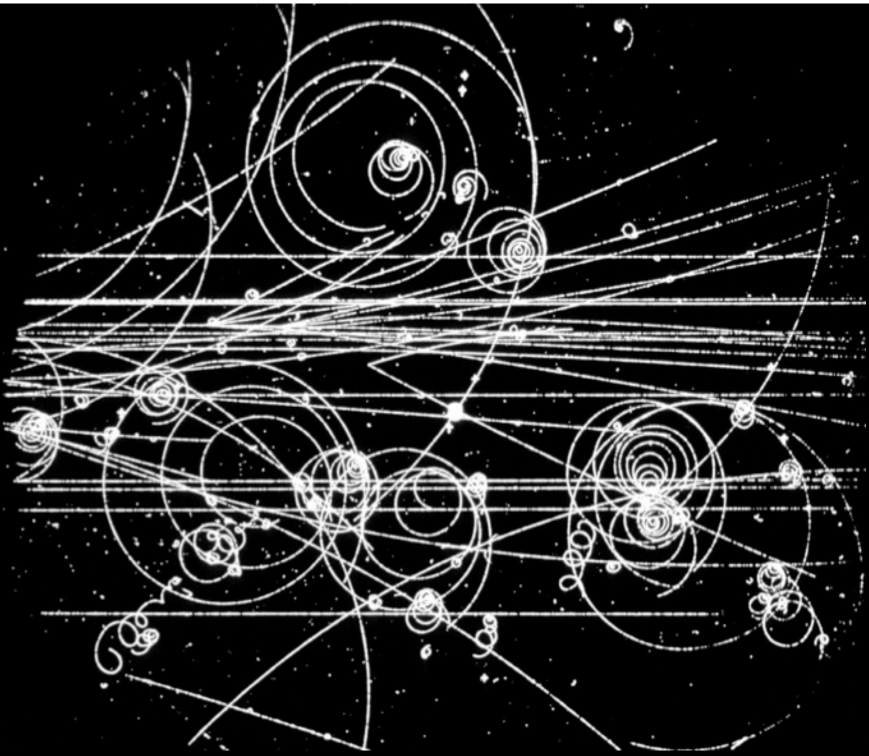
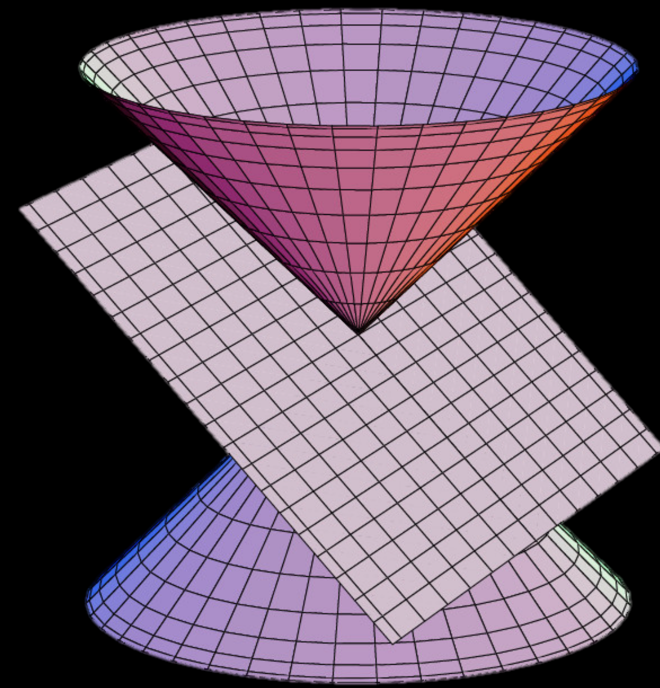




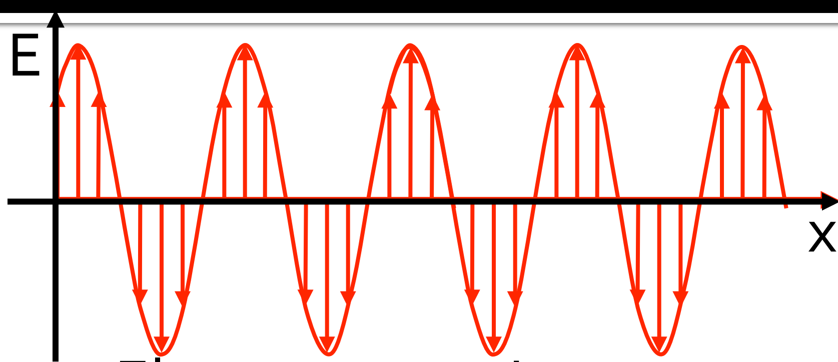
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

Announcements

- Back on regular schedule for the next two weeks – until Spring Break!
  - There **will** be labs and homework due this week and next
  - Labs this week and next are computer labs – meet in Van 201 rather than your usual lab room

Wave Equation



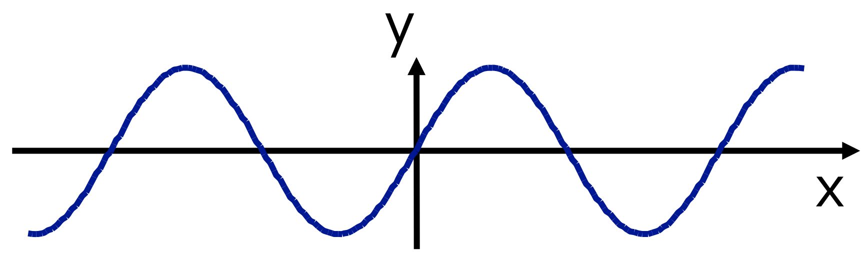
Electromagnetic waves:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

c=speed of light

Solutions: E(x,t)

Magnitude is non-spatial:
= Strength of electric field



Vibrations on a string:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

v=speed of wave

Solutions: y(x,t)

Magnitude is spatial:
= Vertical displacement of string

A New Kind of Wave

EM Waves (light/photons)

- Amplitude E = electric field
- $|E|^2$ tells you the probability of detecting a photon.
- Maxwell's Equations:

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}$$

- Solutions are sine/cosine waves:

Matter Waves (electrons/etc)

- Amplitude Ψ = matter field
- $|\Psi|^2$ tells you the probability of detecting a particle.
- Schrödinger Equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + U(x,t)\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

- Solutions are **complex** sine/cosine waves:

Wave Solutions

- EM Waves

$$E(x,t) = A\sin(kx - \omega t)$$

$$E(x,t) = A\cos(kx - \omega t)$$

Wave Functions

$$\Psi(x,t) = A\exp\left[i\left(kx - \omega t\right)\right]$$

$$= A\left[\cos(kx - \omega t) + i\sin(kx - \omega t)\right]$$

- In either case, wave packets can be formed by superposing waves with different k values.

How to Solve a Differential Equation

How to solve a differential equation in physics:

1) Guess functional form for solution

2) Make sure functional form satisfies Diff EQ
   (find any constraints on constants)
      1 derivative: need 1 soln → $f(x,t)=f_1$
      2 derivatives: need 2 soln → $f(x,t) = f_1 + f_2$

3) Apply all boundary conditions ← The hardest part!
      (find any constraints on constants)

Concept Check: Solutions

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

1) Guess functional form for solution

Which of the following functional forms works as a possible solution to this differential equation?

I. $y(x, t) = Ax^2t^2$,
II. $y(x, t) = A\sin(Bx)$
III. $y(x,t) = A\cos(Bx)\sin(Ct)$

a. I b. III c. II, III d. I, III e. None or some other combo

Concept Check: Solutions

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

1) Guess functional form for solution

Which of the following functional forms works as a possible solution to this differential equation?

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a. I b. III c. II, III d. I, III e. None or some other combo

Trial and Error

1) Guess functional form for solution

II. y(x, t) = Asin(Bx)

$$y(x,t) = A\sin(Bx)$$

LHS: $\frac{\partial^2 y}{\partial x^2} = -AB^2\sin(Bx)$

RHS: $\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$

$$-AB^2\sin(Bx) = 0$$

$$\sin(Bx) = 0$$

III. y(x,t) = Acos(Bx)sin(Ct)

$$y(x,t) = A\cos(Bx)\sin(Ct)$$

LHS: $\frac{\partial^2 y}{\partial x^2} = -AB^2\cos(Bx)\sin(Ct)$

RHS: $\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = -\frac{AC^2}{v^2}\cos(Bx)\sin(Ct)$

$$-AB^2\cos(Bx)\sin(Ct) = \frac{-AC^2}{v^2}\cos(Bx)\sin(Ct)$$

$$B^2 = \frac{C^2}{v^2}$$

Superposition of Solutions

y(x,t)=Asin(kx)cos(ωt) + Bcos(kx)sin(ωt)

Equivalent y(x,t)=Csin(kx-ωt) + Dsin(kx+ωt)

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$y(x,t) = C\sin(kx - \omega t)$$

$$-Ck^2\sin(kx - \omega t) = -\frac{C\omega^2}{v^2}\sin(kx - \omega t)$$

Satisfies wave eqn if: $k^2 = \frac{\omega^2}{v^2}$ $\quad v = \frac{\omega}{k} = f\lambda$

Wave Number and Frequency

$y(x,t)=C\sin(kx-\omega t) + D\sin(kx+\omega t)$

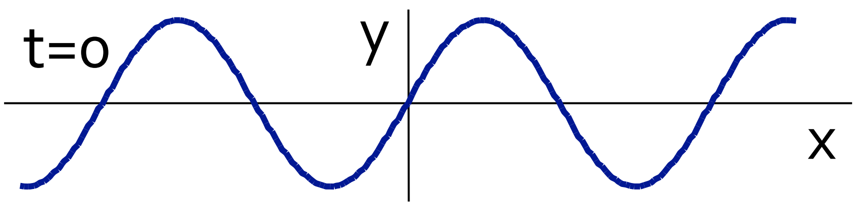
What is the wavelength of this wave? Ask yourself …

→How much does x need to increase to increase kx-ωt by 2π?

$\sin(k(x+\lambda) - \omega t) = \sin(kx - \omega t + 2\pi)$

$k(x+\lambda)=kx+2\pi$

$k\lambda=2\pi$ ➔ $k=2\pi/\lambda$



k=wave number (radians-m^{-1})

What is the period of this wave? Ask yourself …

→How much does t need to increase to increase kx-ωt by 2π?

$\sin(kx-\omega(t+T)) = \sin(kx - \omega t + 2\pi)$

$\omega T=2\pi$ ➔ $\omega=2\pi/T$

$= 2\pi f$

ω= angular frequency

Speed

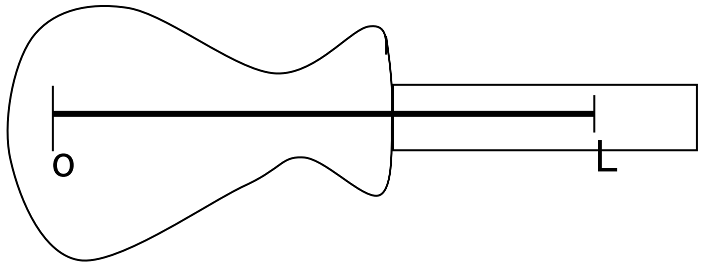
$$v = \frac{\lambda}{T} = \frac{\omega}{k}$$

Concept Check: Boundary Conditions

$y(x,t) = A\sin(kx)\cos(\omega t) + B\cos(kx)\sin(\omega t)$

Boundary conditions?

I. $y(x,t) = 0$ at $x=0$ and $x=L$



At $x=0$: $y(x,t) = B\sin(\omega t) = 0$ → only works if $B=0$

$y(x,t) = A\sin(kx)\cos(\omega t)$

From BC at $x = L$, what are possible values for k?

a. k can have any positive or negative value

b. $\pi/(2L)$, π/L, $3\pi/(2L)$, $2\pi/L$ …

c. π/L

d. π/L, $2\pi/L$, $3\pi/L$, $4\pi/L$ …

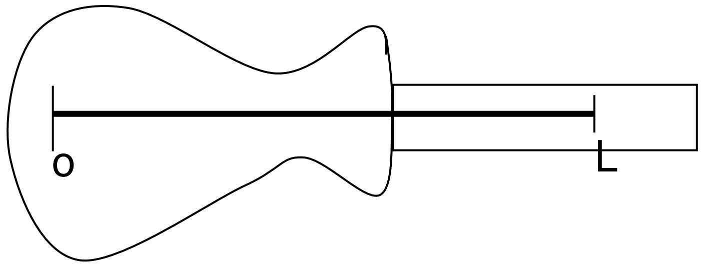
e. $2L$, $2L/2$, $2L/3$, $2L/4$, ….

Concept Check: Boundary Conditions

$y(x,t) = A\sin(kx)\cos(\omega t) + B\cos(kx)\sin(\omega t)$

Boundary conditions?

I. $y(x,t) = 0$ at $x=0$ and $x=L$



At x=0: $y(x,t) = B\sin(\omega t) = 0$ → only works if B=0

$y(x,t) = A\sin(kx)\cos(\omega t)$

From BC at x = L, what are possible values for k?

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c. π/L

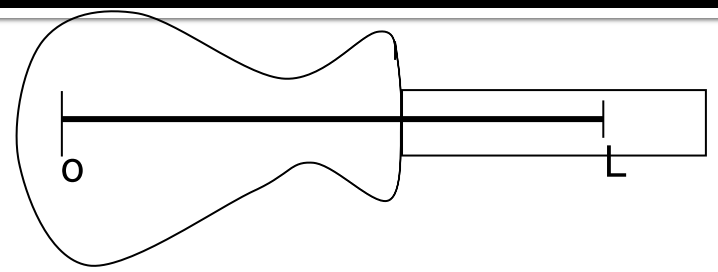
d. π/L, $2\pi/L$, $3\pi/L$, $4\pi/L$ …

e. $2L$, $2L/2$, $2L/3$, $2L/4$, ….

Boundary Conditions

Which boundary conditions need to be satisfied?

I. y(x,t) = 0 at x=0 and x=L



$$y(x,t) = A\sin(kx)\cos(\omega t) + B\cos(kx)\sin(\omega t)$$

At x=0:

$y = B\sin(\omega t) = 0$

→ B=0

At x=L:

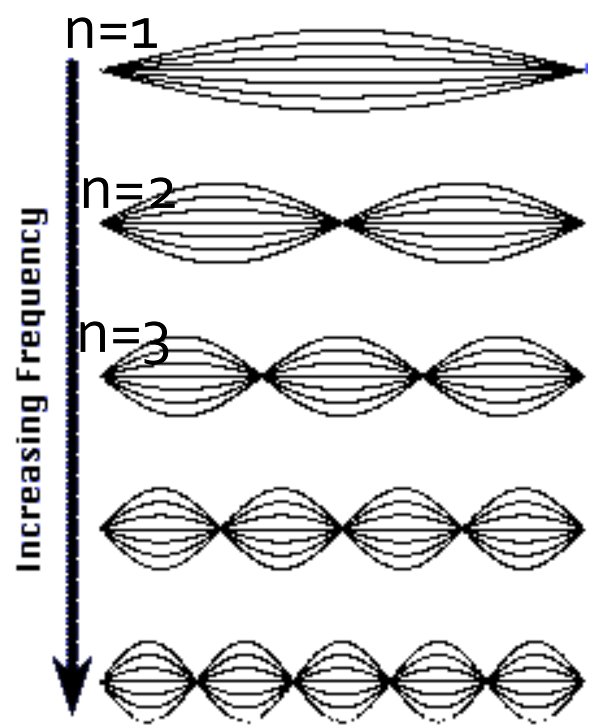
$y = A\sin(kL)\cos(\omega t) = 0$

→ $\sin(kL)=0$

→ $kL = n\pi$ (n=1,2,3, …)

→ $k=n\pi/L$

$$y(x,t) = A\sin(n\pi x/L)\cos(\omega t)$$

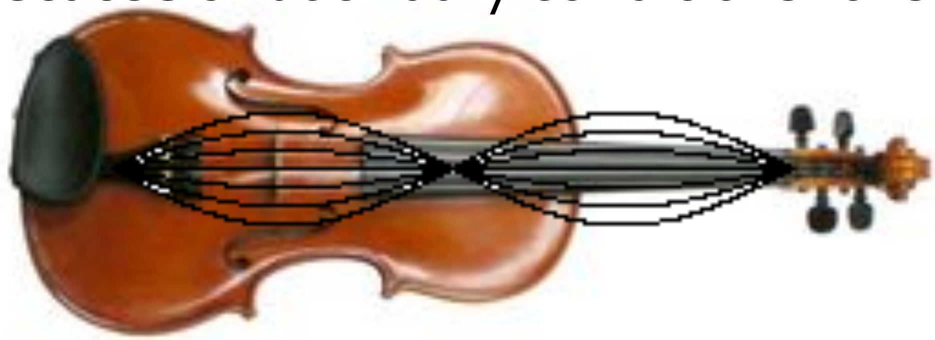


Origin of Quantization

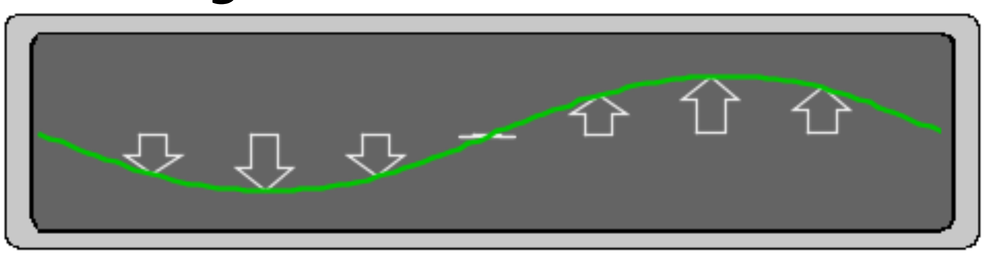
With Wave on Violin String:

Find: Only certain values of k (and thus λ) allowed

→ because of boundary conditions for solution



Same as for electromagnetic wave in microwave oven:



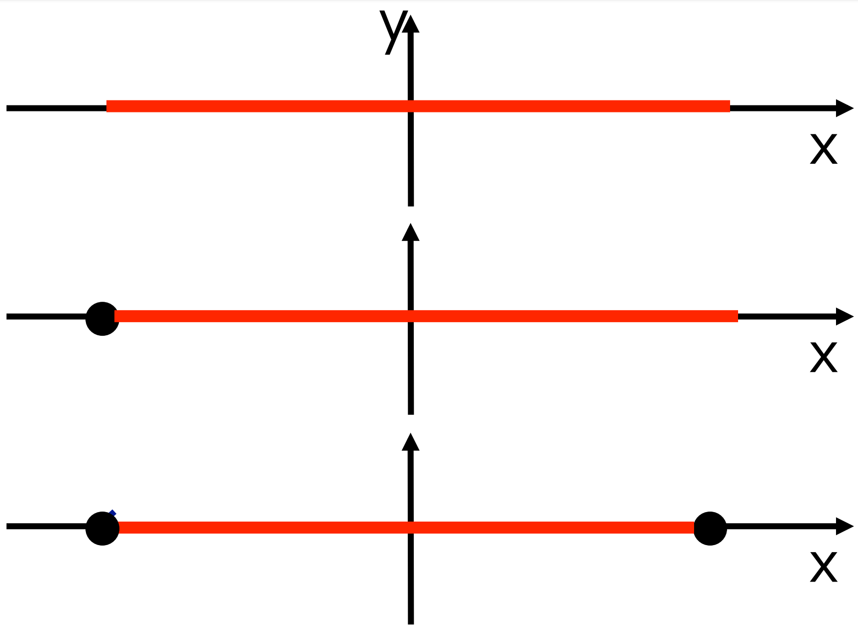
Exactly same for electrons in atoms:

Concept Check

Case I: no fixed ends

Case II: one fixed end

Case III, two fixed ends:



For which of these cases, do you expect to have only certain frequencies or wavelengths allowed… that is for which cases will the allowed frequencies be quantized.

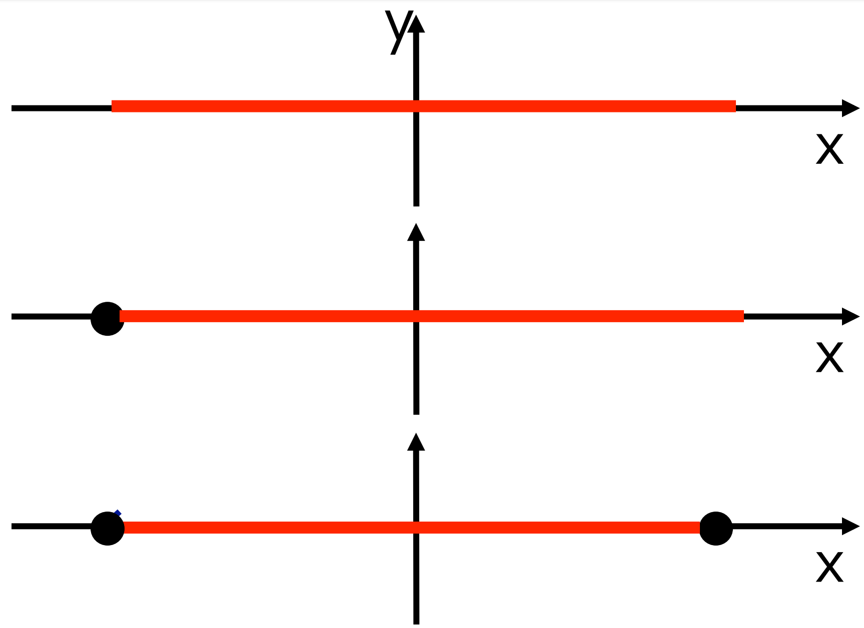
a. I only b. II only c. III only d. more than one

Concept Check

Case I: no fixed ends

Case II: one fixed end

Case III, two fixed ends:

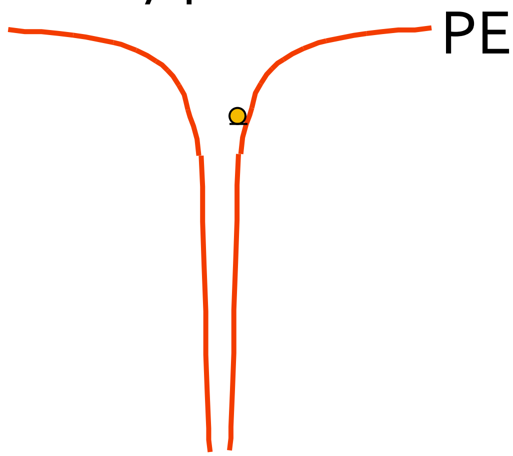


For which of these cases, do you expect to have only certain frequencies or wavelengths allowed… that is for which cases will the allowed frequencies be quantized.

a. I only   b. II only   c. III only   d. more than one

Quantum Mechanics

Electron bound
in atom (by potential energy)



Free electron

Only certain energies allowed
Quantized energies

Boundary Conditions
→ standing waves

Any energy allowed

No Boundary Conditions
→ traveling waves

Concept Check

- A confined electron m in a box has wave numbers $k = n\pi/L$. What are the allowed momenta and energy of the particle?

A. $p = nh/(2L)$, $E = nhc/(2L)$
B. $p = nh/(2L)$, $E = n^2h^2/(8m_eL^2)$
C. $p = hL/(n\pi)$, $E = hcL/(n\pi)$
D. $p = hL/(n\pi)$, $E = h^2L^2/(2n^2\pi^2)$

Concept Check

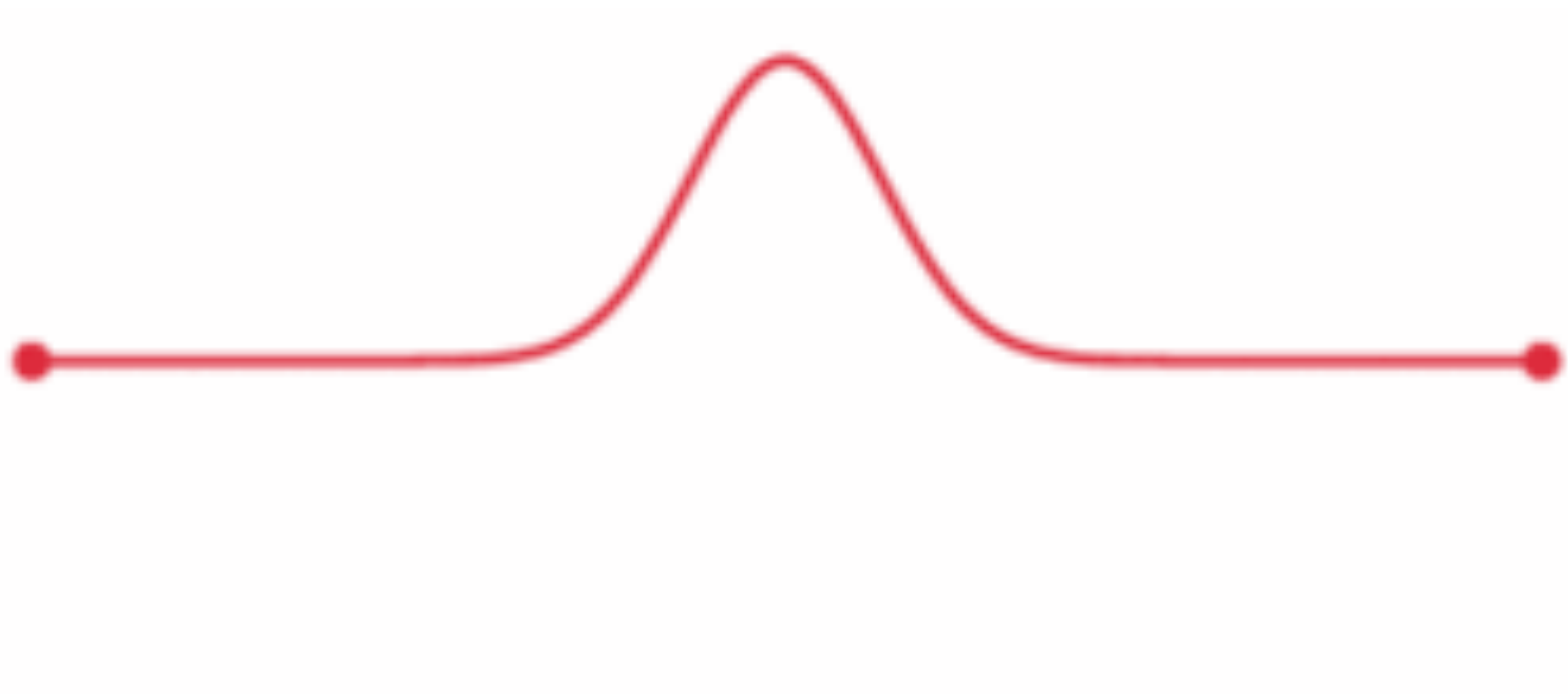
- A confined electron m in a box has wave numbers $k = n\pi/L$. What are the allowed momenta and energy of the particle?

A. $p = nh/(2L)$, $E = nhc/(2L)$
B. $p = nh/(2L)$, $E = n^2h^2/(8m_eL^2)$
C. $p = hL/(n\pi)$, $E = hcL/(n\pi)$
D. $p = hL/(n\pi)$, $E = h^2L^2/(2n^2\pi^2)$

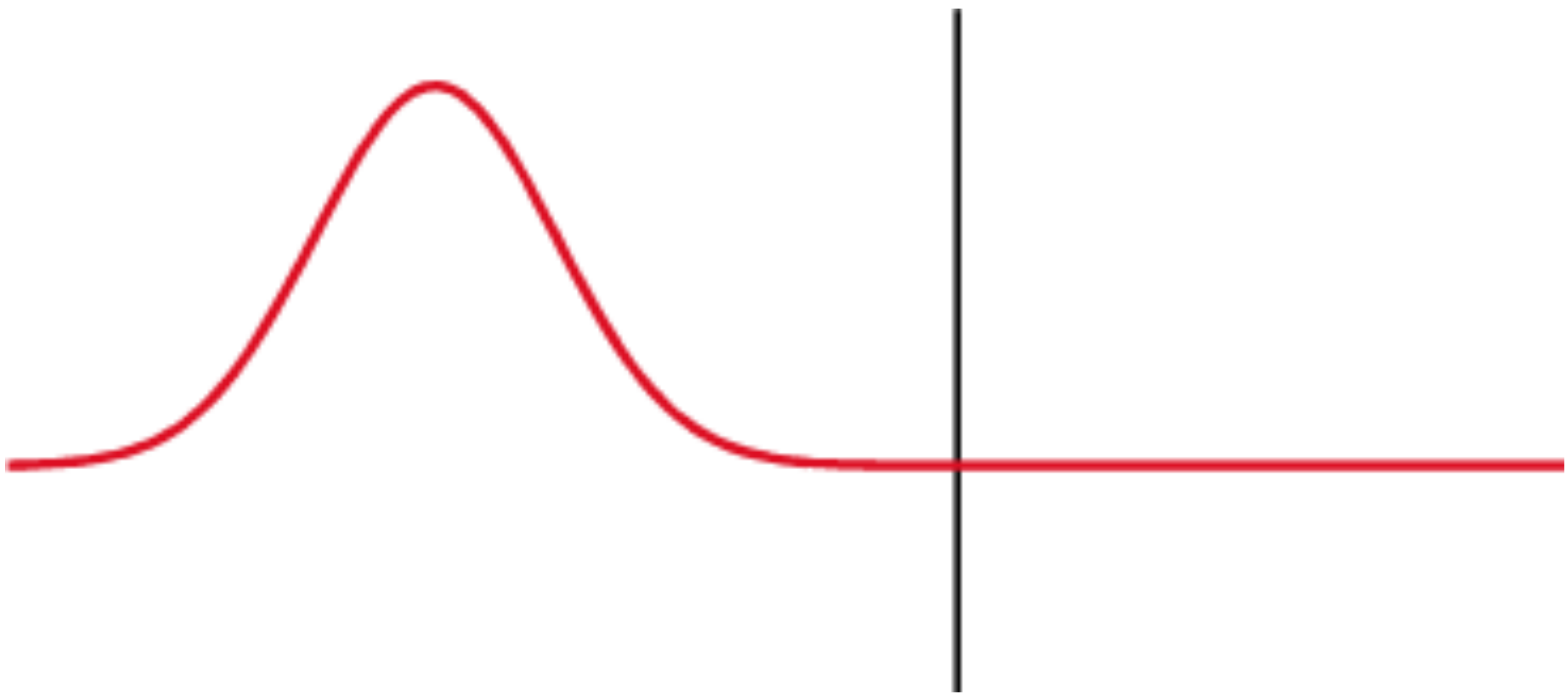
Continuity of Waves

- At a finite boundary, a wave has to be continuous in both value and slope
- At an infinite boundary, a wave can be discontinuous in slope but still must be continuous in value

Single Pulse, Infinite Boundary



Single Pulse, Finite Boundary



Wave Transmission/Reflection

Incident, Reflected, Total Transmitted