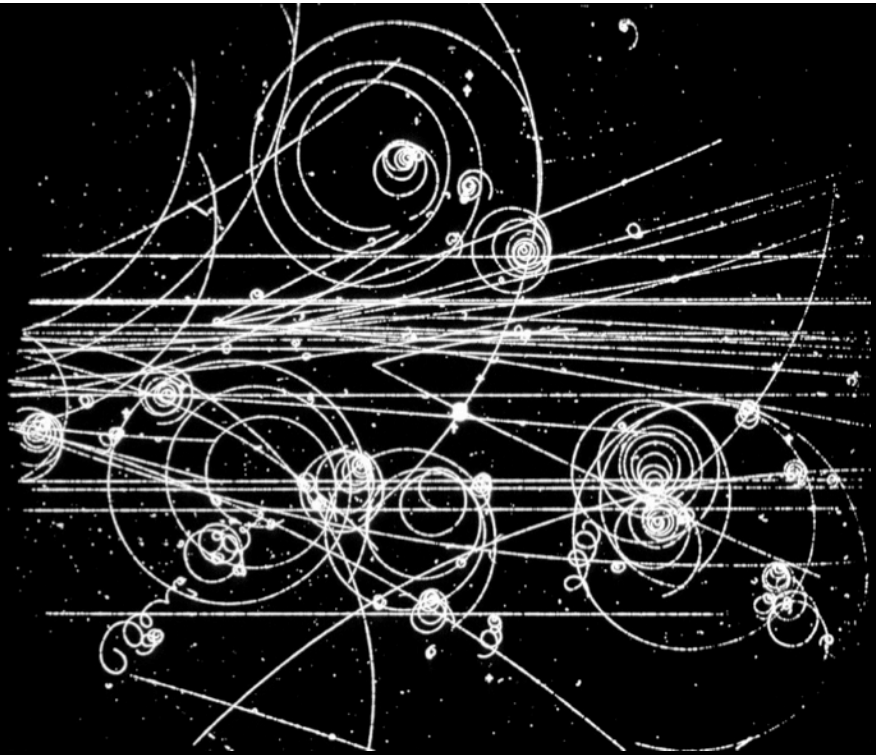
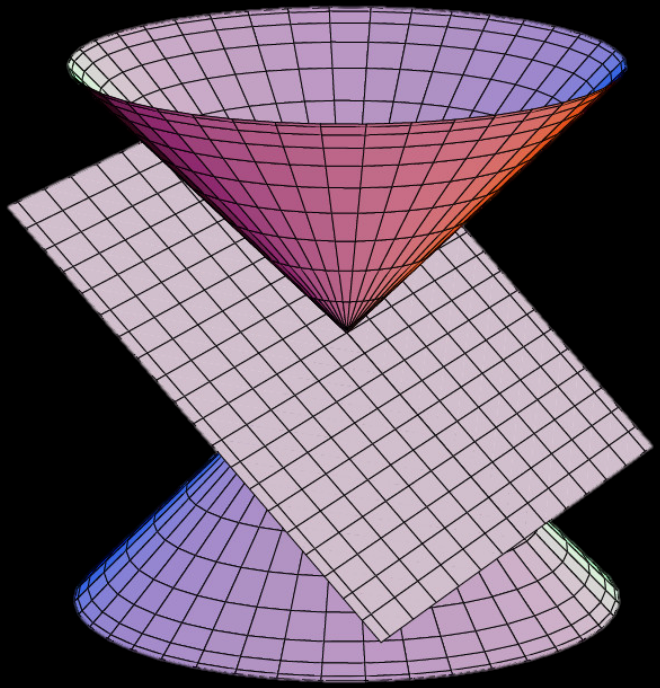
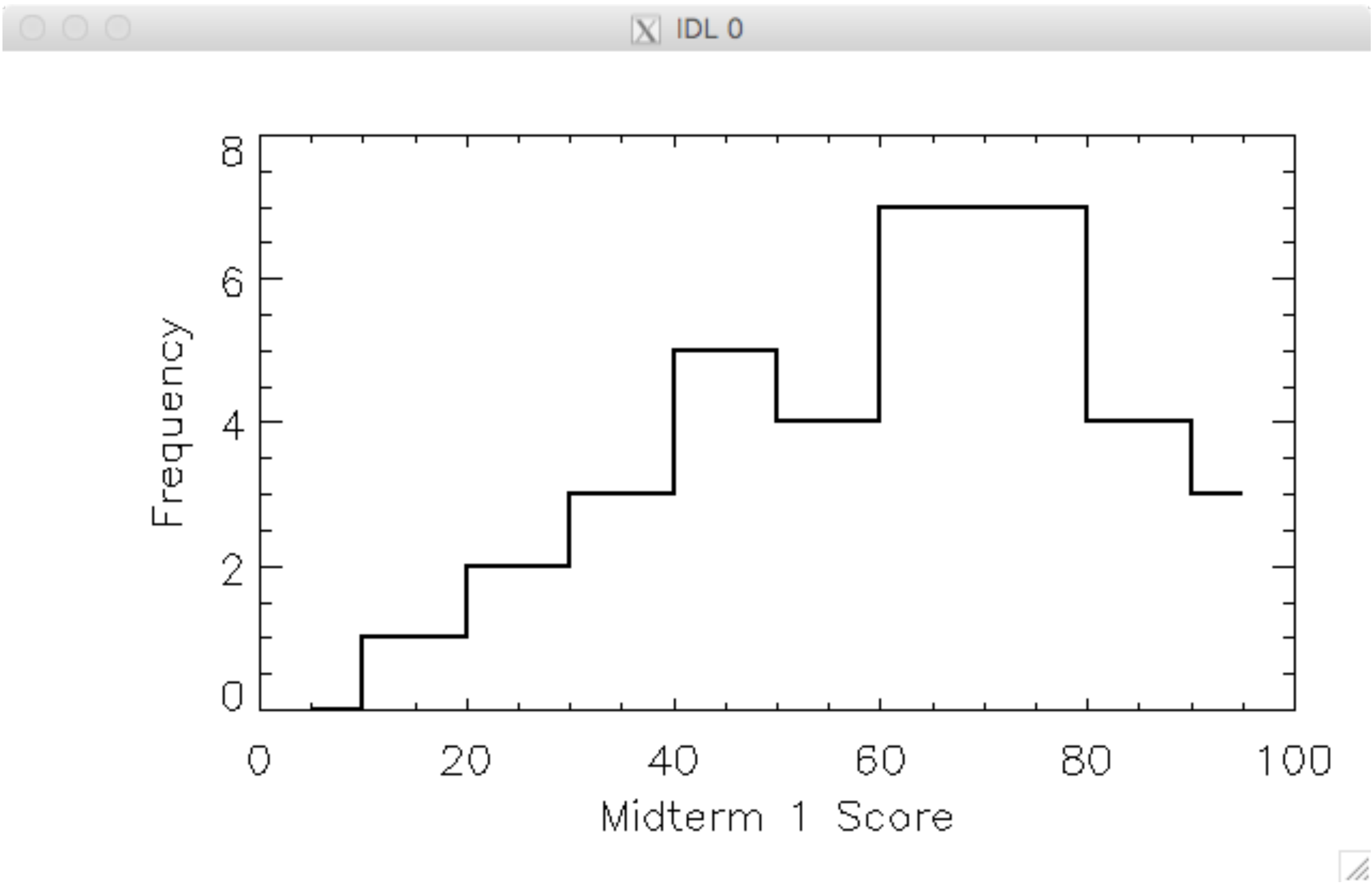




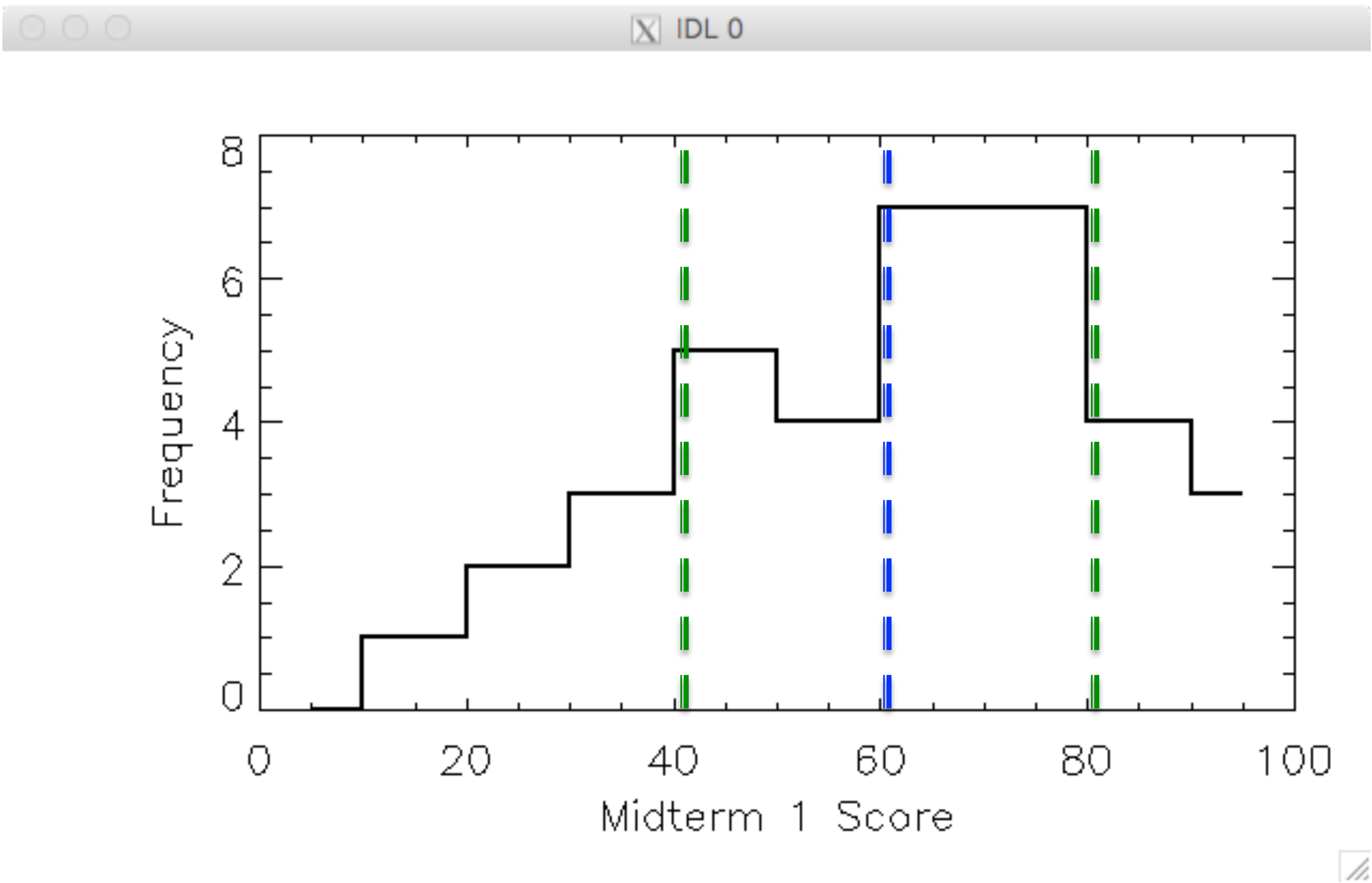
Modern Physics (Phys. IV): 2704

Professor Jasper Halekas
Van Allen 70
MWF 12:30-1:20 Lecture

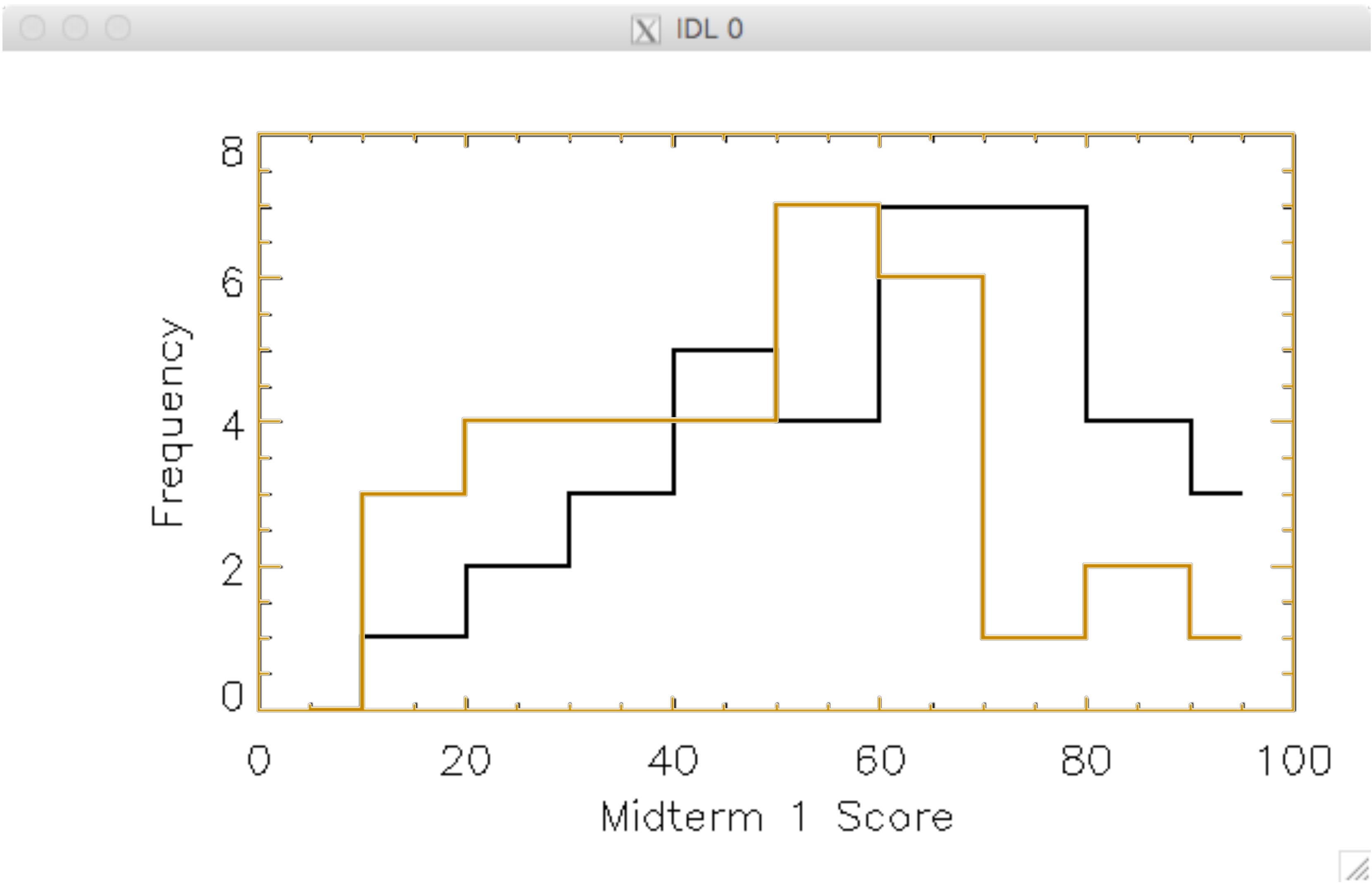
Midterm 1 Score Distribution



Midterm 1 Score Distribution



Comparison to Last Year



Q1: Relativistic Energy

$$\begin{aligned}\text{total energy E} &= \text{rest energy } + \text{KE} \\ &= (mc^2) \quad + \quad (\gamma - 1)\, mc^2 \\ &= \gamma mc^2\end{aligned}$$

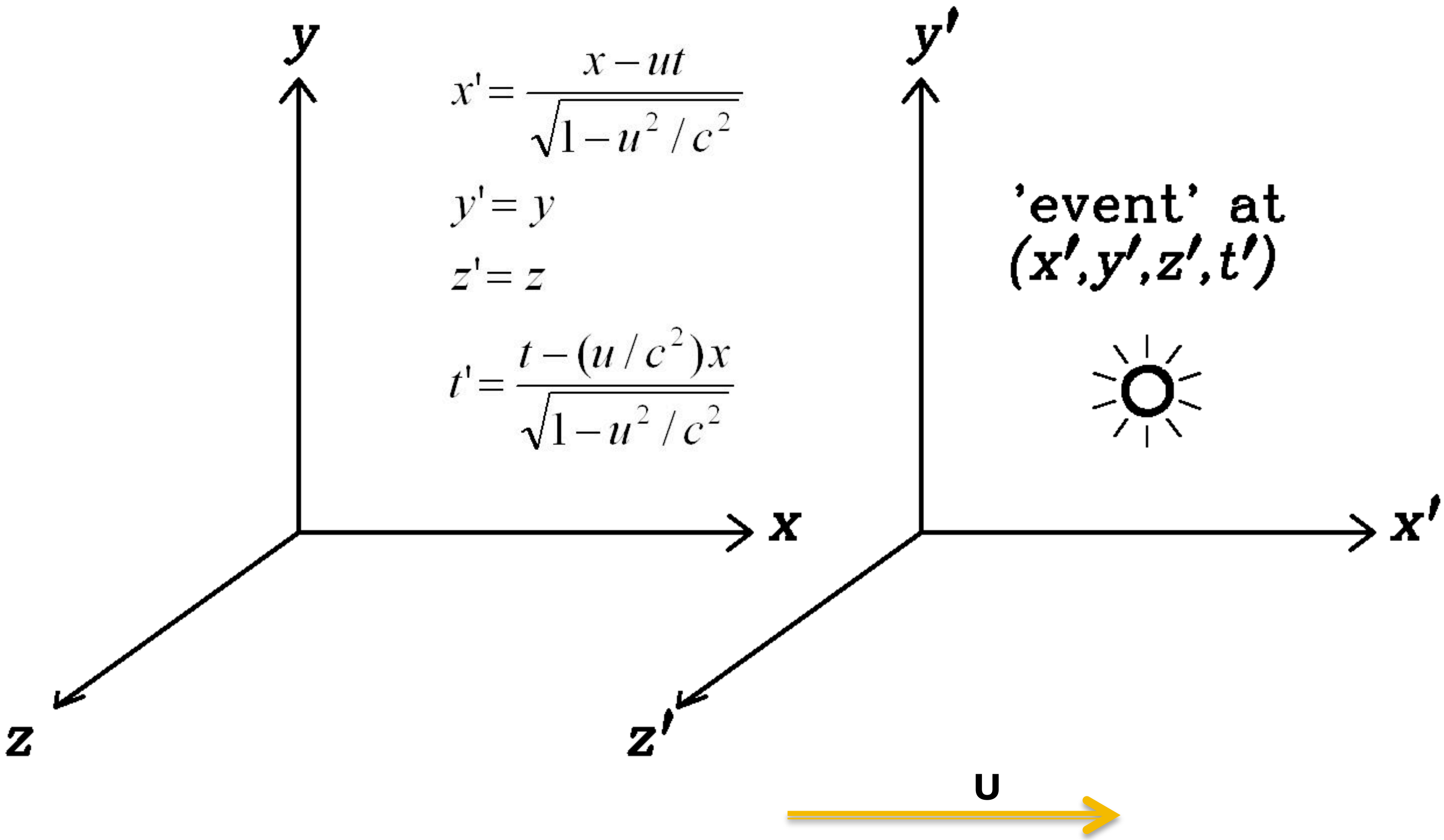
Q2: Lorentz Transformations

$$x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}$$

$$y' = y$$

$$z' = z$$

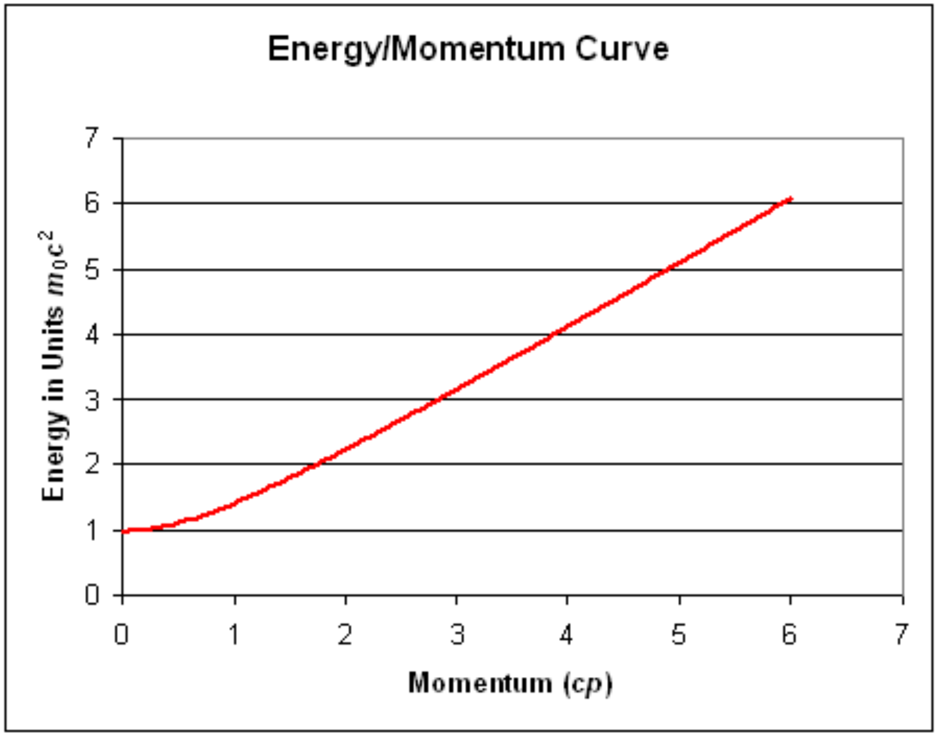
$$t' = \frac{t - (u/c^2)x}{\sqrt{1 - u^2/c^2}}$$



Q3: Conservation of Relativistic Energy and Momentum w/ Photons

$$E = h\nu = h\frac{c}{\lambda}$$

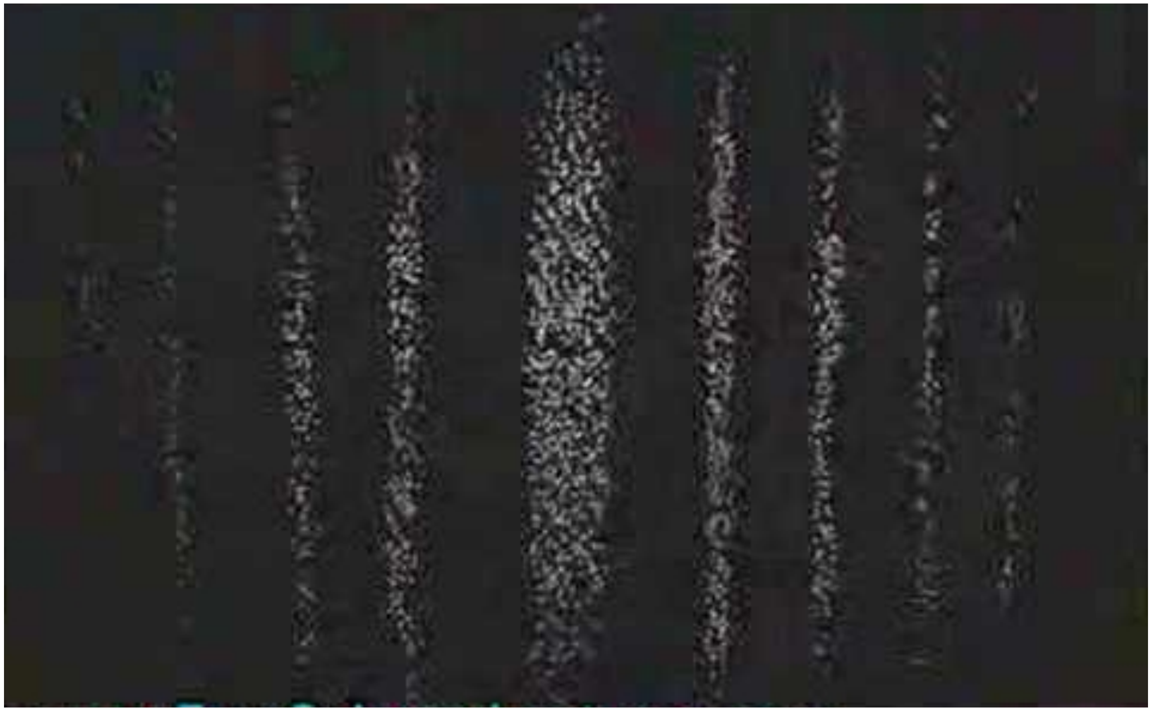
$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$



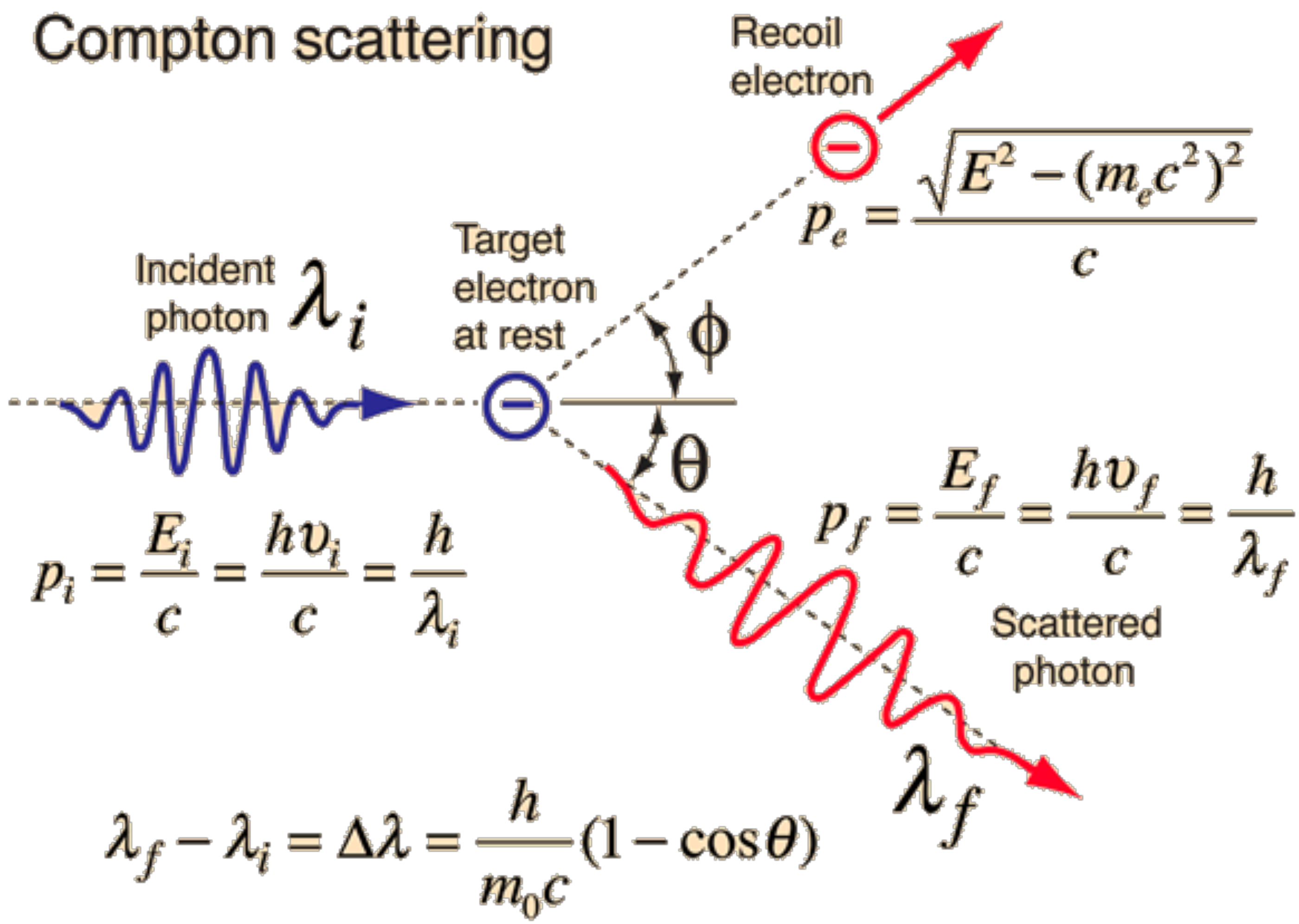
Q4: Electron Diffraction and de Broglie Wavelength

- The *de Broglie wavelength* of a particle is

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

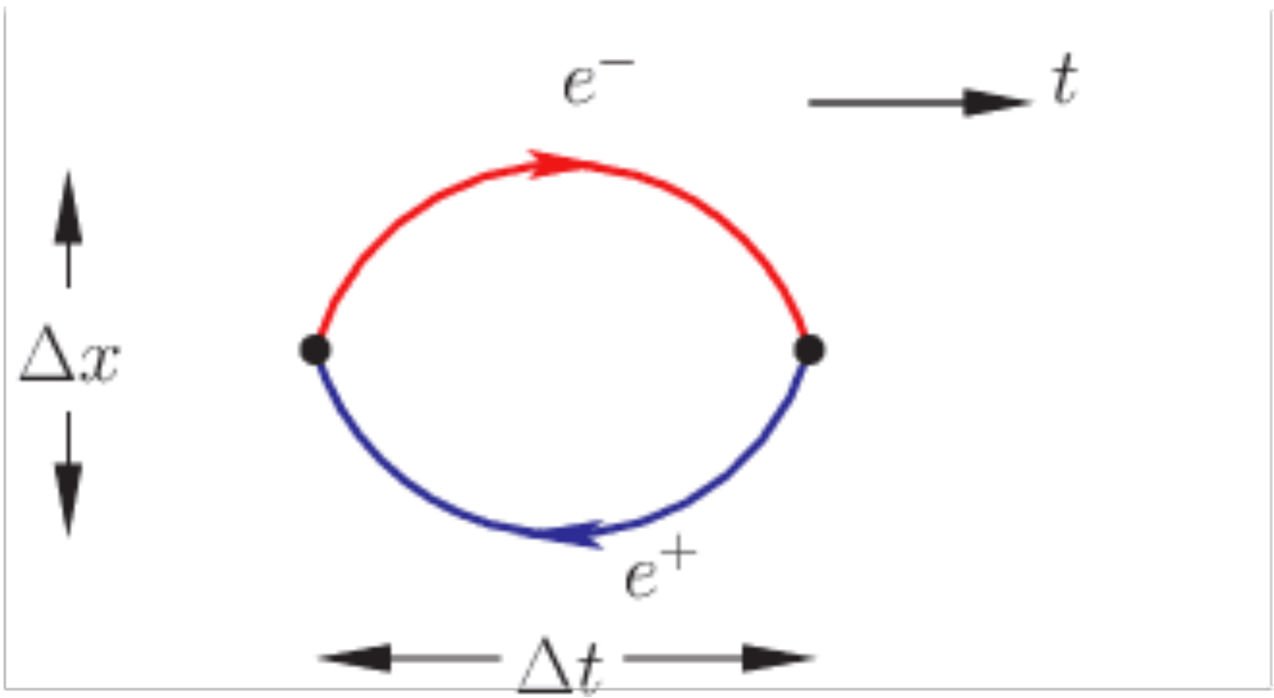


Q5: Compton Scattering

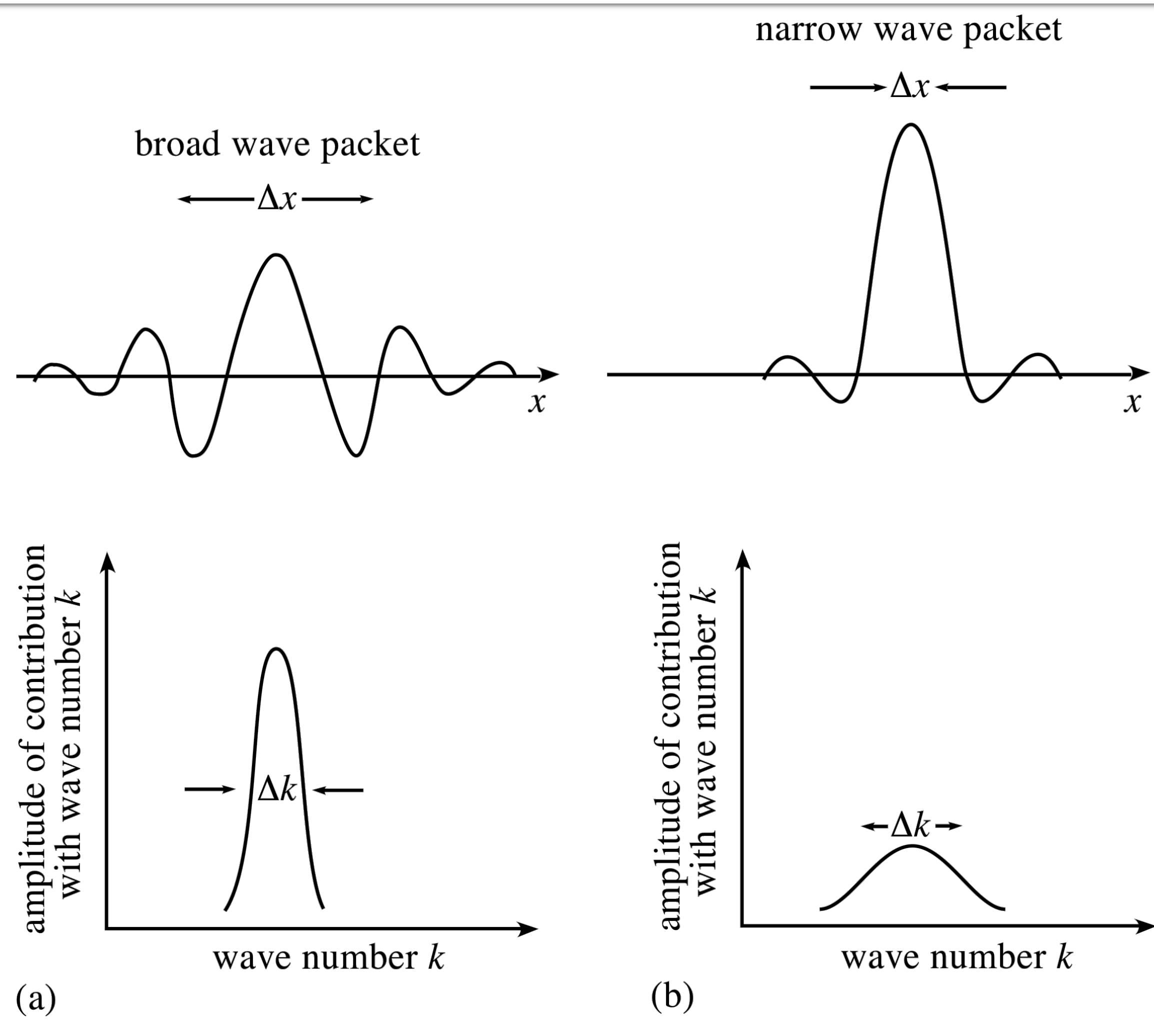


Q6: Uncertainty Principle

$$\Delta E \, \Delta t \;\geq \frac{1}{2}\,\hbar$$



Q7: Wave Packets



1. Total E conserved
Initial KE = 0, final KE > 0
some rest energy was converted to KE. So $\boxed{2m < M}$

2. a. $L' = L_0 = 10\,m$

$L = L'/\gamma = 6\,m$

$ct = cL/v = 7.5\,m$

so $\boxed{x_2 = 0,\ ct_2 = 7.5}$

b. $\boxed{x_2' = -10,\ ct_2' = 12.5}$

c. $\Delta s^2 = 7.5^2 = 56.25$

$\Delta s'^2 = 12.5^2 - 10^2 = 56.25$

$\Delta s'^2 = \Delta s^2$ (always true)

3. $E_i = 2E_\gamma$

$p_{xi} = E_\gamma/c \cos(60°) + E_\gamma/c \cos(60°)$

$= E_\gamma/c$

$p_{yi} = 0$

$E_f = 2E_\gamma,\ p_f = [E_\gamma/c,\ 0]$

$Mc^2 = \sqrt{(2E_\gamma)^2 - (p_f c)^2} = \sqrt{3E_\gamma^2}$

$\Rightarrow \boxed{M = \sqrt{3}\,E_\gamma/c^2}$

4. Faster electrons

$\Rightarrow$ more momentum

$\Rightarrow$ smaller wavelength

$\Rightarrow$ **decreased spacing**

5.a. $\Delta \nu = \Delta E / h$

$= \boxed{250{,}000\ e / h}$

6. $\Delta \lambda = .0024\ \text{nm}\ (1 - \cos\theta)$

$\lambda_i = 0.0024\ \text{nm}$

$\Rightarrow E_i = hc/\lambda_i = 1200/0.0024$

$= 500000\ \text{eV}$

$E_f = 250{,}000\ \text{eV}$

$\Rightarrow \lambda_f = .0048\ \text{nm}$

$\Delta \lambda = 0.0024\ \text{nm} \Rightarrow \cos\theta = 0$

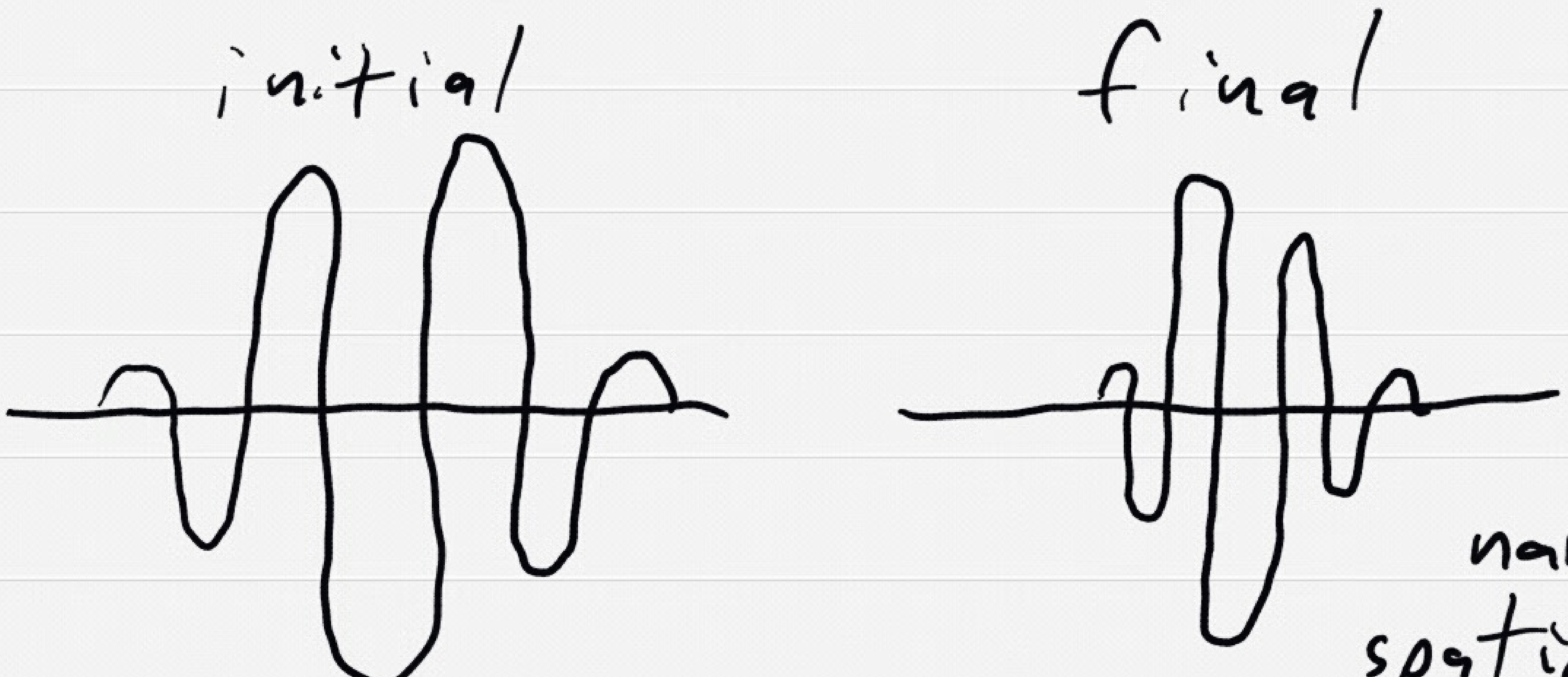
$\Rightarrow \boxed{\theta = 90^\circ}$

6. $\Delta E\, \Delta t \sim \hbar \sim 10^{-15}\ \text{eV s}$

$\Delta t \sim 10^{-15} / (2 \cdot 500000)$

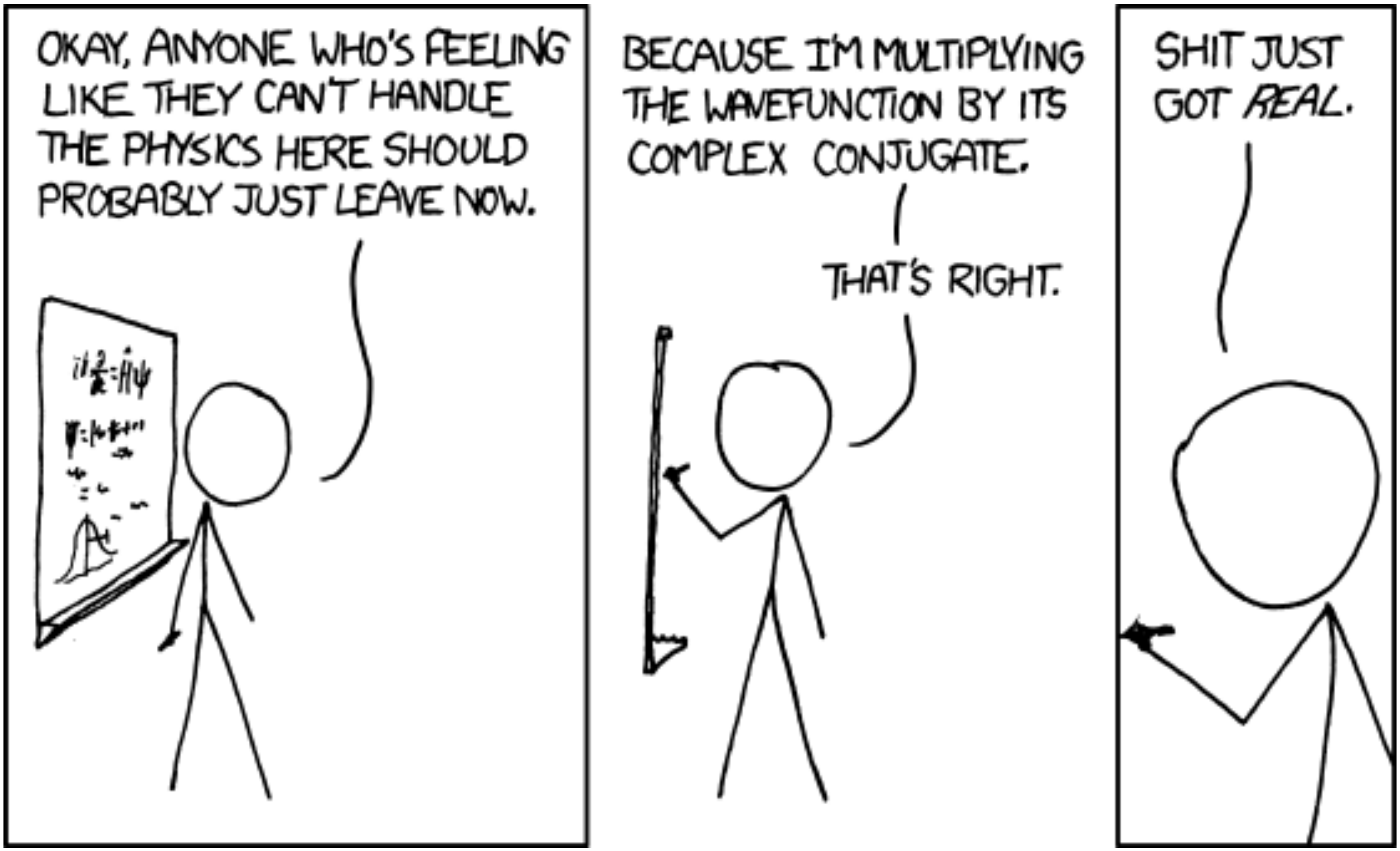
$= \boxed{10^{-21}\ \text{s}}$

7.



2X narrower spatial extent

Complex Numbers



Complex Math

$i = \sqrt{-1}$

$z = x + iy$

$z^* = x - iy$ (complex conjugate)

$$zz^* = x^2 + ixy - ixy - i^2y^2$$
$$= x^2 + y^2$$

$|z| = \sqrt{zz^*}$ (real)

$|z|^2 = [Re(z)]^2 + [Im(z)]^2$

$$e^x = 1 + x + x^2/2! + x^3/3! \cdots$$

$$e^{ix} = 1 + ix - x^2/2! - ix^3/3! \cdots$$

$$= 1 - x^2/2! + x^4/4! \cdots$$
$$+ i[x - x^3/3! + x^5/5!] \cdots$$

$$= \cos(x) + i\sin(x)$$

$$|e^{ikx}|^2 = e^{ikx} e^{-ikx} = e^0 = 1$$

also $$|e^{ikx}|^2 = (\cos kx + i\sin kx)(\cos kx - i\sin kx)$$
$$= \cos^2 kx + \sin^2 kx$$
$$= 1$$

$$e^{ikx} + e^{-ikx} = 2\cos kx$$

$$\Rightarrow \cos kx = (e^{ikx} + e^{-ikx})/2$$

$$e^{ikx} - e^{-ikx} = 2i\sin kx$$

$$\Rightarrow \sin kx = (e^{ikx} - e^{-ikx})/2i$$

$$e^{i(\theta_1+\theta_2)} = \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$$
$$= e^{i\theta_1} e^{i\theta_2} = (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$$

$$= \cos\theta_1\cos\theta_2 + i\sin\theta_1\cos\theta_2$$
$$+ i\sin\theta_2\cos\theta_1 + i^2\sin\theta_1\sin\theta_2$$

collecting terms

$$\cos(\theta_1+\theta_2) = \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$$
$$\sin(\theta_1+\theta_2) = \sin\theta_1\cos\theta_2 + \sin\theta_2\cos\theta_1$$

can always write $z = |z| e^{i\alpha}$

$$= |z| (\cos\alpha + i \sin\alpha)$$

$$= Re(z) + i Im(z)$$

$$|z|^2 = (Re(z))^2 + (Im(z))^2$$

$$\tan\alpha = Im(z) / Re(z)$$

Magnitude and phase

$|z|$ and α