}{r} \end{aligned}$$

$$\begin{aligned} \dot{r} &= \frac{dr}{dt} = \frac{dr}{d\phi} \frac{d\phi}{dt} \\ &= \frac{dr}{d\phi} \cdot \frac{L}{mr^2} \end{aligned}$$

$$u = 1/r \rightarrow du = -\frac{1}{r^2} dr$$

$$\Rightarrow \dot{r} = -\frac{du}{d\phi} \cdot \frac{L}{m}$$

$$K = \frac{L^2}{2m} \left(\frac{du}{d\phi}\right)^2 + \frac{L^2}{2m} u^2 + au$$

Take $d/d\phi$

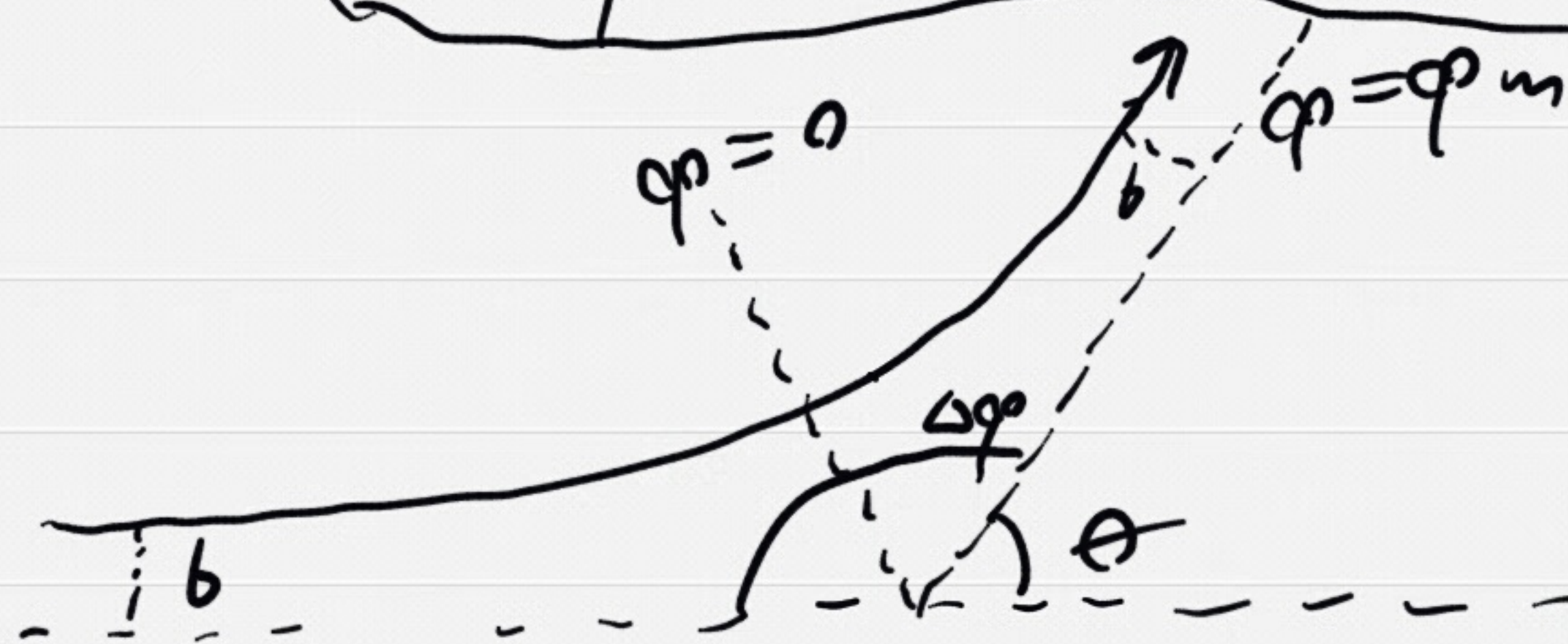
$$\Rightarrow 0 = \frac{L^2}{m} \frac{du}{d\phi} \cdot \frac{d^2u}{d\phi^2} + \frac{L^2}{m} u \frac{du}{d\phi} + a \frac{du}{d\phi}$$

$$\Rightarrow \frac{L^2}{m} \frac{d^2u}{d\phi^2} + \frac{L^2}{m} u + a = 0$$

$$\Rightarrow \frac{d^2 u}{d\varphi^2} + u = -\frac{a_m}{L^2}$$

Solution:

$$u(\varphi) = -\frac{a_m}{L^2} [1 - \epsilon \cos \varphi]$$



Want relation
between b
and Θ
(impact parameter
& scattering angle)

$$\begin{aligned} \Theta &= \pi - \Delta\varphi \\ &= \pi - 2\varphi_m \end{aligned}$$

$$K = \frac{L^2}{2m} \left(\frac{du}{d\varphi} \right)^2 + \frac{L^2}{2m} u^2 + a u$$

$$\frac{du}{d\varphi} = -\frac{a_m}{L^2} \epsilon \sin \varphi$$

$$\begin{aligned} \Rightarrow K &= \frac{L^2}{2m} \left(\frac{a_m \epsilon}{L^2} \right)^2 \sin^2 \varphi \\ &+ \frac{L^2}{2m} \left(\frac{a_m}{L^2} \right)^2 (1 - \epsilon \cos \varphi)^2 \\ &- \frac{a^2 m}{L^2} (1 - \epsilon \cos \varphi) \end{aligned}$$

$$= \frac{a^2 m}{L^2} \left[\frac{1}{2} \epsilon^2 \sin^2 \varphi + \frac{1}{2} - \epsilon \cos \varphi + \frac{1}{2} \epsilon^2 \cos^2 \varphi - 1 + \epsilon \cos \varphi \right]$$

$$= \frac{a^2 m}{L^2} \left[\frac{1}{2} \epsilon^2 - \frac{1}{2} \right]$$

$$\Rightarrow \epsilon = \sqrt{1 + \frac{2KL^2}{a^2 m}}$$

$$L = m v_0 b = \sqrt{2mK} b$$

$$\Rightarrow \epsilon = \sqrt{1 + 4K^2 b^2 / a^2}$$

$$So \quad u(\varphi) = \frac{a}{2\kappa b^2} [\xi \cos \varphi - 1]$$

$$w/\xi = \sqrt{1 + 4\kappa^2 b^2 / a^2}$$

$$\Rightarrow 1 + \frac{2\kappa b^2 u}{a} = \xi \cos \varphi$$

$$\Rightarrow \cos \varphi = \frac{1 + \frac{2\kappa b^2 u}{a}}{\sqrt{1 + 4\kappa^2 b^2 / a^2}}$$

Solve for $\varphi_m = \frac{\pi - \theta}{2} \quad [\begin{matrix} u \rightarrow 0 \\ r \rightarrow \infty \end{matrix}]$

$$\cos \varphi_m = \frac{1}{\sqrt{1 + 4\kappa^2 b^2 / a^2}}$$

$$\cos(\frac{\pi}{2} - \theta/2) = \sin(\theta/2)$$

$$\Rightarrow \sin(\theta/2) = \frac{1}{\sqrt{1 + 4\kappa^2 b^2 / a^2}}$$

$$\Rightarrow \frac{4\kappa^2 b^2}{a^2} = \frac{1}{\sin^2(\theta/2)} - 1$$
$$= \cot^2(\theta/2)$$

$$\Rightarrow b = \frac{a}{2\kappa} \cot(\theta/2)$$

$$\Rightarrow \boxed{b = \frac{z z e^2}{8\pi \epsilon_0 \kappa} \cot(\theta/2)}$$

Rutherford Scattering

