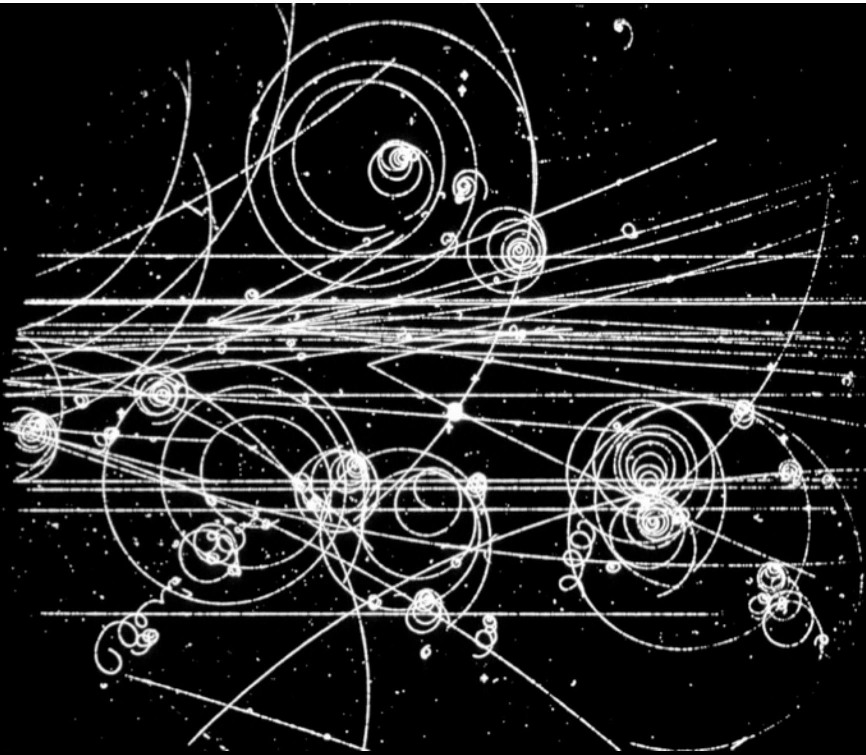
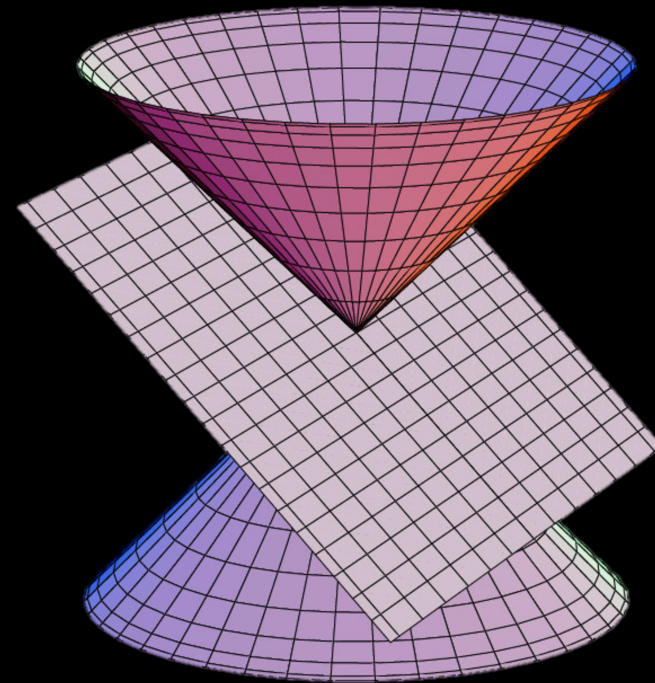



Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Traveling Wave Solution

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x) \qquad \psi(x) = A\exp(ikx)$$

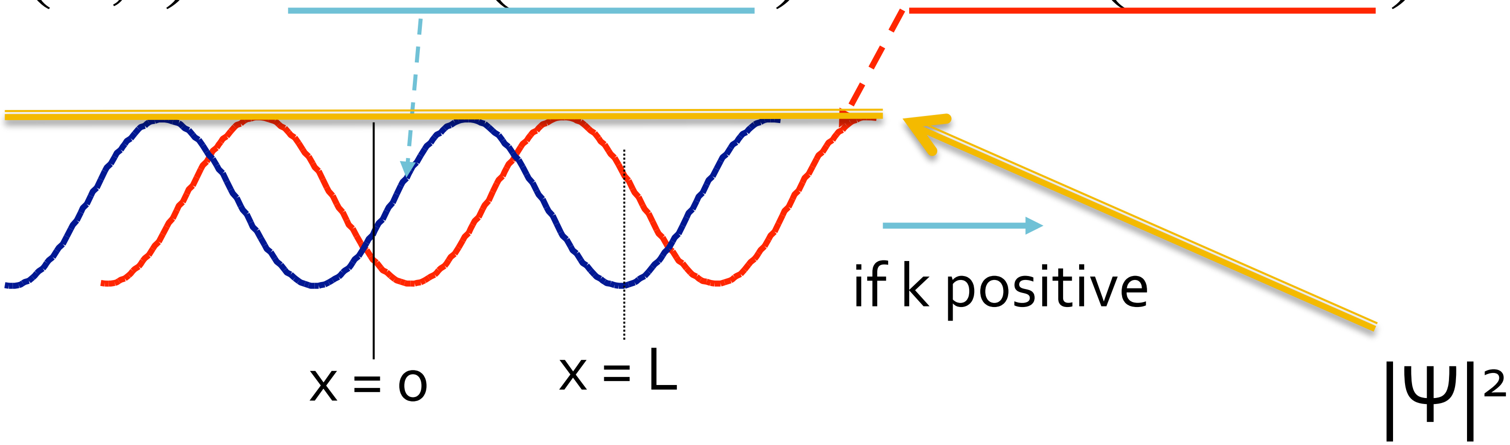
$$\frac{\hbar^2 k^2}{2m} = E$$

$$\Psi(x,t) = \psi(x)\phi(t) \qquad \phi(t) = e^{-iEt/\hbar}$$

$$\Psi(x,t) = A\exp[i(kx - \omega t)]$$

Traveling Wave Probability

$$\Psi(x,t) = A\cos(kx-\omega t) + Ai\sin(kx-\omega t)$$



If $U(x,t) = U(x)$

$$\Psi(x,t) = \psi(x) e^{-i\omega t}$$

$$|\Psi(x,t)|^2$$

$$= \psi^*(x) e^{i\omega t} \psi(x) e^{-i\omega t}$$

$$= \psi^*(x)\psi(x)$$

$$= |\psi(x)|^2 \quad \text{(no time dependence)}$$

$$= A^2 \quad \text{if } \psi(x) = Ae^{ikx}$$

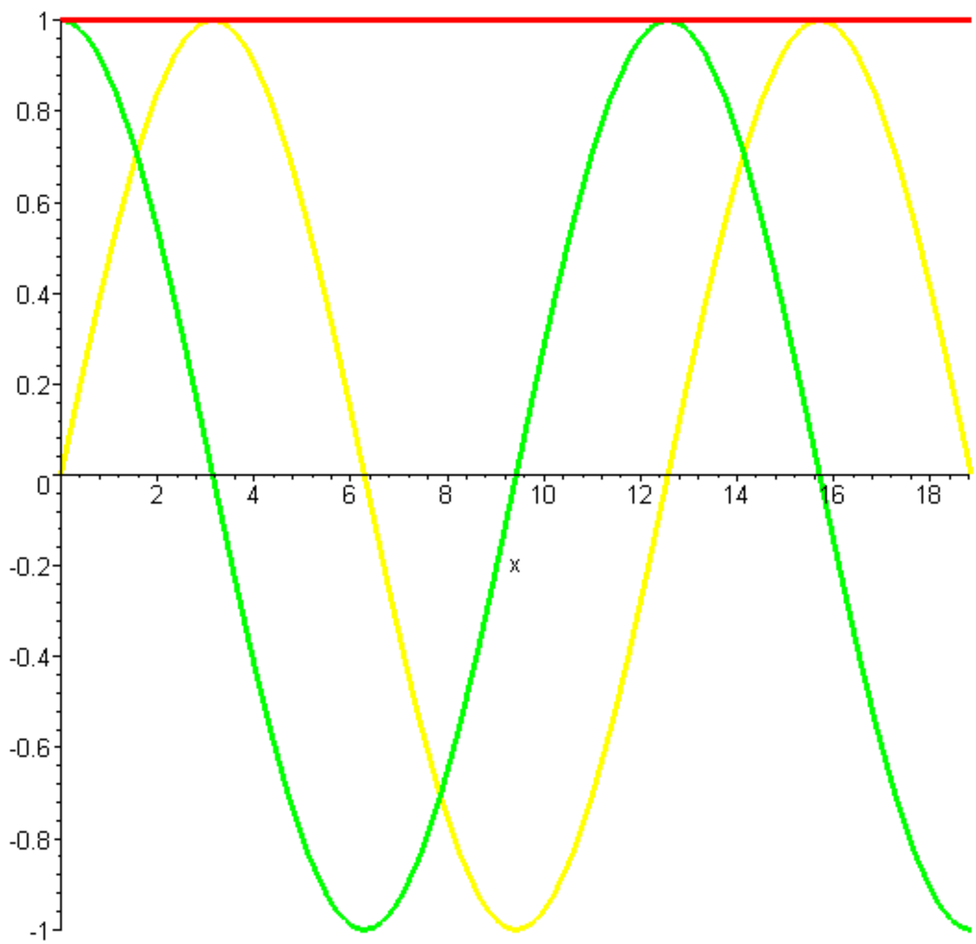
- Not true if wave has multiple components

i.e. $$\Psi(x,t) = A\psi_1(x) e^{-i\omega_1 t} + B\psi_2(x) e^{-i\omega_2 t}$$

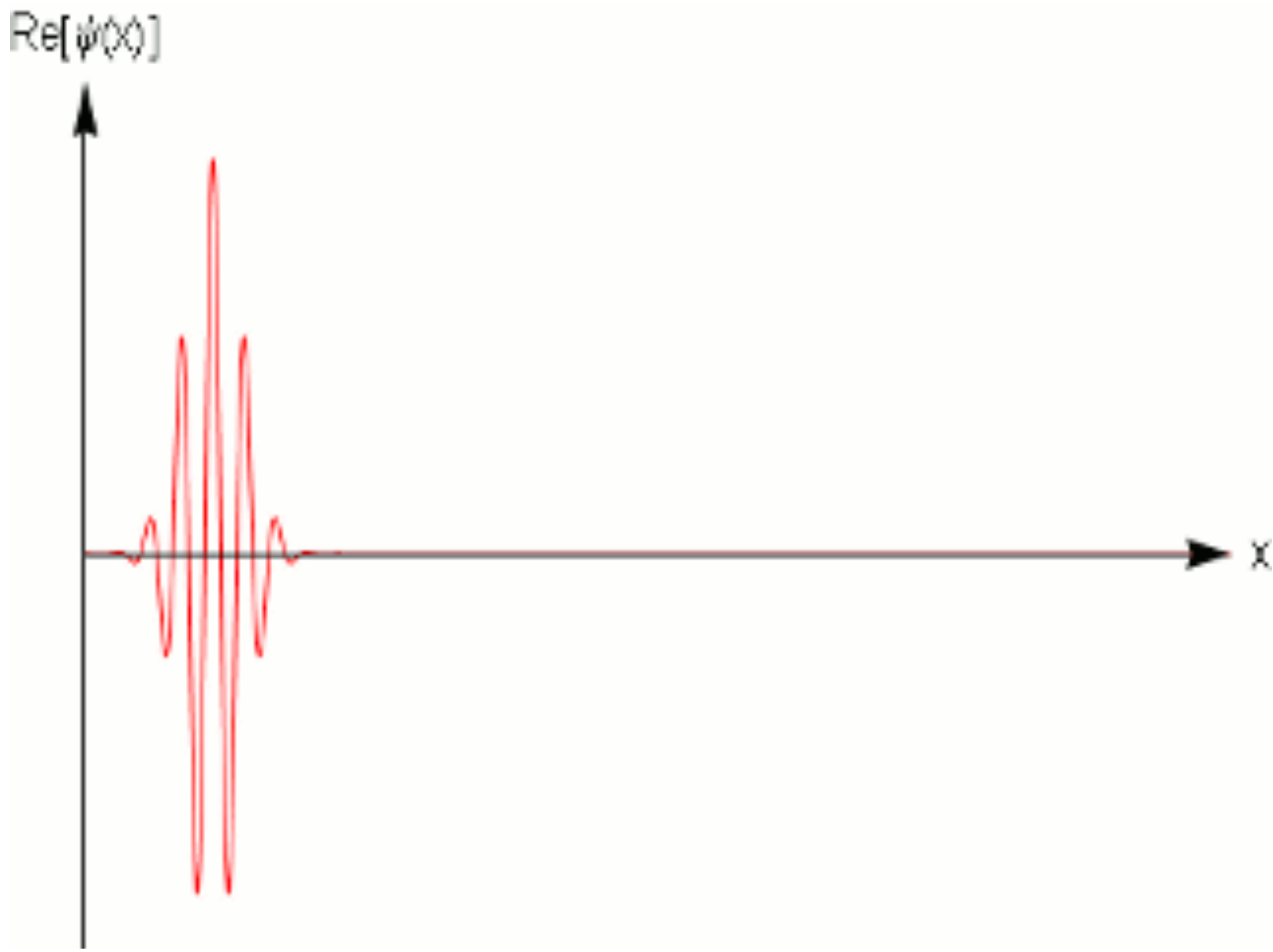
$$|\Psi(x,t)|^2 = A^2|\psi_1(x)|^2 + B^2|\psi_2(x)|^2 + AB\,\psi_1^*(x)\psi_2(x)\, e^{-i(\omega_1-\omega_2)t} + AB\,\psi_2^*(x)\psi_1(x)\, e^{-i(\omega_2-\omega_1)t}$$

- Cross terms ⇒ beat patterns

Traveling Wave Functions



Traveling Wave Packet

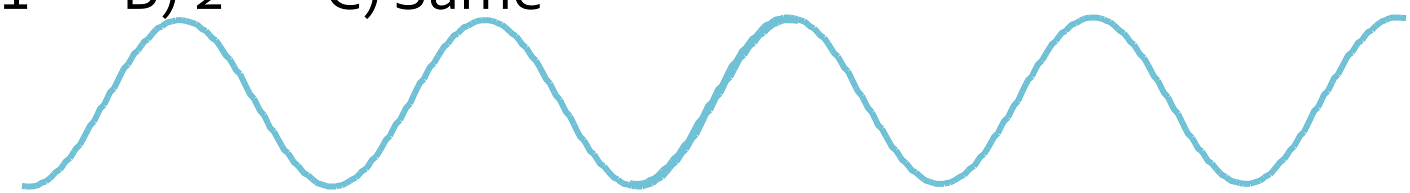


Concept Check

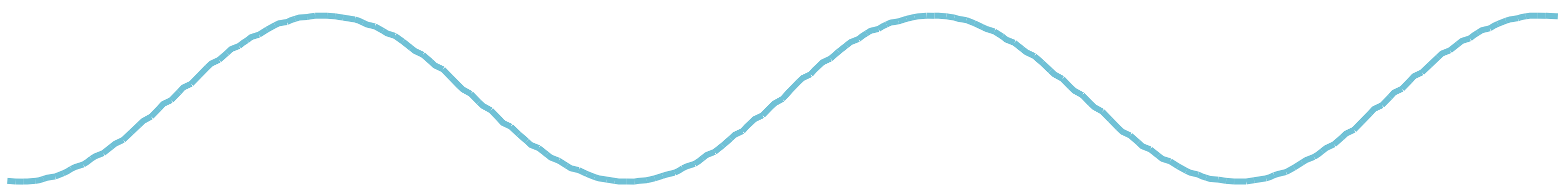
Given the wave functions below, which free electron has more kinetic energy?

A) 1 B) 2 C) Same

1.



2.

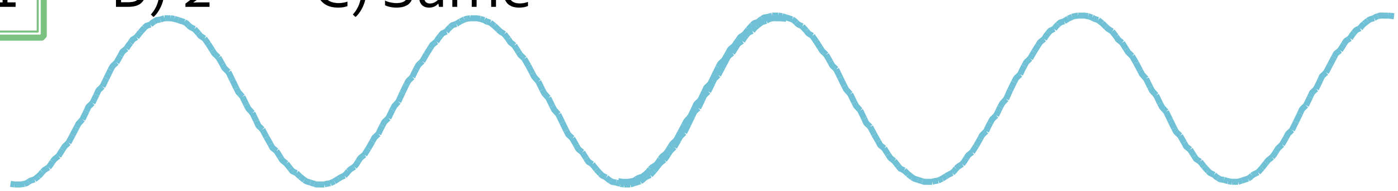


Concept Check

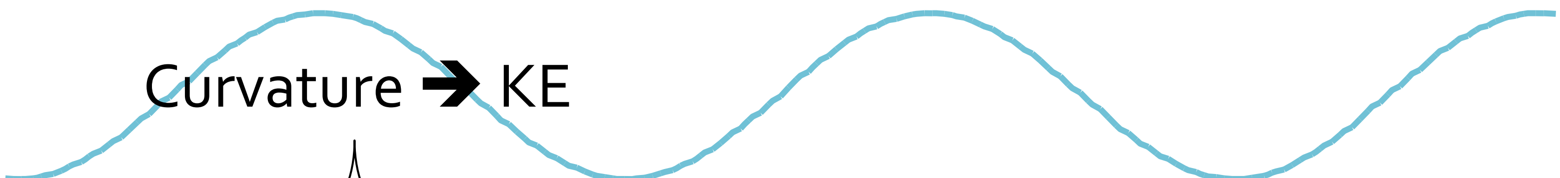
Given the wave functions below, which free electron has more kinetic energy?

A) 1   B) 2   C) Same

1.



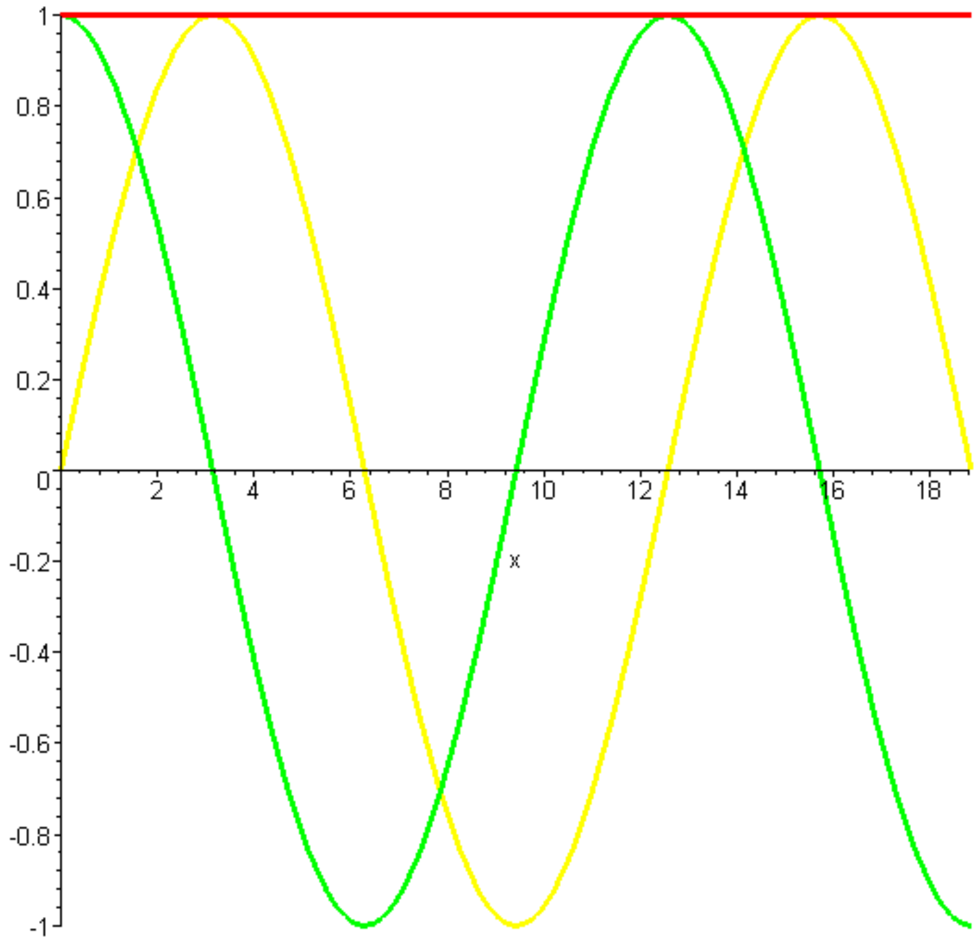
2.



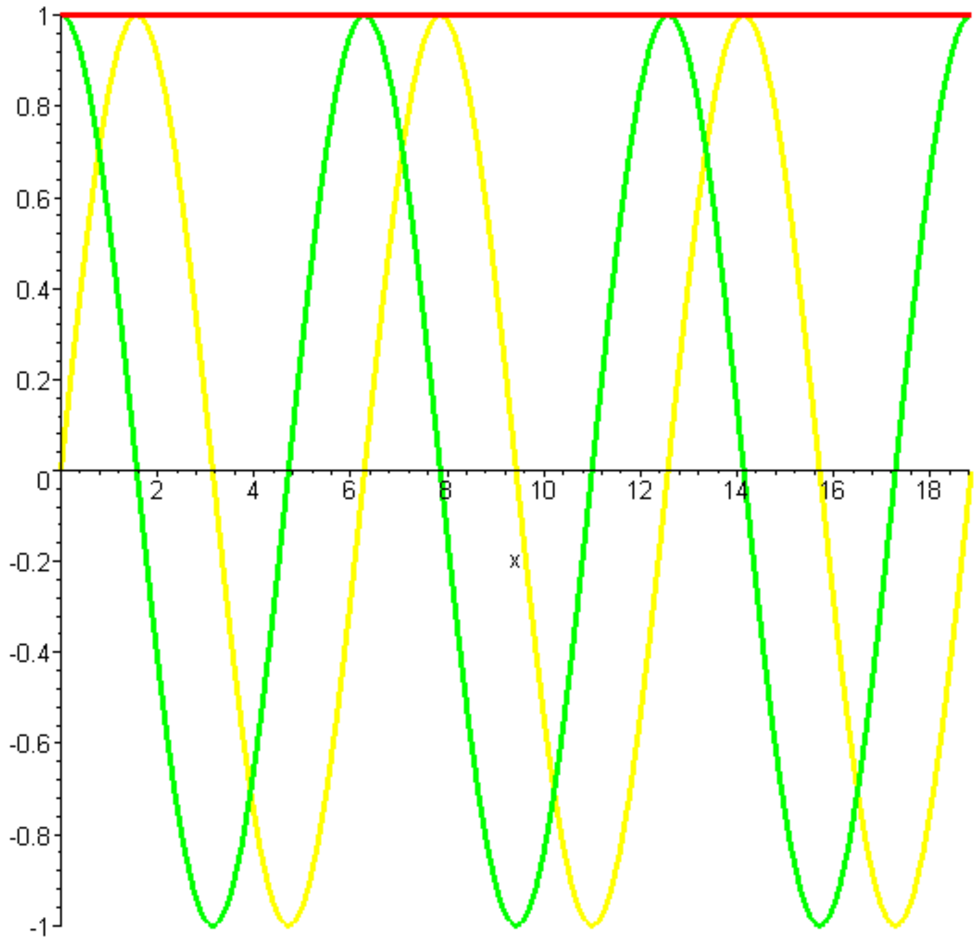
Curvature ➔ KE

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = E\psi(x)$$

Traveling Wave Functions



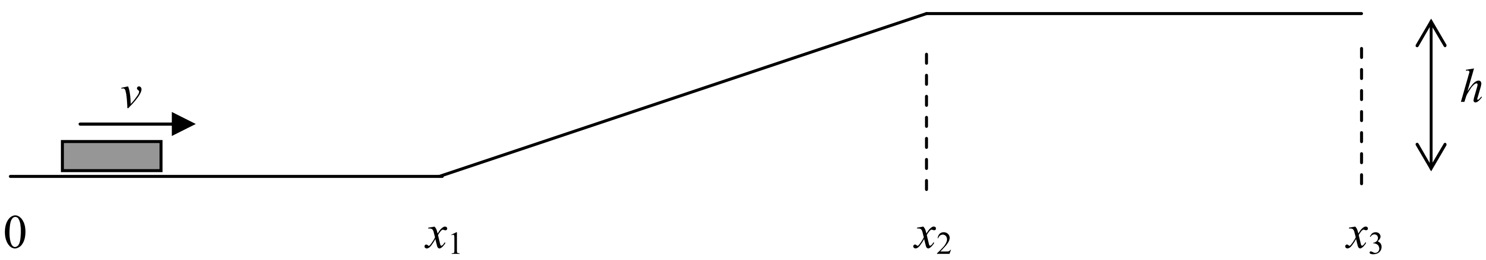
Traveling Wave Functions



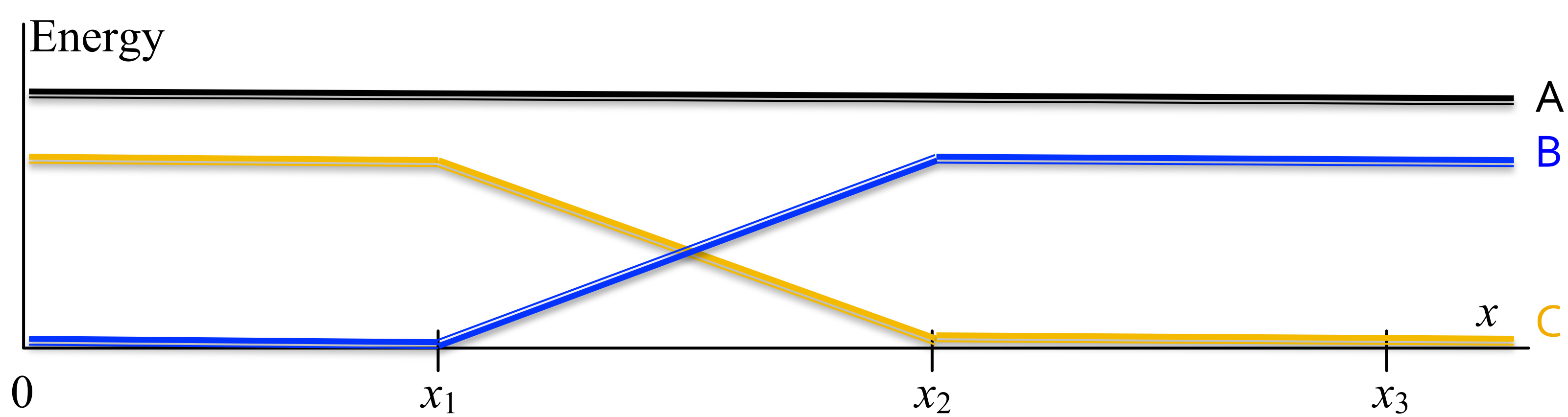
2x momentum
4x kinetic energy

Energy Diagrams

1. A small puck is gliding with initial speed v across a frictionless horizontal surface. It glides up a small hill and then moves on a horizontal surface that is a distance h above the first surface.

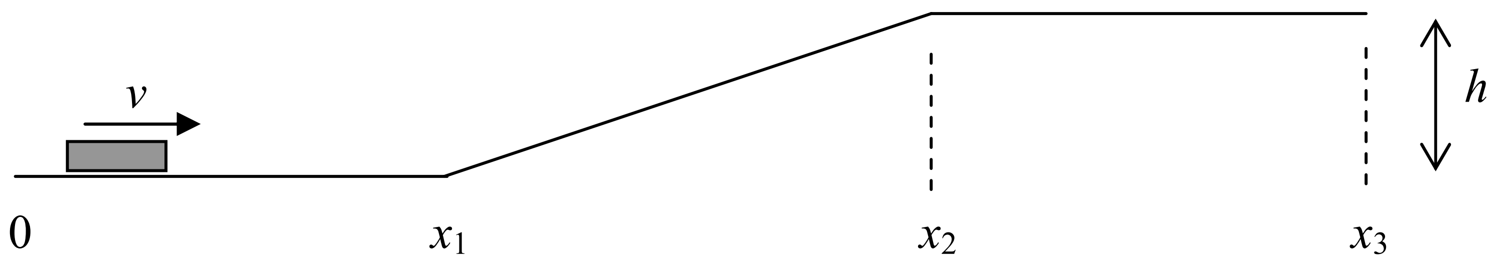


Which is the correct plot for the puck's potential energy?

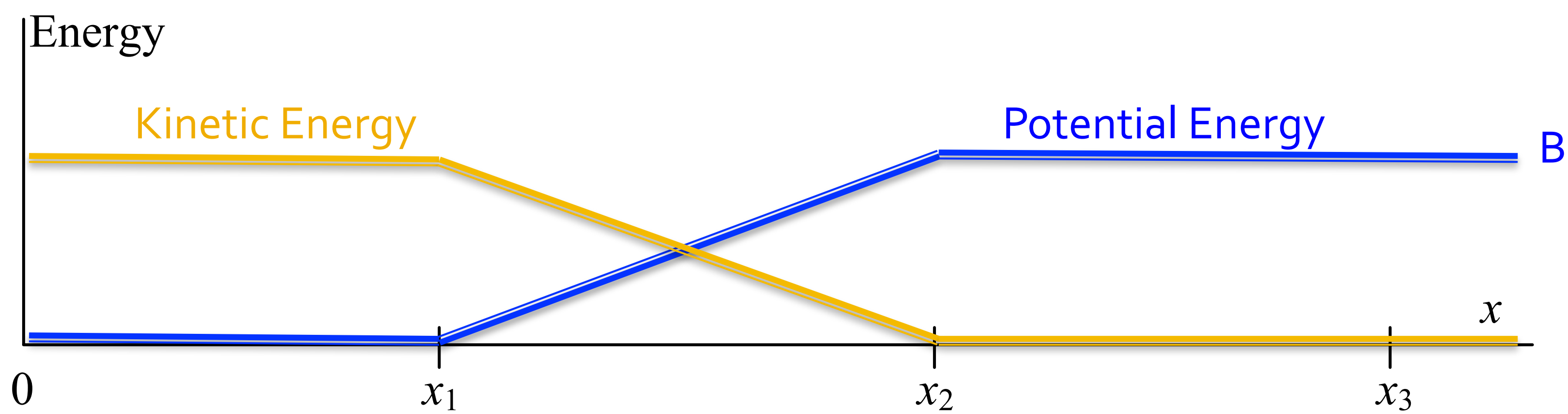


Energy Diagrams

1. A small puck is gliding with initial speed v across a frictionless horizontal surface. It glides up a small hill and then moves on a horizontal surface that is a distance h above the first surface.

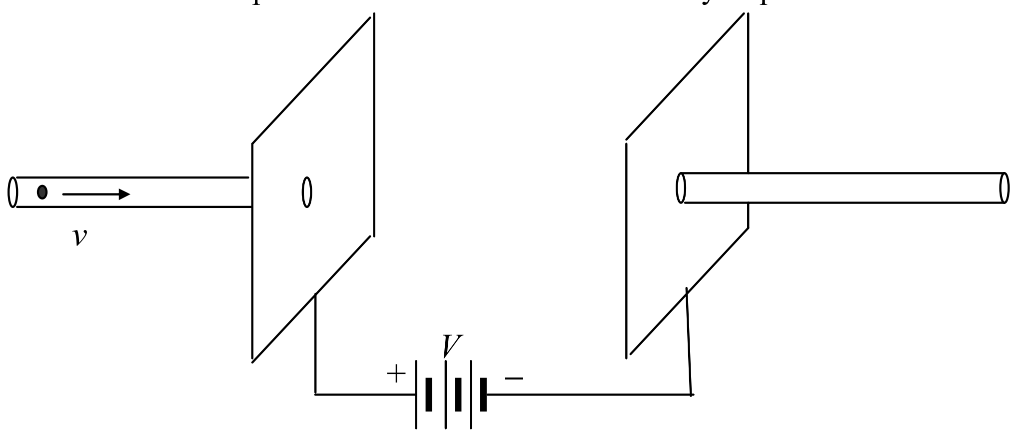


Which is the correct plot for the puck's potential energy?

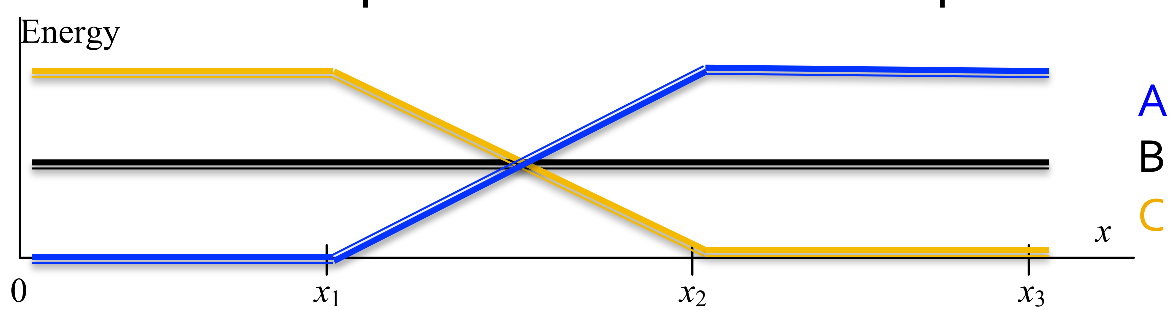


Energy Diagrams

2. An electron is moving with initial speed v inside a thin hollow metal tube. It emerges from the tube through a hole in a large metal plate and continues through a hole in a second plate into another thin tube. The two plates are connected across a battery of potential difference V.

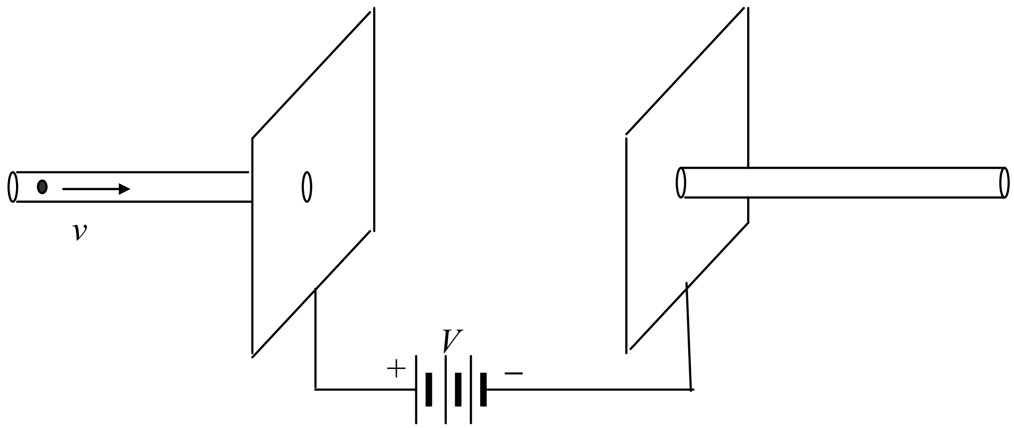


Which is the correct plot for the electron's potential energy?

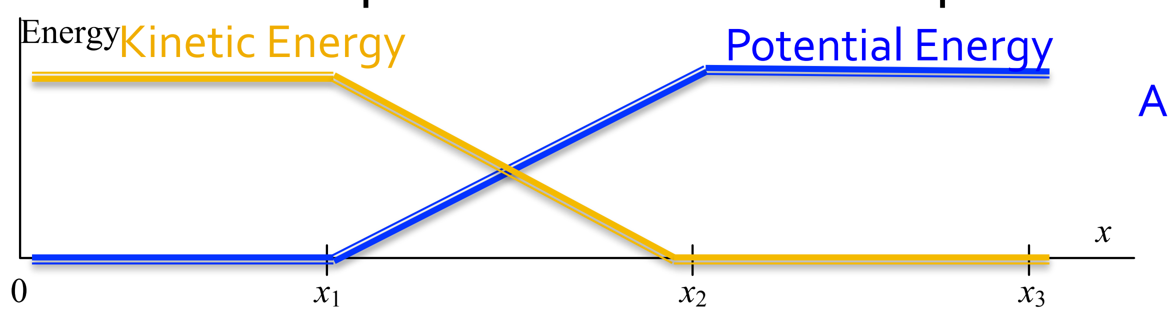


Energy Diagrams

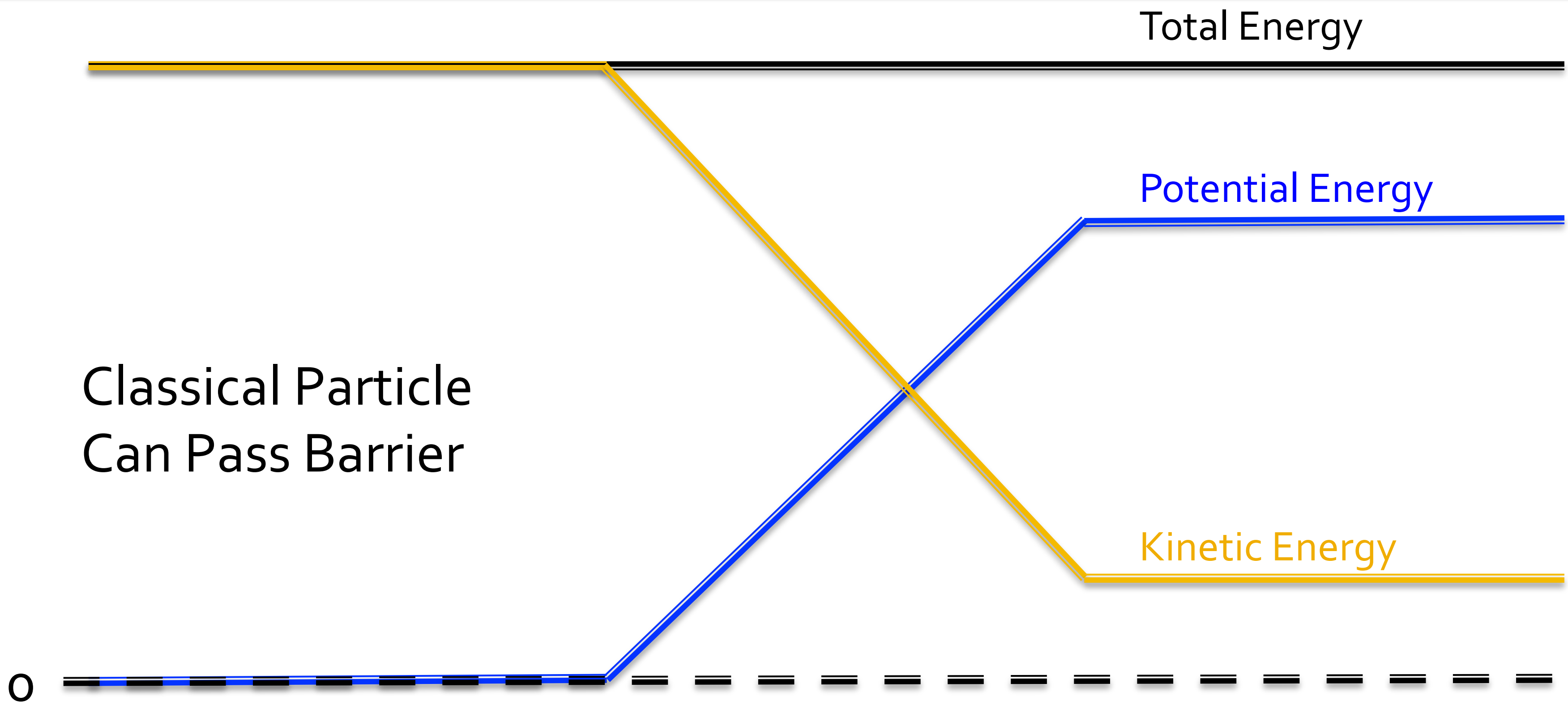
2. An electron is moving with initial speed v inside a thin hollow metal tube. It emerges from the tube through a hole in a large metal plate and continues through a hole in a second plate into another thin tube. The two plates are connected across a battery of potential difference V.



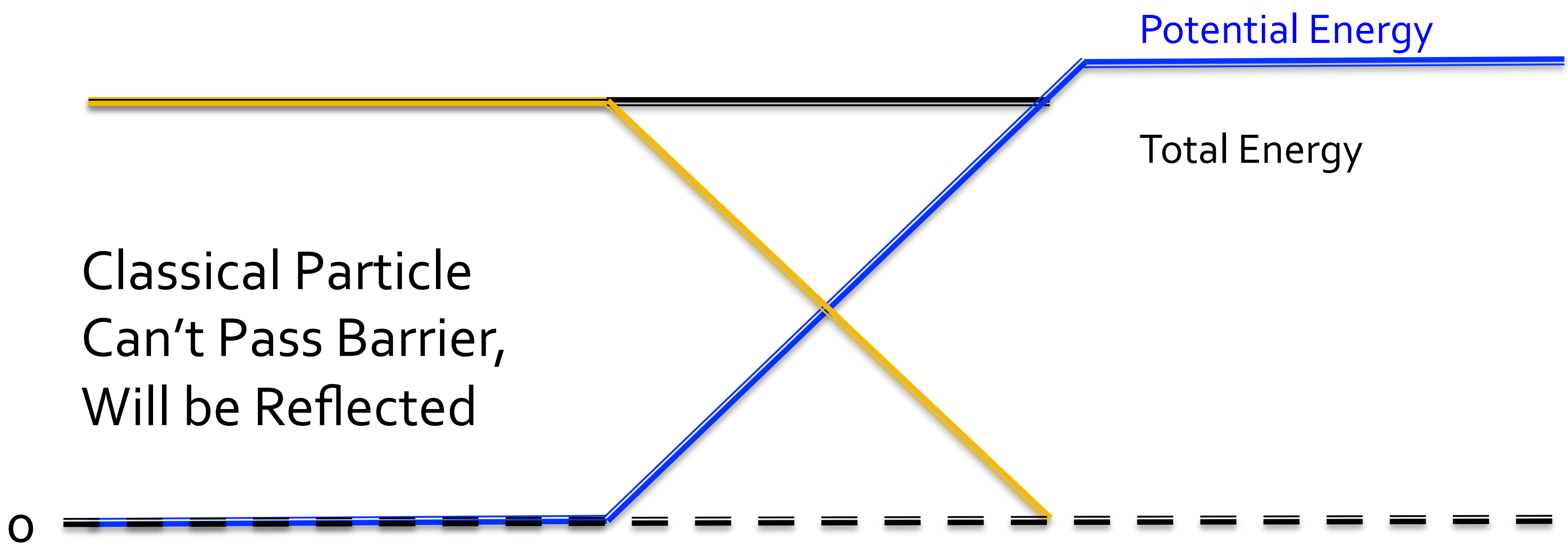
Which is the correct plot for the electron's potential energy?



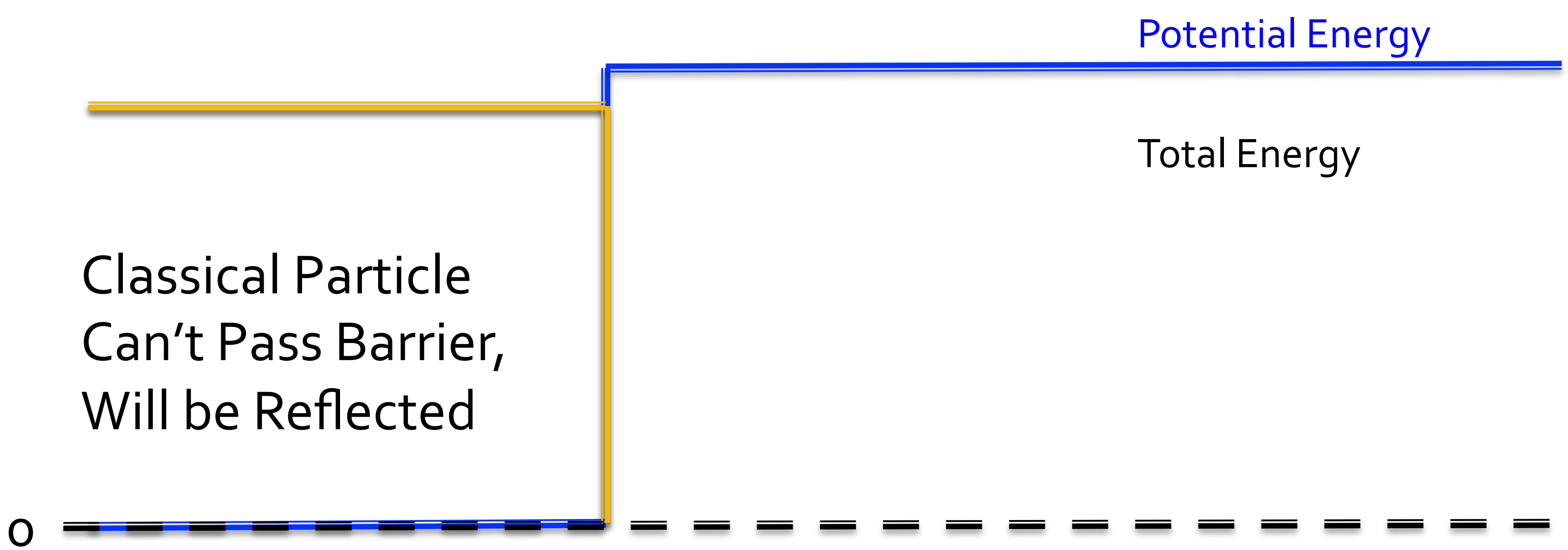
Energy Diagrams



Energy Diagrams



Sharp Barrier Energy Diagram



Schrödinger Equation for constant potential

$$U(x) = U \neq 0$$

$$-\hbar^2/2m \, \partial^2/\partial x^2 \, \psi + U\psi = E\psi$$

$$-\hbar^2/2m \, \partial^2/\partial x^2 \, \psi = (E-U)\psi$$

$$\partial^2\psi/\partial x^2 = \frac{-2m}{\hbar^2}(E-U)\psi$$

$E > U$: put $k^2 = \dfrac{2m(E-U)}{\hbar^2}$

$$\partial^2\psi/\partial x^2 = -k^2\psi$$

solutions $\psi(x) = e^{ikx}, e^{-ikx}$

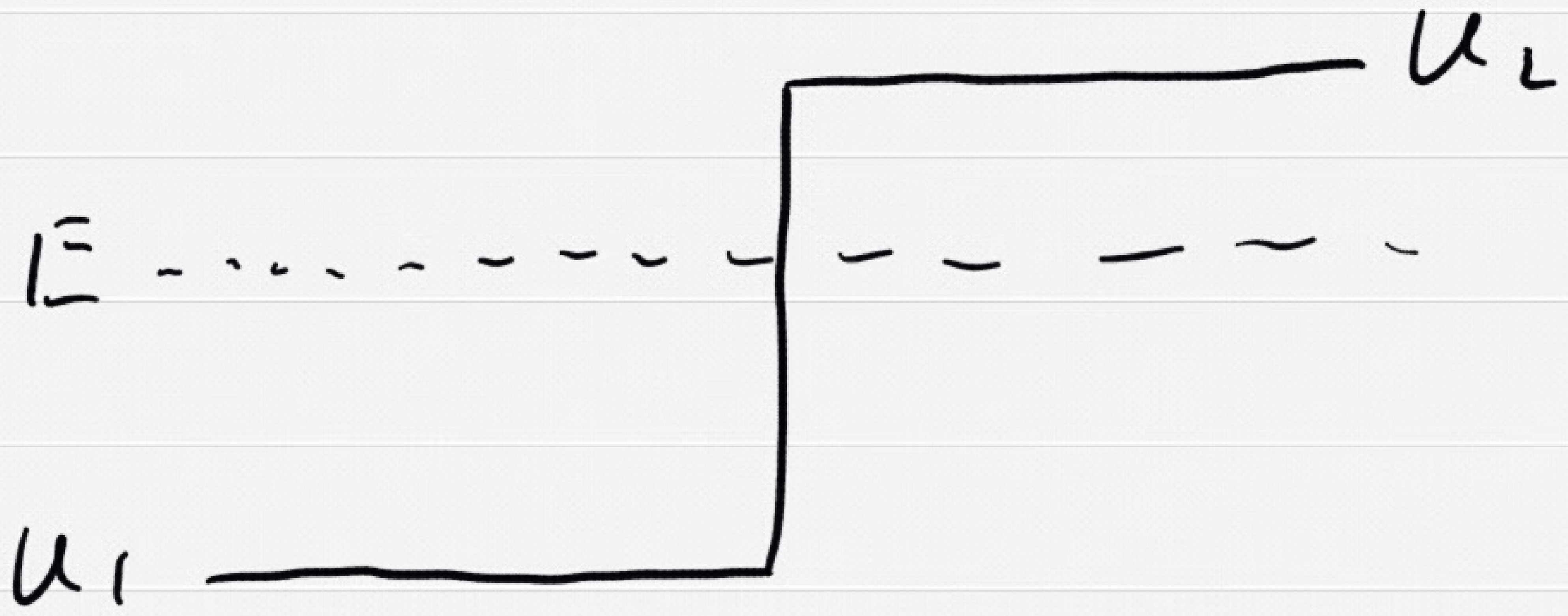
$E < U$: put $k^2 = \dfrac{2m(U-E)}{\hbar^2}$

$$\partial^2\psi/\partial x^2 = k^2\psi$$

solutions $\psi(x) = e^{kx}, e^{-kx}$

(i.e. e^{ikx}, e^{-ikx} w/ imaginary k)

Step Potential



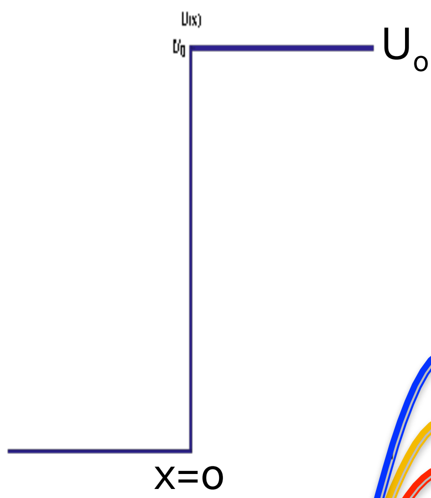
$$\psi(x) = Ae^{ik_1x} + Be^{-ik_1x}$$ on left

w/ $k_1 = \sqrt{2m(E-U_1)/\hbar^2}$

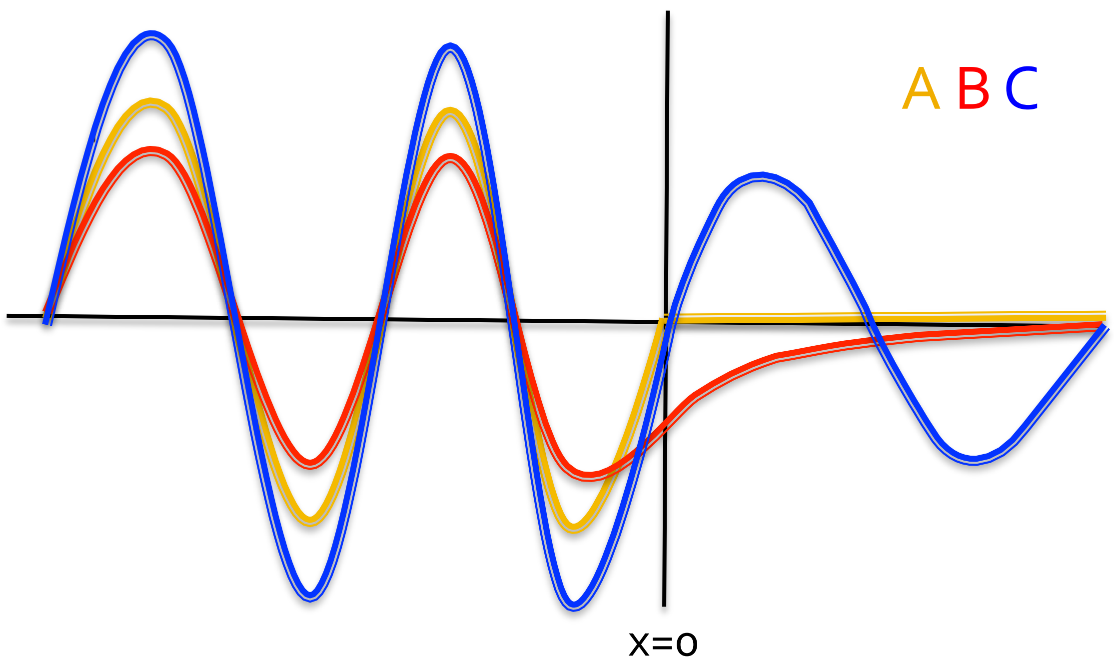
$$\psi(x) = Ce^{-\kappa_2 x}$$ on right

w/ $\kappa_2 = \sqrt{2m(U_2-E)/\hbar^2}$

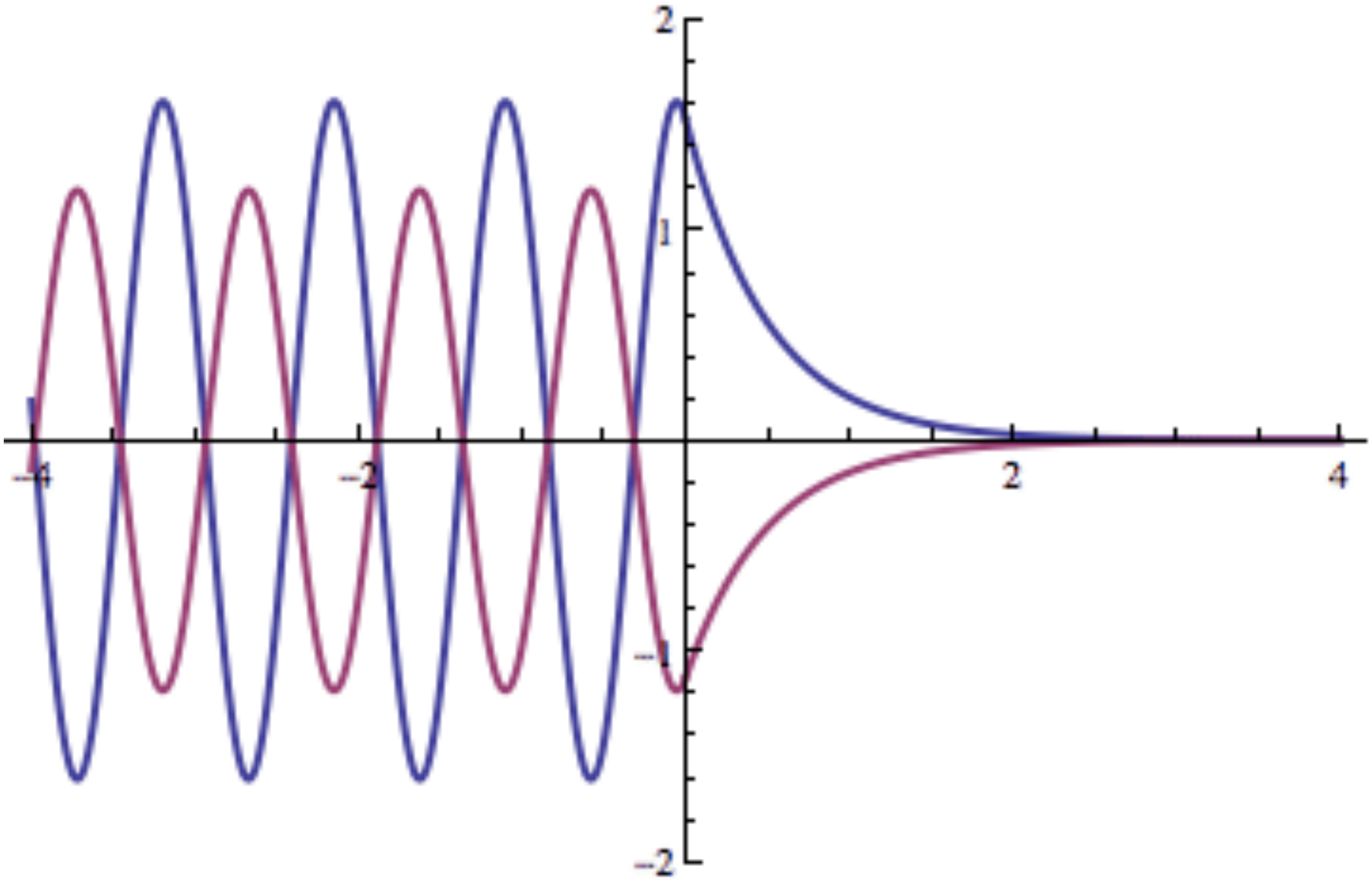
Wave Approaching a Potential Barrier



What does the wavefunction look like for an electron with $E < U_o$ (U_o = the height of the step)?



Actual Wave Function



Wave Function Components