Scattering Equivalences in Nuclear Physics

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Outline

- **H. Ekstein - Physical Review 117,1590(1960)**
- **What are scattering equivalences?**
- **Nuclear matter**
- **Three-body forces**
- **Electromagnetic currents**
- **Relativistic quantum mechanics**
- **Cluster properties**
- **Implications**

Ekstein: To what extent does the S-matrix determine the Hamiltonian?

H. Ekstein - Physical Review 117,1590(1960)

Scattering 101

$$S(H, I, H_0) := \Omega_+^\dagger(H, I, H_0)\Omega_-(H, I, H_0)$$

$$\Omega_\pm(H, I, H_0) := s - \lim_{t\to\pm\infty} e^{iHt} I e^{-iH_0 t}$$

Let W be unitary and define:

$$H' := WHW^\dagger.$$

When is

$$S' := S(H', I, H_0) = S(H, I, H_0) =: S$$

?

$$S' = \Omega_+^\dagger(H', I, H_0)\Omega_-(H', I, H_0)$$

$$= \Omega_+^\dagger(H, W^\dagger, H_0)W^\dagger W\Omega_-(H, W^\dagger, H_0)$$

$$= \Omega_+^\dagger(H, W^\dagger, H_0)\Omega_-(H, W^\dagger, H_0).$$

A sufficient condition for S' to equal S is

$$\Omega_\pm(H, W^\dagger, H_0) = \Omega_\pm(H, I, H_0)$$

$$\Updownarrow$$

$$0 = \lim_{t\to\pm\infty} \|e^{iHt}(I - W^\dagger)e^{-iH_0t}|\psi\rangle\| = \lim_{t\to\pm\infty} \|(W - I)e^{-iH_0t}|\psi\rangle\|.$$

Unitary operators W satisfying this condition are called scattering equivalences.

Conversely assume $S = S'$

$$\Omega_+^\dagger(H, I, H_0)\Omega_-(H, I, H_0) = S = S' = \Omega_+^\dagger(H', I, H_0)\Omega_-(H', I, H_0).$$

Define

$$W = \Omega_-(H', I, H_0)\Omega_-^\dagger(H, I, H_0) = \Omega_+(H', I, H_0)\Omega_+^\dagger(H, I, H_0).$$

Then

$$H'W = WH \qquad W\Omega_\pm(H, I, H_0) = \Omega_\pm(H', I, H_0)$$

$$\Downarrow$$

$$W\Omega_\pm(H, I, H_0) = W\Omega_\pm(H, W^\dagger, H_0)$$

$$\Downarrow$$

$$\lim_{t\to\pm\infty} \|(W - I)e^{-iH_0 t}|\psi\rangle\| = 0.$$

For scattering theories formulated in terms of wave operators

$$\lim_{t \to \pm\infty} \|(W - I)e^{-iH_0 t}|\psi\rangle\| = 0$$

$$\Updownarrow$$

$$S(H, I, H_0) = S(H', I, H_0).$$

What is significant is that H_0 does not change so the scattering asymptotic conditions and the description of free particles do not change.

This means that $H = H_0 + V$ and $H' = H_0 + V'$ are equivalent Hamiltonians.

Elementary examples:

$$W = \frac{I + iK}{I - iK} \qquad K = K^\dagger$$

$$\lim_{t \to \pm\infty} \|(W - I) e^{-iH_0 t} |\psi\rangle\| = 0$$

$$\Uparrow$$

$$\lim_{t \to \pm\infty} \|K e^{-iH_0 t} |\psi\rangle\| = 0$$

$$\Uparrow$$

$$\langle P, p | K | P', p' \rangle = \delta(P - P') \langle p | \bar{K} | p' \rangle \qquad \bar{K} \textbf{ compact}$$

or

$$\langle p_1, \cdots, p_n | K | p_n', \cdots, p_1' \rangle = \delta(P_{ij} - P_{ij}') \langle p_{ij} | \bar{K} | p_{ij}' \rangle \prod_{k \neq \{i,j\}} \delta(p_k = p_k')$$

$\bar{K}$ **compact.**

Algebraic properties:

$$\mathcal{S} := \{\textbf{set of scattering equivalences}\}$$

$$I \in \mathcal{S}$$

$$W \in \mathcal{S} \Rightarrow W^{-1} \in \mathcal{S}$$

$$W_1, W_2 \in \mathcal{S} \Rightarrow W = W_1 W_2 \in \mathcal{S}$$

$\mathcal{S}$ is a group, also unitary elements of a $*$-algebra of asymptotic constants.

Essential "counter" example

Let H and H' be any two repulsive two-body potentials that lead to different phase shifts.

$$W = W_+ := \Omega_+(H', I, H_0)\Omega_+^\dagger(H, I, H_0)$$

$$WHW^\dagger = H'$$

however

$$S(H, I, H_0) \neq S(H', I, H_0)$$

because

$$W_+ \neq W_- = \Omega_-(H', I, H_0)\Omega_-^\dagger(H, I, H_0).$$

Application 1: Nuclear matter

Coester, Cohen, Day, Vincent, Phys. Rev. C. 1,769(1970).

Motivated by Ekstein's observation that two-body scattering and bound state observables do not uniquely fix the nucleon-nucleon interaction, CCDV suggested that nuclear matter calculations could be used to put additional constraints on the nucleon-nucleon interaction.

The paper considers models with no three-body interactions.

$$H = WHW^{\dagger}$$

$$\langle r'|W - I|r\rangle = -\{1 - \cos(\theta)[|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|]+$$
$$\sin(\theta)[|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|]\}$$

$$\Downarrow$$

CCDV observed a correlation between the binding energy and the density of nuclear matter for different choices of $|g_i\rangle$

$$\Downarrow$$

"Coester Line".

Application 2: Three-body forces
W.P. and W. Glöckle - Few-Body Systems, 9,97(1990).

$$H = \sum_i H_i + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk} + \cdots + V_{1,\cdots,N}$$

$$K = \sum_{ij} K_{ij} + \sum_{ijk} K_{ijk} + \cdots + K_{1,\cdots,N}$$

$$\|K_a\| < \infty \qquad K_a = K_a^\dagger$$

$$\lim_{t\to\pm\infty} \|K_a e^{-i\sum_j H_j t}|\psi\rangle\| = 0$$

Theorem:

$$H' = WHW^\dagger \qquad W = \frac{I - iK}{I + iK}$$

$$\Downarrow$$

$$\sigma(H) = \sigma(H') \qquad S(H) = S(H').$$

Can we use this freedom to eliminate three (many)-body interactions?

Induction on particle number:

$$N = 3$$

$$H = \sum_i H_i + \sum_{ij} V_{ij} + V_{123}$$

$$H' = W(K) H W^\dagger(K) = \sum_i H_i + \sum_{ij} V'_{ij}(K) + V'_{123}(K)$$

$$F[K] := \mathrm{Tr}\left[V'_{123}(K)^2\right]$$

$$F[K] = F[K]^* \geq 0 \qquad F[K_0] = 0 \Rightarrow V'_{123}(K_0) = 0$$

$$\frac{\delta}{\delta K} F[K] = 0 \qquad \frac{\delta^2}{\delta^2 K} F[K] > 0.$$

- **The problem of finding a minimal three-body interaction is a well-posed problem.**
- **It is not known if $V'_{123} = 0$ is a possible solution.**
- **This process can be continued by induction to recursively minimize all of the many body $(N \geq 3)$ interactions.**
- **R. Lazauskas and J. Carbonell - use non-local two-body interactions - do not need a three-body interaction. Phys. Rev. C70,044002(2004).**

Application 3: Electromagnetic and weak observables

$$\{U(\Lambda, a), J^\mu(x)\}$$

$$\Downarrow$$

$$\{U'(\Lambda, a), J'^\mu(x)\} := W(K)\{U(\Lambda, a), J^\mu(x)\}W^\dagger(K)$$

$$\Downarrow$$

$$\langle\phi|J^\mu(x)|\psi\rangle = \langle\phi'|J'^\mu(x)|\psi'\rangle$$

$J'^\mu(x)$ **and** $J'^\mu(x)$ **differ by exchange currents.**

Possible Problems:

- **Use the freedom to choose $W(K)$ to find a representation that minimizes the exchange current:**

$$F[K] := \sum_{\mu=0}^{3} Tr[(J^{\mu}(0) - J_0^{\mu}(0))(J^{\mu}(0) - J_0^{\mu}(0))].$$

This has a lower bound which is not zero.

- **Relate current operators in models that have identical spectra and cross sections:**

$$W := \Omega'_{-}\Omega_{-}^{\dagger} = \Omega'_{+}\Omega_{+}^{\dagger}$$

$$J'^{\mu}(x) := W J^{\mu}(x) W^{\dagger}.$$

F. Coester and A. Ostebee, Phys. Rev. C11,1836(1975). Motivated by the inability of the nucleon-nucleon scattering and bound state observables to uniquely (Ekstein) determine the nucleon-nucleon interaction - C&O suggested measuring polarization observables in electron-deuteron scattering to put additional constraints on the nucleon-nucleon interaction.

Constituent quark models
Evan Sengbusch and W. P., Phys. Rev. C70,058201(2004).

$$M = \sqrt{m_q^2 + k^2} + \sqrt{m_{\bar{q}}^2 + k^2} + V_c$$

$$V_c(\lambda_1, \cdots, \lambda_4) = \lambda_1 + \frac{\lambda_2}{r} + \lambda_3 r + \delta \vec{s}_q \cdot \vec{s}_{\bar{q}} e^{-\frac{r^2}{2\lambda_4^2}}$$

$$W = I + \Delta \qquad \Delta := -\rho(|m_\pi\rangle - |\bar{m}_\pi\rangle)(\langle m_\pi| - \langle \bar{m}_\pi|)$$

where

$$\rho = \frac{\cos(\theta) + 1}{\sin^2(\theta)} \qquad \cos(\theta) = \langle m_\pi | \bar{m}_\pi \rangle.$$

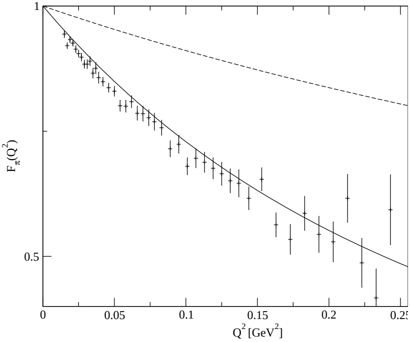

Figure: Pion form factor at low Q^2.

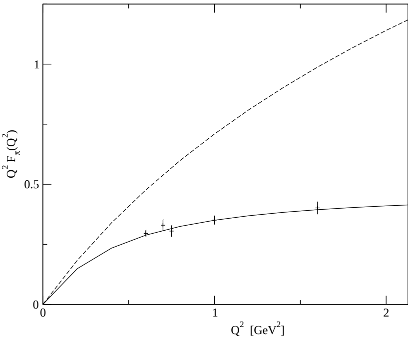

Figure: Pion form factor at high Q^2.

The scattering equivalence W is used to define a transformed mass operator

$$M' := W^\dagger (M_0 + V_c) W = M_0 + V_c'$$

which leads to an equivalent quark model with confining interaction

$$V_c' = V_c + \Delta M + M\Delta^\dagger + \Delta M \Delta^\dagger =$$

$$V_c + \rho(|m_\pi\rangle - |\bar{m}_\pi\rangle)\langle \bar{m}_\pi|(V_c - \bar{V}_c)$$

$$+\rho(V_c - \bar{V}_c)|\bar{m}_\pi\rangle(\langle m_\pi| - \langle \bar{m}_\pi|)$$

$$+\rho^2(|m_\pi\rangle - |\bar{m}_\pi\rangle)\langle \bar{m}_\pi|(V_c - \bar{V}_c)|\bar{m}_\pi\rangle(\langle m_\pi| - \langle \bar{m}_\pi|).$$

- The scattering equivalence does not change the constituent quark kinetic energy - it only transforms the confining interaction.
- The light front kinematic subgroup of the Poincaré group is preserved under W.
- The spectrum of the original mass operator is preserved.
- The new representation is chosen so the light front-impulse approximation provides a good description of the pion electromagnetic form factor.

Application 4: Equivalence of Dirac's forms of dynamics

Quantum probabilities independent of inertial coordinate system.

$\Updownarrow$

E. P. Wigner, Ann. Math. 40,149(1939).

$\Updownarrow$

Unitary representation of the Poincaré group $U(\Lambda, a)$

$\Updownarrow$

$$\{H, \vec{P}, \vec{J}, \vec{K}\}.$$

Single particles

$$\{H, \vec{P}, \vec{J}, \vec{K}\} \qquad (10)$$

$$\Downarrow$$

$$\{M, j, X(4)\,\Delta X(4)\} \rightarrow |(m,j)\,x\,\rangle$$

$$\Downarrow$$

$$U(\Lambda, a)|(m,j)\,x\rangle = \sum{}' \,|(m,j)\,x'\rangle\, D^{mj}_{x'x}(\Lambda, a)$$

$$D^{mj}_{x'x}(\Lambda, a) := \langle (m,j)\,x'|U(\Lambda, a)|(m,j)\,x\rangle =$$

mass m, spin j
irreducible representation of the Poincaré group.

Two particles - Poincaré Clebsch-Gordan coefficients

$$|(m_1, j_1)\, x_1'\rangle \otimes |(m_2, j_2)\, x_2'\rangle$$

$$\Downarrow$$

$$\langle (M, j)\, X; d | (m_1, j_1)\, x_1'; (m_2, j_2)\, x_2'\rangle$$

$$|(M, j)\, X; d\rangle$$

$$U_x(\Lambda, a)|(M, j)\, X; d\rangle = \sum{}' |(M, j)\, X', d\rangle\, D^{Mj}_{X'X}(\Lambda, a).$$

Dynamics

$$M_I := M + V_x$$

$$\langle (M,j)\,X; d|V_x|(M',j')\,X'; d'\rangle := \delta(X - X')\delta_{jj'}\langle M, d\|V^j\|M', d'\rangle$$

$$M_I|(\lambda,j)X\,\rangle = \lambda|(\lambda,j)X\,\rangle$$

$$U(\Lambda,a)|(\lambda,j)\,X\rangle = \sum{}' \,|(\lambda,j)\,X'\rangle\, D^{\lambda j}_{X'X}(\Lambda,a).$$

Different choices of irreducible vector labels, x, give different forms of dynamics. Given $\langle \cdot \| V^j \| \cdot \rangle$ independent of x and y:

$$x \neq y \Leftrightarrow V_x \neq V_y \Leftrightarrow M_x \neq M_y$$

$$\sigma(M_x) = \sigma(M_y) \qquad j_x = j_y$$

$$\Downarrow$$

$$\exists W$$

$$\Downarrow$$

$$U_x(\Lambda, a) = W U_y(\Lambda, a) W^\dagger$$

scattering equivalent.

Application 5: Cluster properties

"There have been many attempts to formulate a relativistically invariant theory that would not be a local field theory, and it is indeed possible to construct theories that are not field theories and yet yield a Lorentz invariant S-matrix for two particle scattering, but such efforts have always run into trouble in sectors with more than two particles: either the three particle S matrix is not Lorentz invariant, or else it violates the cluster decomposition principle." **(S. Weinberg)**

Application 5: Cluster properties

$$U(\Lambda, a) = U_{12}(\Lambda, a) \otimes U_3(\Lambda, a)$$

Alternative construction - F. Coester, Helv. Phys. Acta 38,7(1965).

$$\prod_{i=1}^{3} |(m_i, j_i) x_i\rangle$$

$$\Updownarrow$$

$$|(M(m,q), j)\, X; D, m, j_{12}, d\rangle$$

$$\langle \cdot | \bar{V}_x | \cdot' \rangle = \delta(X - X') \delta_{jj'} \delta(q - q') \delta_{DD'} \langle m, d \| V^{j_{12}} \| m', d' \rangle$$

$$\Downarrow$$

$$M_I = M + V_x \neq \bar{M}_I = M + \bar{V}_x$$

$$\sigma(M_I) = \sigma(\bar{M}_I) \qquad S = \bar{S}.$$

$$\lim_{\lambda\to\infty} e^{i(\lambda(p_{12}-p_3)\cdot r}\bar{V}_x e^{-i(\lambda(p_{12}-p_3)\cdot r} = 0$$

$$\Downarrow$$

- $\bar{U}(\Lambda, a)$ **violates cluster properties.**
- $U(\Lambda, a)$ **satisfies cluster properties.**
- $\bar{U}(\Lambda, a)$ **scattering equivalent to** $U(\Lambda, a)$.

$$\Downarrow$$

- **This suggests that scattering equivalences can restore cluster properties without changing the** S **matrix.**
- **For the solution see F. Coester and W.P., Phys. Rev. D26, 1348(1982).**

Group of scattering equivalences has a subgroup of cluster equivalences.

Summary remarks

Implications - Why should Ekstein be taught in QM classes?

- Wave functions cannot be measured.
- Multiparticle correlations in a wave function are not observable.
- Unique potentials cannot be derived from “field theory”; implicit assumptions are made that select a particular element of a class of scattering equivalent interactions. There is no physics in these assumptions.
- Potential surfaces are not observable.
- Off-shell properties of transition operators are not observable.

- **Three-body interactions come in scattering equivalent classes that depend on the choice of the two-body interaction. A three-body interaction is only defined after first specifying the associated two-body interaction.**
- **Dirac's forms of dynamics have no observable consequences. Any result obtained from a model in one form of dynamics can be obtained from an equivalent model in another form.**
- **Cluster properties of the S matrix do not require cluster properties of the dynamics.**
- **There is nothing fundamental about local potentials; the same physical observables can be reproduced by non-local potentials.**