

Three-body scattering in Poincaré invariant quantum mechanics

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Outline

- Motivation
- Poincaré invariant quantum mechanics
- Construction of the three-body dynamics
- Solving the Faddeev equations
- Results
- Observations

(T. Lin et. al. Phys. Rev. C76,014010(2007))

Motivation

- Extend NR QM so it is applicable at energy scales where sub-nucleon degrees of freedom may be relevant:

$$\Delta pc \geq \frac{\hbar c}{(\Delta x)} \gtrsim m_n c^2$$

- Poincaré invariance is required for a consistent treatment of dynamics.
- What are the observable consequences of Poincaré symmetry in the three-nucleon system?

Poincaré invariant quantum mechanics

(Wigner - 1939)

$$P = |\langle \Psi_f | \Psi_i \rangle|^2 = |\langle \Psi'_f | \Psi'_i \rangle|^2 = P'$$



$$|\Psi'\rangle = U(\Lambda, a)|\Psi\rangle$$



$$P^\mu, J^{\mu\nu}$$

Single-nucleon Hilbert space, $U(\Lambda, a) \rightarrow U_n(\Lambda, a)$

||

$(m = m_n, j = \frac{1}{2})$ irreducible representation space

$$P_n^\mu, J_n^{\mu\nu} \Rightarrow m, j, h_i, \Delta h_i \quad 1 \leq i \leq 4$$

$$\mathcal{H}_n = L^2(\sigma(h)) \quad |(m, j) \mathbf{h}\rangle$$

$$U_n(\Lambda, a) |(m, j) \mathbf{h}, \rangle = \int \mathcal{D}\mathbf{h}' |(m, j) \mathbf{h}', \rangle D_{\mathbf{h}'\mathbf{h}}^{mj}(\Lambda, a)$$

eg. $\mathbf{h} = (\mathbf{p}, \hat{\mathbf{z}} \cdot \mathbf{j}_c)$

N-nucleon Hilbert space

$$\mathcal{H} = \bigotimes_{i=1}^N \mathcal{H}_n$$

Tensor product basis

$$|(m_n, j_n), \mathbf{h}_1, \dots, (m_n, j_n), \mathbf{h}_N\rangle$$

Kinematic representation of the Poincaré group

$$U_0(\Lambda, a) := \bigotimes_{i=1}^N U_n(\Lambda, a)$$

$$U_0(\Lambda, a) |(m_n, j_n), \mathbf{h}_1, \dots, (m_n, j_n), \mathbf{h}_N\rangle =$$

$$\int |(m_n, j_n), \mathbf{h}'_1, \dots, (m_n, j_n), \mathbf{h}'_N\rangle \prod_{i=1}^N d\mathbf{h}'_i D_{\mathbf{h}'_i \mathbf{h}_i}^{m_n j_n}(\Lambda, a)$$

This is reducible!

N free particle irreducible representations

Poincaré Clebsch-Gordan coefficients

$$|(m, j), \mathbf{h}; \mathbf{d}\rangle = \sum |(m_n, j_n), \mathbf{h}_1, \dots, (m_n, j_n), \mathbf{h}_N\rangle \langle CG \rangle$$

$$\bigotimes_{i=1}^N D_{\mathbf{h}'_i \mathbf{h}_i}^{m_n j_n}(\Lambda, a) \langle CG \rangle = \int_{\oplus m, j, d} \langle CG \rangle D_{\mathbf{h}' \mathbf{h}}^{mj}(\Lambda, a)$$

↓

$$U_0(\Lambda, a) |(m, j), \mathbf{h}; \mathbf{d}\rangle = \sum |(m, j), \mathbf{h}'; \mathbf{d}\rangle d\mathbf{h}' D_{\mathbf{h}' \mathbf{h}}^{mj}(\Lambda, a)$$

Dynamics

$$M = M_0 + V \geq 0$$

$$[V, j^2] = [V, \mathbf{h}] = [V, \Delta \mathbf{h}] = 0$$

Diagonalize M in free particle irreducible basis, $| (m, j), \mathbf{h}, \mathbf{d} \rangle$



Simultaneous eigenstates of M, j^2, \mathbf{h}

$$| (\lambda, j), \mathbf{h}, \mathbf{d}_I \rangle$$

(Bakamjian & Thomas - 1953)

Dynamical representation of the Poincaré group

$$\{ |(\lambda, j), \mathbf{h}, \mathbf{d}_I\rangle \}$$

complete and

$$U(\Lambda, a) |(\lambda, j), \mathbf{h}, \mathbf{d}_I\rangle = \int d\mathbf{h}' D_{\mathbf{h}'\mathbf{h}}^{\lambda j}(\Lambda, a) |(\lambda, j), \mathbf{h}', \mathbf{d}_I\rangle$$

$U(\Lambda, a) =$ solution of dynamical problem

Scattering Theory

$$S = \Omega_+^\dagger(H, H_0)\Omega_-(H, H_0)$$

$$\Omega_\pm(H, H_0) = \Omega_\pm(M, M_0) = \Omega_\pm(M^2, M_0^2)$$

(Kato - Birman - 1966)

$$\bar{M} := WMW^\dagger \quad \bar{M}_0 := M_0 \quad WW^\dagger = I$$

$$S(M, M_0) = S(\bar{M}, M_0)$$



$$\lim_{t \rightarrow \pm\infty} \|(W - I)e^{-iM_0 t}|\psi\rangle\| = 0$$

(Ekstein - 1960)

$$N = 2$$

$$V = 4m_n V_{nr} \quad M_0^2 = 4(\mathbf{k}^2 + m_n^2) \quad M^2 = M_0^2 + V = 4m_n(H_{nr} + m_n)$$

$$\frac{d\sigma}{d\Omega}(k) = \frac{(2\pi)^4}{v_{nr}} |\langle f | T_{H_{nr}} | i \rangle|^2 k^2 \frac{dk}{dE_{nr}} = \frac{(2\pi)^4}{v_r} |\langle f | T_M | i \rangle|^2 k^2 \frac{dk}{dE_r}$$

No relativistic corrections for $N = 2$! ($k_{nr} = k_r$)

No loss of generality in choosing $V = 4m_n V_{nr}$!

(Coester, Pieper & Serduke - 1975)

$$N = 3$$

Structure of M dictated by two-body M and cluster properties

$$M = W(V)\bar{M}(V)W^\dagger(V) \quad \bar{M}(V) = M_0 + V_{12} + V_{23} + V_{31}$$

$$V_{12} = \sqrt{M_{12}^2 + q_3^2} - \sqrt{M_{012}^2 + q_3^2} \quad \dots$$

$$M_{12}^2 = 4(\mathbf{k}^2 + m_n^2) + 4m_n V_{NN} \quad M_{012}^2 = 4(\mathbf{k}^2 + m_n^2)$$

$W(V)$ not required to calculate S !

(Coester 1965, Sokolov 1977, F.C & W.P. 1982)

Predictions

Binding energies

$$M(V)|\Psi\rangle = \lambda|\Psi\rangle$$

$$|\Psi\rangle = W(V)|\bar{\Psi}\rangle \quad \bar{M}(V)|\bar{\Psi}\rangle = \lambda|\bar{\Psi}\rangle$$

Cross sections

$$S = \langle\Psi_f^+|\Psi_i^i\rangle = \langle\bar{\Psi}_f^+|\bar{\Psi}_i^i\rangle$$

Current matrix elements

$$\langle\Psi_f|J^\nu(0)|\Psi_i\rangle = \langle\bar{\Psi}_f|W^\dagger(V)J^\nu(0)W(V)|\bar{\Psi}_i\rangle$$

Faddeev Equations

$$T^{ab}(z) = \bar{\delta}^{ab}(z - M_0) + \sum_{c \neq a} V_c(z - M_0 - V_c)^{-1} T^{cb}(z)$$

$$a, b, c \in \{(12)(3), (23)(1), (31)(2)\}$$

Kernel

$$\int' \langle a|c' \rangle dc' \langle c'| V_c(z - M_0 - V_c)^{-1} |c \rangle$$

Solution by direct integration - no partial waves !

(Liu, Elster, Glöckle 2005)

Technical considerations

$\langle a|c\rangle =$ Poincaré Group Racah coefficient

(Changes order of coupling irreducible representations.)

$$\langle a|c\rangle = \delta[\mathbf{h}; \mathbf{h}'] \delta(M - M') \delta_{JJ'} R^{MJ}(\mathbf{d}_a, \mathbf{d}_c)$$

Become NR permutation operators in static limit

Technical considerations

$$z = z_c$$

$$\langle c' | V_c(z_c - M_0 - V_c)^{-1} | c \rangle =$$

$$\frac{4m_n}{m'_c + m_c} \langle c' | t_{NN}(z_c)(z_c - M_0)^{-1} | c \rangle$$

NR half-shell transition matrix elements can be used to directly construct half-shell Faddeev kernel

Technical considerations

$$z \neq z_c$$

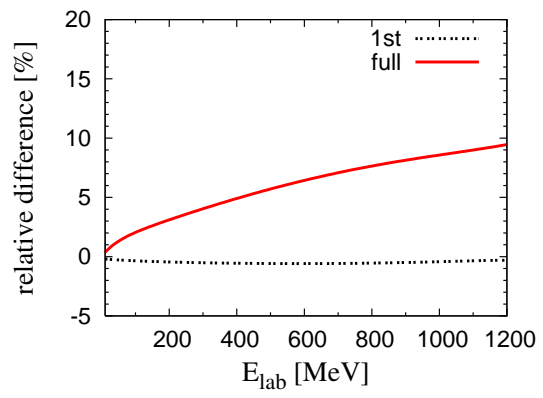
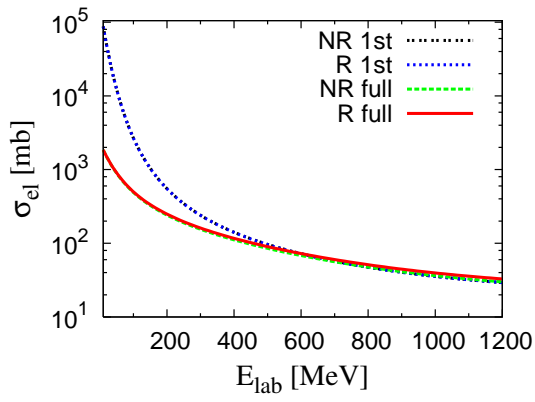
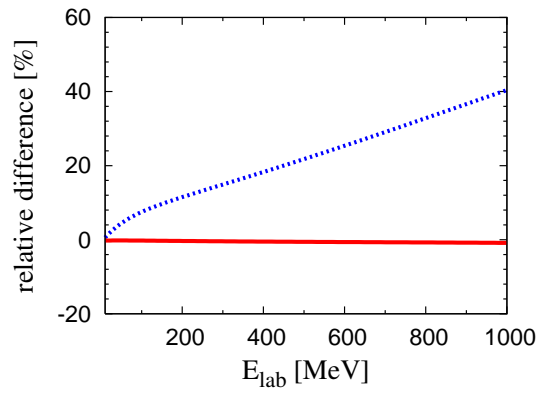
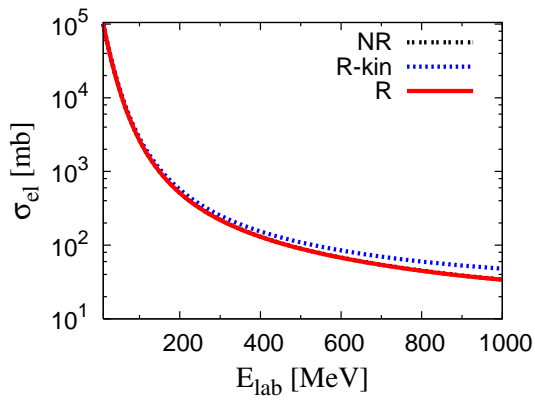
First resolvent equation gives fully off-shell Faddeev kernel

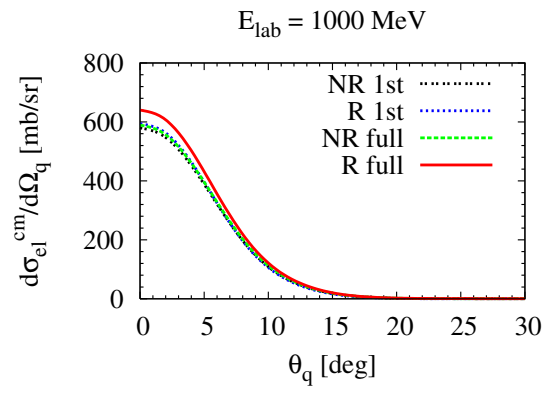
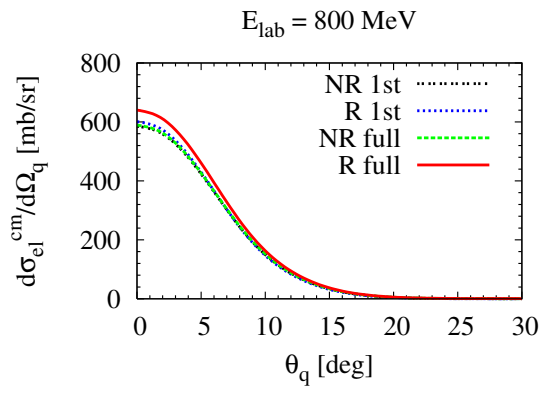
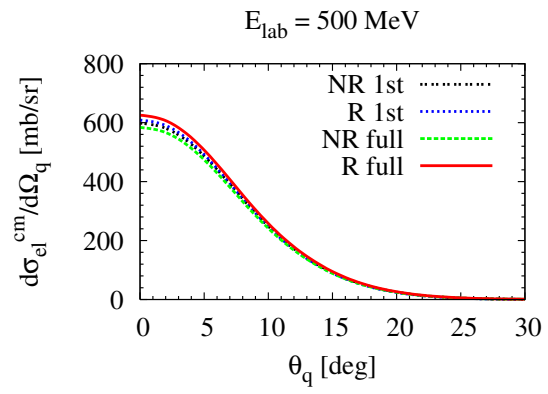
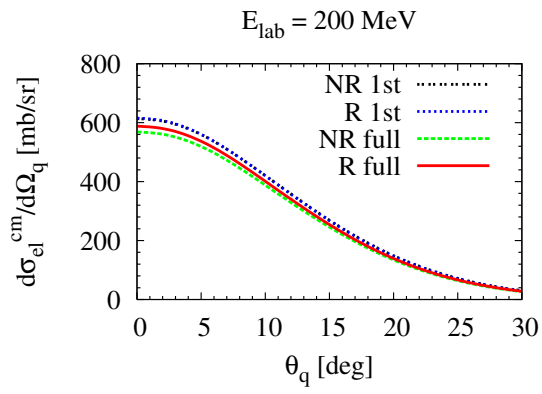
$$T_c(z) = T_c(z_c) + T_c(z) \frac{(z - z_c)}{(z - M_0)(z_c - M_0)} T_c(z_c)$$

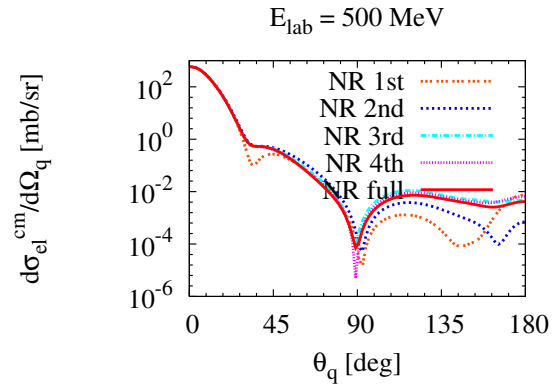
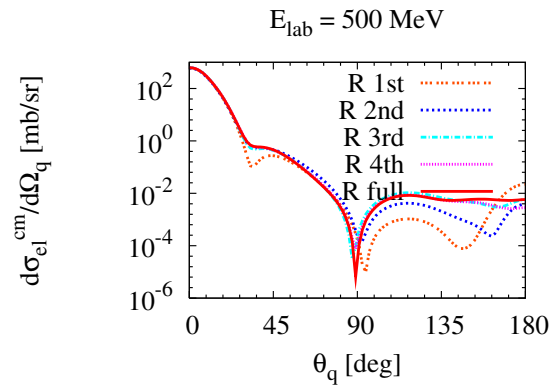
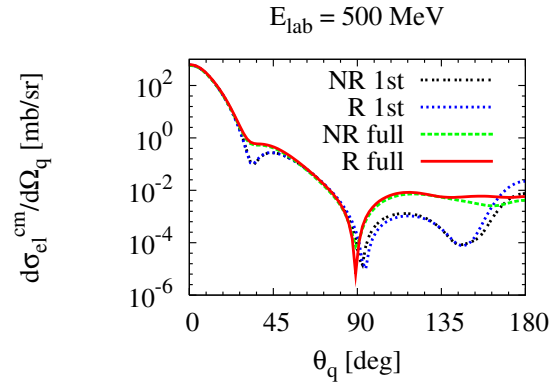
$$\langle c' | T_c(z_c) | c \rangle = \frac{4m_n}{m_c + m'_c} \langle c' | t_{NN}(z_c) | c \rangle$$

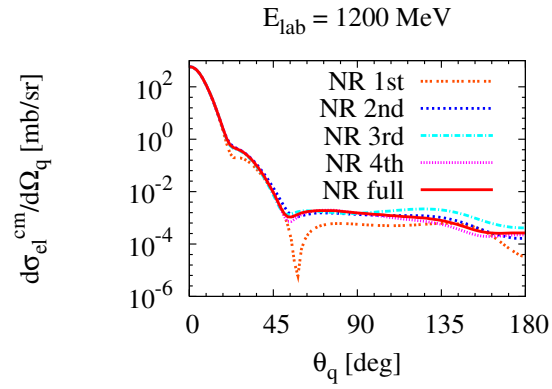
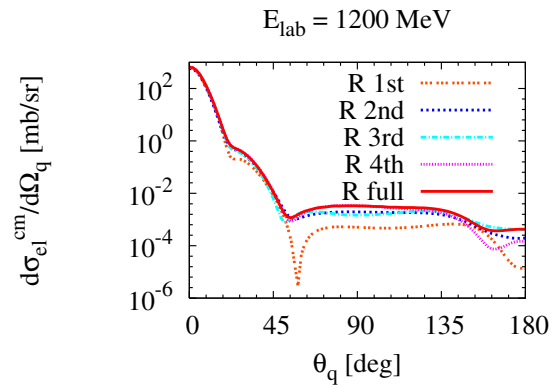
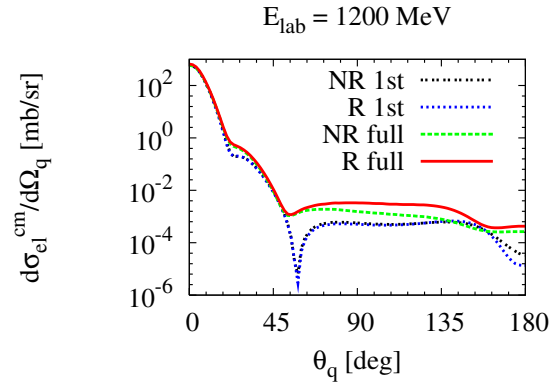
Input to Faddeev equations

(B. Keister and W.P. 2006)









$E_{lab} = 1000 \text{ MeV}, \theta_1 = 6 \text{ deg}, \text{Neumann}$

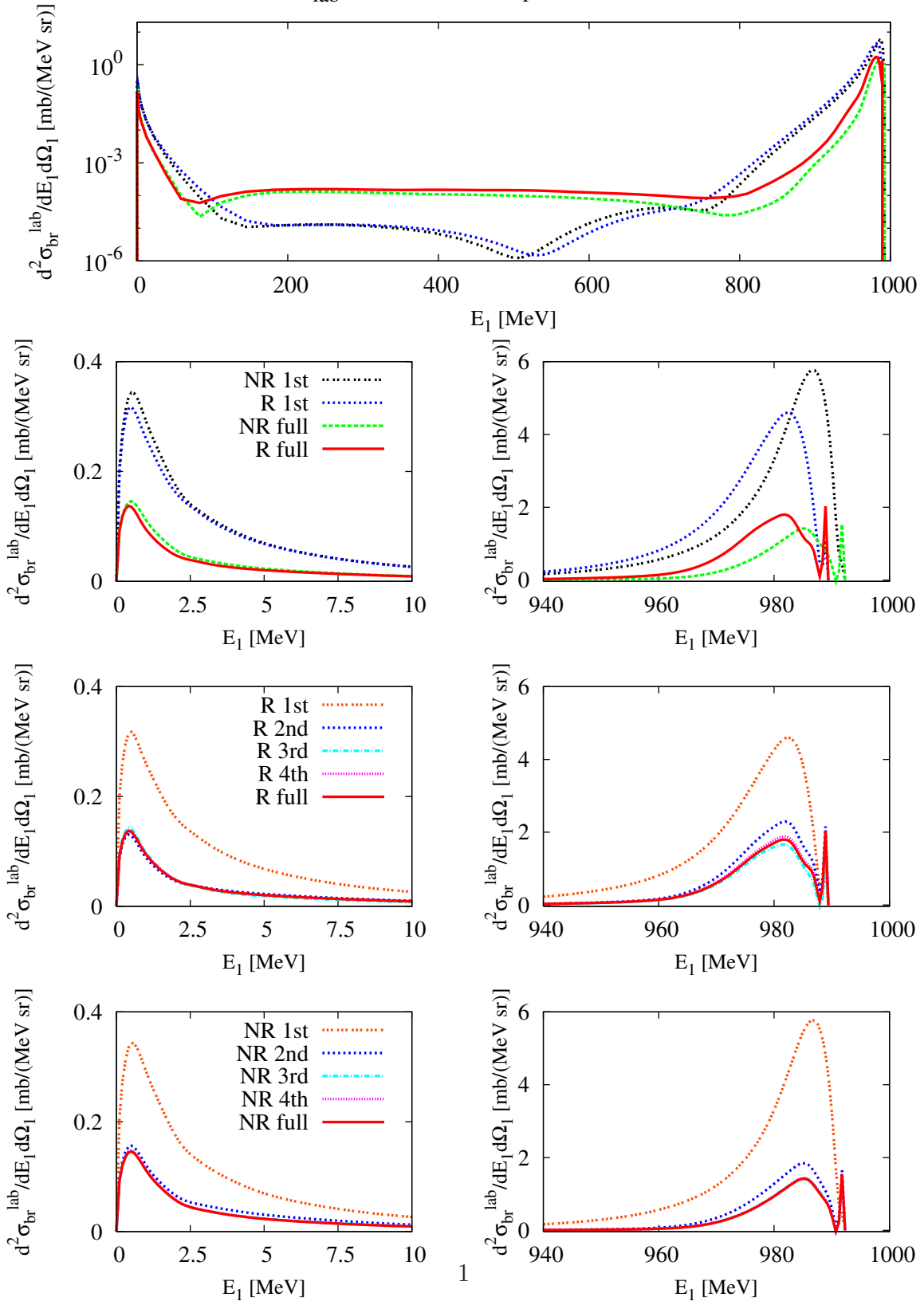


Figure 1: The incls differential cross section at projectile energy $E_{lab} = 1000\text{MeV}$ and $\theta_1 = 6^\circ$. Neumann

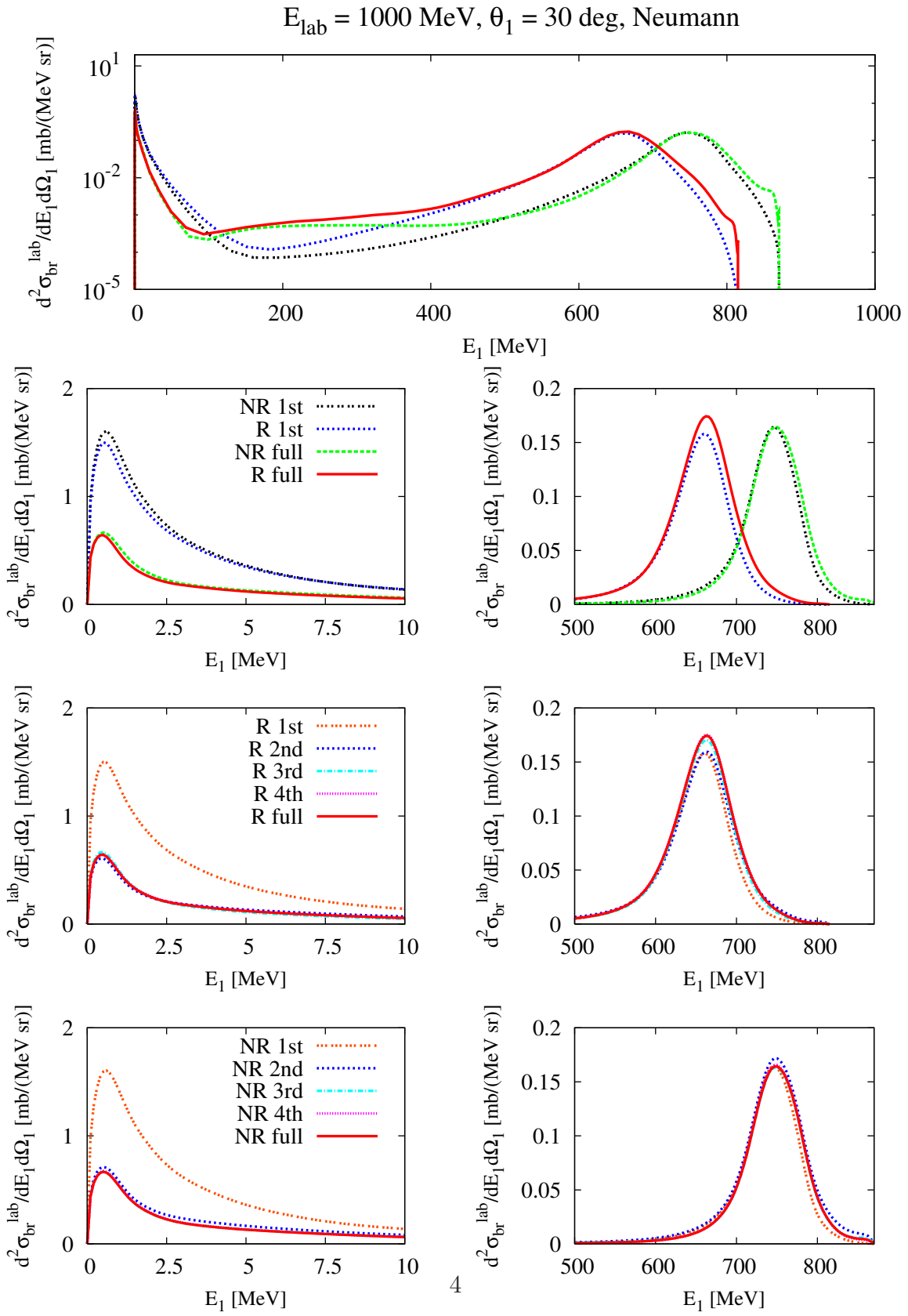


Figure 4: The incls differential cross section at projectile energy $E_{lab} = 1000\text{MeV}$ and $\theta_1 = 30^\circ$. Neumann

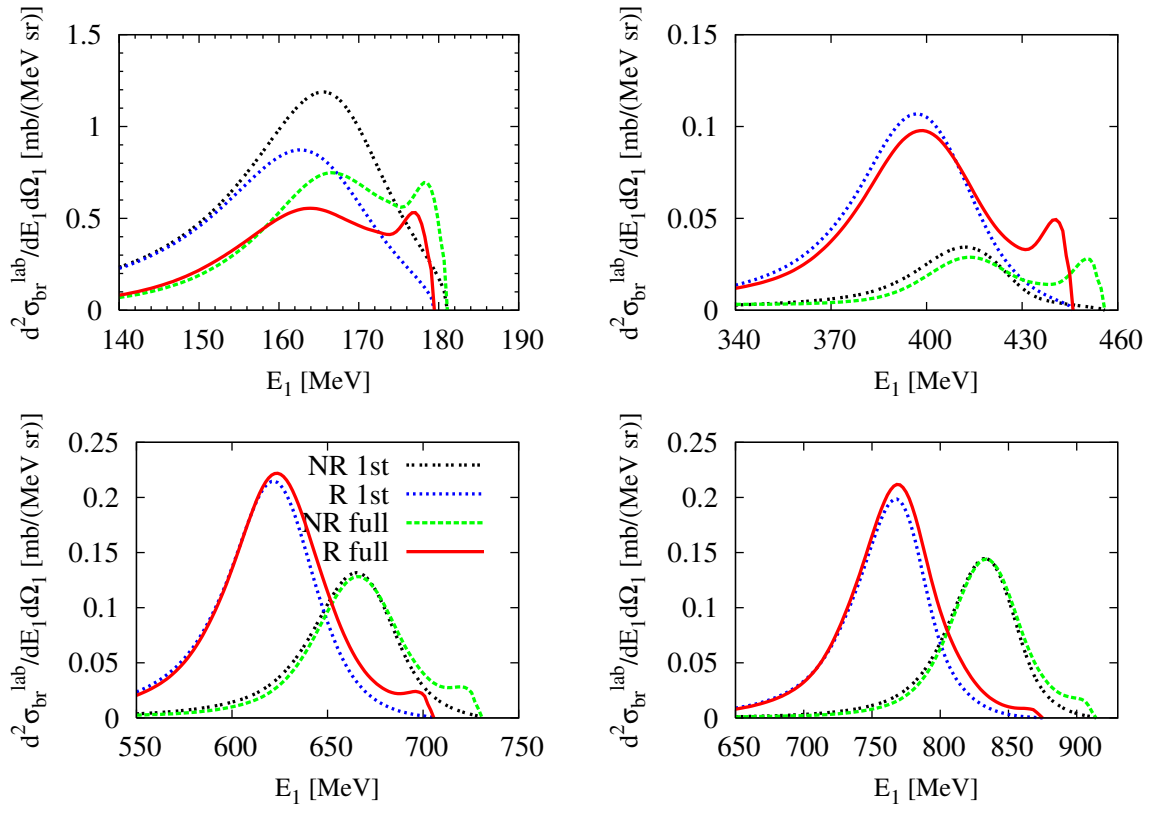
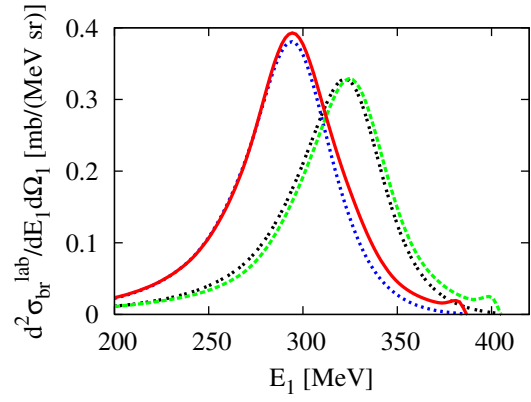
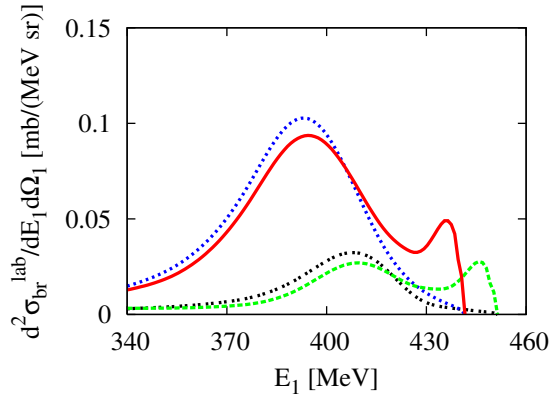
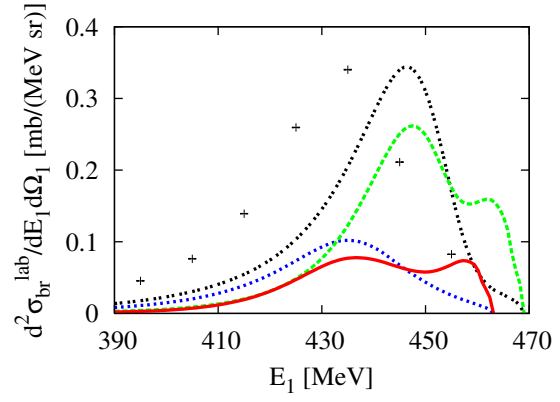
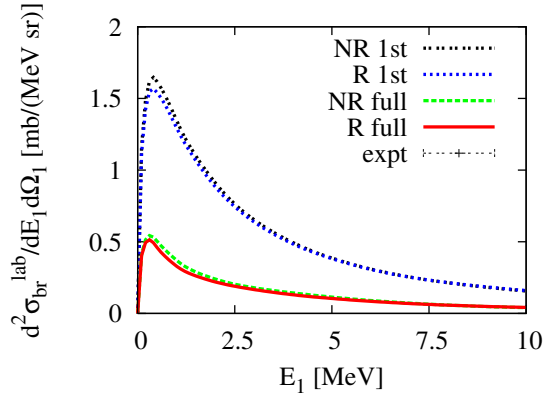


Figure 1: The quasifree peak of inclusive breakup scattering at $\theta_1 = 24^\circ$. From left to right, the corresponding projectile energies are: upper panel 200 and 500 MeV; lower panel 800 and 1000 MeV.

$$E_{lab} = 495 \text{ MeV}$$



Observations

- Differences in the relativistic and non-relativistic calculations with same two-body input appear (1) in the Poincaré Racah coefficients and (2) in the embedding of the two-body interaction in the three-body mass operator.
- The computational method is stable beyond 1 GeV.
- High precision NN interactions can be used as input **without** modification. Calculations shown are with a Malfliet-Tjon potential.
- With the same two-body input there are measurable differences in the NR and relativistic three-body cross sections. In some parts of phase space there are differences even at 200 MeV.
- Watson series requires 3-4 iterations for convergence near quasi-elastic peak at 1 GeV for forward angles.
- Kinematic effects of relativity are largely canceled by dynamical effects.

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