

Osterwalder-Schrader Stability

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Outline

- Time-Ordered Green Functions
- The Bethe-Salpeter Equation
- The Mandelstam-Method - Physics
- Quasi-Schwinger Functions - (analytic continuation)
- Quasi-Wightman Functions - (analytic continuation)

Outline

- OS Reconstruction of RQM
- Reflection Positivity
- Stability and the BS equation
- Null Spaces
- The Instability

Time-ordered Green Functions

$$G(x_1, \dots, x_n) := \langle 0 | T (\phi(x_1) \cdots \phi(x_n)) | 0 \rangle$$

$$\begin{aligned} T(\phi(x_1)\phi(x_2)) &:= \\ \theta(t_1^0 - t_2^0)\phi(x_1)\phi(x_2) &\pm \theta(t_2^0 - t_1^0)\phi(x_2)\phi(x_1) \\ &\vdots \end{aligned}$$

Cluster Decomposition

$$I = |0\rangle\langle 0| + \Pi_p$$

$$\begin{aligned} W_2(x_1, x_2) &:= \langle 0 | \phi(x_1) \phi(x_2) | 0 \rangle \\ &= \langle 0 | \phi(x_1) (|0\rangle\langle 0| + \Pi_p) \phi(x_2) | 0 \rangle \\ &= \langle 0 | \phi(x_1) |0\rangle\langle 0| \phi(x_2) | 0 \rangle + \langle 0 | \phi(x_1) \Pi_p \phi(x_2) | 0 \rangle \end{aligned}$$

$$W_2(x_1, x_2) = W_1(x_1)W_1(x_2) + W_t(x_1, x_2)$$

Cluster Decomposition

$$W_2(x_1, x_2) = W_1(x_1)W_1(x_2) + W_t(x_1, x_2)$$

$$W_t(x_1, x_2) \sim \int \frac{d^3p}{2\omega(p)} \langle 0 | \phi(0) | p \rangle e^{ip \cdot (x_1 - x_2)} \langle p | \phi(0) | 0 \rangle$$

\Downarrow

$$\lim_{(x_1 - x_2)^2 \rightarrow \infty} W_t(x_1 - x_2) \rightarrow 0$$

Cluster Decomposition for G_4

$$G_1(x) = \text{constant} = 0$$



$$G_4(x_1, x_2, x_3, x_4) = \\ G_2(x_1, x_4)G_2(x_2, x_3) + G_2(x_1, x_3)G_2(x_2, x_4) + \\ G_2(x_1, x_2)G_2(x_3, x_4) + G_t(x_1, x_2, x_3, x_4)$$



$$G_4 = G_0 + G_t$$

Bethe-Salpeter Equation

- Treat G_4 as the kernel of an integral operator

$$K := G_0^{-1} - G_4^{-1}$$

⇓

$$G_4 - G_0 = G_0 K G_4 = G_t$$

- BS equation determines G_t from G_0 and K

Homogeneous BS equation

$$G_4(x_1, x_2, x'_2, x'_1) \rightarrow G'_4(X - X', x, x') :$$

$$X = \frac{1}{2}(x_1 + x_2) \quad x = x_1 - x_2$$

$$X' = \frac{1}{2}(x'_1 + x'_2) \quad x' = x'_2 - x'_1$$

Homogeneous BS equation

$$G'_4(X - X', x, x') =$$

$$\theta(X^0 - X'^0) \langle 0 | T(\phi(X + \frac{x}{2}) \phi(X - \frac{x}{2})) \times$$

$$T(\phi^\dagger(X' + \frac{x'}{2}) \phi^\dagger(X' - \frac{x'}{2})) | 0 \rangle + \text{other time orderings}$$



Homogeneous BS equation

$$\langle 0 | T(\phi(X + \frac{x}{2})\phi(X - \frac{x}{2}))T(\phi^\dagger(X' + \frac{x'}{2})\phi^\dagger(X' - \frac{x'}{2})) | 0 \rangle$$

$$= \sum_n \int \langle 0 | T(\phi(\frac{x}{2})\phi(-\frac{x}{2})) | p_n \rangle e^{iP_n \cdot (X - X')} \frac{d^3 p_n}{2\omega_n(p)}$$

$$\langle p_n | T(\phi^\dagger(\frac{x'}{2})\phi^\dagger(-\frac{x'}{2})) | 0 \rangle$$

⇓

Homogeneous BS equation

- Use existence of complete sets of physical states and the representation

$$\theta(X^0) = \frac{1}{2\pi i} \int ds \frac{e^{isX^0}}{s - i0^+}$$



Homogeneous BS equation

$$\Rightarrow \frac{1}{2\pi i} \sum \int d^4 p \frac{\chi_{P_n}(x) \chi_{P_n}^\dagger(x')}{P^0 - \omega_n - i0^+} e^{-iP \cdot (X - X')} + \dots$$

$$\chi_{P_n}(x) := \frac{\langle 0 | T(\phi(\frac{1}{2}x) \phi(-\frac{1}{2}x)) | p, n \rangle}{\sqrt{2\omega_n(\vec{p})}}$$

$$\chi_{P_n}^\dagger(x') := \frac{\langle p, n | T(\phi^\dagger(\frac{1}{2}x') \phi^\dagger(-\frac{1}{2}x')) | 0 \rangle}{\sqrt{2\omega_n(\vec{p})}}.$$

Homogeneous BS equation

$$\begin{aligned}\tilde{G}_P(x, x') &= \frac{1}{(2\pi)^2} \int d^4 X e^{iP \cdot X} G'(X, x, y) \\ &= -2\pi i \sum_n \frac{\chi_{P_n}(x) \chi_{P_n}^\dagger(x')}{P^0 - \omega_n - i0^+} + \dots\end{aligned}$$

Homogeneous BS equation

$$\lim_{P_0 \rightarrow \omega_n} (P^0 - \omega_n) \int \tilde{G}_P^{-1}(x, y) \tilde{G}_P(y, x') = 0$$

\Downarrow

$$\int d^4 x' \tilde{G}_{P_n}^{-1}(x, x') \chi_{P_n}(x') = 0$$

BS normalization condition

$$\frac{\partial}{\partial P_0} \left((P_0 - \omega_n) \int \tilde{G}_P^{-1}(x, y) \tilde{G}_P(y, x') d^4 y \right) \rightarrow$$

↓

$$1 = -2\pi i (\chi_{P_n} \frac{\partial \tilde{G}_P^{-1}}{\partial P_0} \chi_{P_n})$$

Mandelstam and Physics

- Goal: compute $\langle p_n | O(y) | p'_m \rangle$
- Input:

$$G_4(x_1, x_2, x_3, x_4) := \langle 0 | T (\phi(x_1) \cdots \phi^\dagger(x_4)) | 0 \rangle$$

$$G_5(x_1, x_2, y, x_3, x_4) :=$$

$$\langle 0 | T (\phi(x_1) \phi(x_2) O(y) \phi^\dagger(x_3) \phi^\dagger(x_4)) | 0 \rangle$$

Mandelstam

- Using the same methods with G_5 gives

$$\tilde{G}_5(p, x, 0, q, x') = (-2\pi i)^2 \int \frac{\chi_{P_n}(x) \langle p_n | O(0) | p'_m \rangle \chi_{P'_m}^\dagger(x')}{(P^0 - \omega_n - i0^+)(P'^0 - \omega'_m - i0^+)} + \dots$$

Mandelstam

- The desired physical matrix element can be extracted using the normalization condition

$$\begin{aligned} \langle p_n | O(y) | p'_m \rangle = \\ \lim (P_0 - \omega_n) (P'^0 - \omega'_m) \times \\ \left(\chi_{P_n}^\dagger, \frac{\partial \tilde{G}_P^{-1}}{\partial P^0} \tilde{G}_5(p, \cdot, y, p', \cdot) \frac{\partial \tilde{G}_{P'}^{-1}}{\partial P'^0} \chi_{P'_m} \right) \end{aligned}$$

Mandelstam

- For **good** Green functions this method can be generalized to extract any matrix element of O .
- Assumes that model behaves as if it **has physical intermediate states**.
- Consistency between G_4 and G_5 required.
- The underlying quantum theory has disappeared.

Quasi-Schwinger Functions

- Analyticity of G_n in times follows from spectral properties:

$$\hat{f}(t) = \int_0^{\infty} e^{iEt} f(E) dE$$

$$\text{supp}(f) \in [0, \infty)$$

$$\hat{f}(t) \rightarrow \hat{f}(t + i\tau) \quad \tau > 0$$

$$\hat{f}(t) \rightarrow \hat{f}(e^{i\phi}t) \quad t > 0, \quad 0 \leq \phi \leq \frac{\pi}{2}$$

Quasi-Schwinger Functions

$$\begin{aligned} S_n(\vec{x}_1, x_1^0, \dots, \vec{x}_n, x_n^0) &= \\ &= \lim_{\phi \rightarrow \frac{\pi}{2}} G_n(\vec{x}_1, e^{i\phi} x_1^0, \dots, \vec{x}_n, e^{i\phi} x_n^0) \end{aligned}$$

$$G_n \Rightarrow S_n$$

Complex Lorentz Group

$$X := x^\mu \sigma_\mu = \begin{pmatrix} x^0 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & x^0 - x^3 \end{pmatrix}$$

$$\sigma_\mu := (I, \sigma_1, \sigma_2, \sigma_3);$$

$$X := x^\mu \sigma_{e\mu} = \begin{pmatrix} ix^0 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & ix^0 - x^3 \end{pmatrix}.$$

$$\sigma_{e\mu} := (iI, \sigma_1, \sigma_2, \sigma_3)$$

Complex Lorentz Group

$$\det(X) := (x^0)^2 - (\vec{x})^2$$

$$\det(X) := -(x^0)^2 - (\vec{x})^2$$

$$A \times B \in SL(2, C) \times SL(2, C)$$

$$X' = AXB^t \quad X' = AXB^t$$

$$\det(X) = \det(X') \quad \det(X) = \det(X')$$

Complex Lorentz Group

$$\Lambda(A, B)^\mu{}_\nu := \frac{1}{2} \text{Tr}[\sigma_\mu^\dagger A \sigma_\nu B^t]$$

$$E(A, B)^\mu{}_\nu := \frac{1}{2} \text{Tr}[\sigma_{e\mu}^\dagger A \sigma_{e\nu} B^t].$$

$$(A, B)(A', B') = (AA', BB')$$

Extended Tubes

$$z_k = E^{-1}(A, B)x_k - a$$

\Downarrow

$$S_n(z_1, \dots, z_n) :=$$

$$S_n(x_1, \dots, x_n) \prod_m D_m[E(A, B)]$$

- This defines quasi-Schwinger functions in a complex Euclidean invariant domain

Quasi-Wightman Functions

- quasi-Wightman functions can be recovered from quasi-Schwinger functions as boundary values of the analytic functions

$$W_n(\vec{x}_1, x_1^0, \dots, \vec{x}_N, x_N^0) = \lim_{x_1^0 > x_2^0 > \dots > x_N^0 \rightarrow 0} S_n(\vec{x}_1, x_1^0 + ix_1^0, \dots, \vec{x}_N, x_N^0 + ix_N^0)$$

- Order of Euclidean times = order of fields in W

$$S_n \Rightarrow W_n$$

Reconstruction of QM

- The reconstruction of an RQM is normally done in terms of quasi-Wightman functions, which behave kernels of the physical Hilbert space scalar product.
- Osterwalder and Schrader found essentially equivalent construction based directly on the quasi-Schwinger functions.

Reconstruction of QM: quasi-Wightman

$$\langle x|f\rangle :=$$

$$\{f_0, f_1(x_{11}), f_2(x_{21}, x_{22}), \dots, f_k(x_{k1}, \dots, x_{kk})\}$$

$$\langle f|g\rangle =$$

$$\sum_{m,n} \int d^4x_1 \cdots d^4x_{m+n} f_n^*(x_n, \dots, x_1) \times$$

$$W_{m+n}(x_1, \dots, x_{m+n}) g_m(x_{n+1}, \dots, x_{n+m}).$$

Reconstruction of RQM: quasi-Wightman

$$\langle x|f\rangle \rightarrow \langle x|f'\rangle := \langle \Lambda^{-1}(x - a)|f\rangle$$

$$W_m(\Lambda x_1 + a, \dots, \Lambda x_m + a) = W_m(x_1, \dots, x_m)$$

\Downarrow

$$\langle f|g\rangle = \langle f'|g'\rangle$$

Reconstruction of QM - OS

$$\langle \mathbf{x} | f \rangle :=$$

$$\{f_0, f_1(\mathbf{x}_{11}), f_2(\mathbf{x}_{21}, \mathbf{x}_{22}), \dots, f_k(\mathbf{x}_{k1}, \dots, \mathbf{x}_{kk})\}$$

$$\theta(\mathbf{x}^0, \vec{\mathbf{x}}) := (-\mathbf{x}^0, \vec{\mathbf{x}})$$

$$\langle \mathbf{x} | \Theta f \rangle :=$$

$$\{f_0, f_1(\theta \mathbf{x}_{11}), f_2(\theta \mathbf{x}_{21}, \theta \mathbf{x}_{22}), \dots, f_k(\theta \mathbf{x}_{k1}, \dots, \theta \mathbf{x}_{kk})\}.$$

Euclidean form

$$(\Theta f, Sg) := (f, \Theta Sg)$$

$$\sum_{m,n} \int d^4x_1 \cdots d^4x_{m+n} f_n^*(\theta x_n, \cdots, \theta x_1) \times \\ S_{m+n}(x_1, \cdots, x_{m+n}) g_m(x_{n+1}, \cdots, x_{n+m}).$$

Reflection Positivity

$$\text{supp} \langle x|f \rangle : t_{k1} > t_{k2} \cdots > t_{kk} > 0 \quad (\langle x|f \rangle \in \mathcal{S}_{>})$$

$$(\Theta f, Sg) \geq 0$$

Physical Hilbert Space

$$f, g \in \mathcal{S}_>$$

$$\Downarrow$$

$$(\Theta(f - g), S(f - g)) := 0 \Leftrightarrow f \sim g$$

$$\Downarrow$$

$$\langle [f]_{\sim} | [g]_{\sim} \rangle := (\Theta f, Sg) \quad f \in [f]_{\sim}, \quad g \in [g]_{\sim}$$

Reflection Positivity

- RP \Rightarrow RP must hold subspaces

$$\int d^4x_1 \cdots d^4x_4 f_2^*(\theta x_2, \theta x_1) S_4(x_1, x_2, x_3, x_4) f_2(x_3, x_4) \geq 0$$

$$\int d^4x_1 d^4x_2 f_1^*(\theta x_1) S_2(x_1, x_2) f_1(x_2) \geq 0.$$

- These conditions constrain G_4 and S_4

Poincaré Group

$e^{i\vec{J}\cdot\vec{\theta}}$ 3-dimensional rotations

$e^{i\vec{P}\cdot\vec{a}}$ 3-dimensional translations

e^{-Hx^0} positive Euclidean time translations

$e^{\vec{B}\cdot\vec{\theta}}$ real rotations in Euclidean space-time planes

Poincaré Group

$$\langle \mathbf{x} | \vec{P} | f \rangle :=$$

$$\left\{ 0, -i \frac{\partial}{\partial \vec{x}_{11}} f_1(\mathbf{x}_{11}), \left(-i \frac{\partial}{\partial \vec{x}_{21}} - i \frac{\partial}{\partial \vec{x}_{22}} \right) f_2(\mathbf{x}_{21}, \mathbf{x}_{22}), \dots \right\}$$

$$\langle \mathbf{x} | \vec{J} | f \rangle :=$$

$$\left\{ 0, -i \vec{x}_{11} \times \frac{\partial}{\partial \vec{x}_{11}} f_1(\mathbf{x}_{11}), \left(-i \vec{x}_{21} \times \frac{\partial}{\partial \vec{x}_{21}} - i \vec{x}_{22} \times \frac{\partial}{\partial \vec{x}_{22}} \right) f_2(\mathbf{x}_{21}, \mathbf{x}_{22}), \dots \right\}$$

Poincaré Group

$$\langle \mathbf{x} | \mathbf{H} | f \rangle :=$$

$$\left\{ 0, \frac{\partial}{\partial \mathbf{x}_{11}^0} f_1(\mathbf{x}_{11}), \frac{\partial}{\partial \mathbf{x}_{21}^0} + \frac{\partial}{\partial \mathbf{x}_{22}^0} f_2(\mathbf{x}_{21}, \mathbf{x}_{22}), \dots \right\}$$

$$\langle \mathbf{x} | \vec{\mathbf{B}} | f \rangle :=$$

$$\left\{ 0, \vec{\mathbf{x}}_{11} \frac{\partial}{\partial \mathbf{x}_{11}^0} - \mathbf{x}_{11}^0 \frac{\partial}{\partial \vec{\mathbf{x}}_{11}} f_1(\mathbf{x}_{11}), \right.$$

$$\left. \left(\vec{\mathbf{x}}_{21} \frac{\partial}{\partial \mathbf{x}_{21}^0} - \mathbf{x}_{21}^0 \frac{\partial}{\partial \vec{\mathbf{x}}_{21}} + \vec{\mathbf{x}}_{22} \frac{\partial}{\partial \mathbf{x}_{22}^0} - \mathbf{x}_{22}^0 \frac{\partial}{\partial \vec{\mathbf{x}}_{22}} \right) f_2(\mathbf{x}_{21}, \mathbf{x}_{22}), \dots \right\}.$$

Dynamics - Euclidean RQM

- $H, \vec{P}, \vec{J}, \vec{B}$ well defined and self-adjoint on the Physical Hilbert space
- $H, \vec{P}, \vec{J}, \vec{B}$ satisfy Poincaré commutation relations
- $\langle \cdot | \cdot \rangle, H, \vec{P}, \vec{J}, \vec{B}$ defines a relativistic quantum theory.

Euclidean BS

$$G_4 = G_0 + G_0 K G_4$$

\Downarrow

$$S_4 = S_0 + S_0 K_e S_4$$

$$G_4 \Leftrightarrow S_4 \Leftrightarrow W_4$$

OS Stability

$$S_4 = S_0 + S_0 K_e S_4$$

$$\Pi_{>} : S \rightarrow S_{>}$$

$$(f, \Pi_{>} \Theta S_0 \Pi_{>} f) \geq 0$$

K_e small, Euclidean covariant

?

$$(f, \Pi_{>} \Theta S_4 \Pi_{>} f) \geq 0$$

Cases

- Case 1:

$$(f, \Pi_{>} \Theta S_0 \Pi_{>} f) > 0 \quad f \neq 0$$

- Case 2:

$$(f, \Pi_{>} \Theta S_0 \Pi_{>} f) = 0 \quad \text{for some } f \neq 0$$

$\pm t$ model

$$\Pi_{>} := \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Theta := \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\begin{aligned} (f, \Theta S f) &= (f, 0) \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \begin{pmatrix} s(\vec{0}, 0) & s(\vec{0}, 2t) \\ s(\vec{0}, -2t) & s(\vec{0}, 0) \end{pmatrix} \begin{pmatrix} f \\ 0 \end{pmatrix} \\ &= f^2 s(\vec{0}, -2t). \end{aligned}$$

$\pm t$ model

$$\begin{pmatrix} s_{11} & s_{21} \\ s_{12} & s_{22} \end{pmatrix} = \begin{pmatrix} s_{011} & s_{021} \\ s_{012} & s_{022} \end{pmatrix} + \begin{pmatrix} s_{011} & s_{021} \\ s_{012} & s_{022} \end{pmatrix} \begin{pmatrix} k_{11} & k_{21} \\ k_{12} & k_{22} \end{pmatrix} \begin{pmatrix} s_{11} & s_{21} \\ s_{12} & s_{22} \end{pmatrix}.$$

$\pm t$ model

$$S \geq 0; \quad \Pi_{>} \Theta S \Pi_{>} > 0$$

\Downarrow

$$k_{11} = k_{22} < \frac{s_{11}}{\det(S_0)} = \frac{s_{11}}{s_{011}^2 - s_{012}^2}$$

$$k_{12} = k_{21} > -\frac{s_{012}}{\det(S_0)} = -\frac{s_{012}}{s_{011}^2 - s_{012}^2}.$$

$$k_{11} + k_{12} < \frac{1}{s_{011} + s_{012}}$$

$\pm t$ model

- The inequalities on k_{ij} are satisfied in an open neighborhood containing the origin.
- This indicates that this trivial model is OS-stable.
- Parts of the analysis can be generalized more complicated systems.
- The special property is that $\Pi_{>} \Theta S \Pi_{>}$ is bounded from below on $S_{>}$.

Null Spaces

$$S_4 = S_0 + S_0 T_e S_0$$

$$T_e = K_e + K_e S_0 T_e.$$

$$\Pi_{>} \Theta S_4 \Pi_{>} = \Pi_{>} \Theta S_0 \Pi_{>} + \Pi_{>} S_0 \Theta T_e S_0 \Pi_{>}$$

Null Spaces

$$(\Pi_{>} f, \Theta S_0 \Pi_{>} f) = 0$$

\Downarrow

$$(\Pi_{>} f, S_4 \Pi_{>} f) = (S_0 \Pi_{>} f, (\Theta T) S_0 \Pi_{>} f).$$

$$\chi := S_0 \Pi_{>} f \quad f \neq 0 \Rightarrow \chi \neq 0$$

\Downarrow

$$(\Pi_{>} f, \Theta S_4 \Pi_{>} f) = (\chi, \Theta T \chi)$$

Null Spaces

$$(\chi, \Theta T \chi) < 0 \Rightarrow \text{instability}$$

$$(\chi, \Theta T \chi) > 0$$



$$(\chi, \Theta(-T)\chi) < 0$$



instability

Null Spaces

- Invariance \Rightarrow Lehmann representation

$$S_0(\mathbf{x} - \mathbf{x}') =$$
$$\frac{1}{(2\pi)^2} \int d^4 p dm \frac{\rho(m) e^{ip^0(t-t') + i\vec{p} \cdot (\vec{x} - \vec{x}')}}{(p^0 + i\omega_m(\vec{p}))(p^0 - i\omega_m(\vec{p}))}.$$
$$\Downarrow$$

Null Spaces

$$(f, \Theta S_0 f) = \int \left| \int dt \tilde{f}(\mathbf{p}, t) \frac{2\pi e^{-\omega_m(\vec{\mathbf{p}})t}}{\sqrt{\omega_m(\vec{\mathbf{p}})}} \right|^2 d^3\mathbf{p} dm \rho(m)$$

$$\tilde{f}(\mathbf{p}, t) = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{\mathbf{p}} \cdot \vec{\mathbf{x}}} f(\vec{\mathbf{x}}, t) d^3\mathbf{x}$$

Null Spaces

$$(f, \Theta S_0 f) = 0$$

\Downarrow

$$\int dt \tilde{f}(\mathbf{p}, t) e^{-\omega_m(\vec{\mathbf{p}})t} = 0$$

\Downarrow

$\forall \vec{\mathbf{p}}$ and all $m \in \text{supp}(\rho)$.

Null Spaces

- S_0 free $\Rightarrow m$ is a fixed number

$$\tilde{f}(\vec{p}, t) = \tilde{g}(\vec{p})\xi(t) \quad \int_a^b \xi(t)dt = 1 \quad \text{supp}(\xi) \in [a, b]$$

$$\hat{\xi}(\vec{p}) := \int_a^b e^{-\omega_m(\vec{p})t} \xi(t) dt$$

$$\tilde{f}(\vec{p}, t) = \tilde{g}(\vec{p})\xi(t)[1 - e^{\omega_m(\vec{p})t}\hat{\xi}(\vec{p})]$$

\Downarrow

$$(f, \Theta S_0 f) = 0$$

Null Spaces

- For **free** particles $S_0 \rightarrow S_1 S_2$ (product of free two-point Schwinger functions).
- The supports of $f = f_1 f_2$ can be designed to be in $S_{>}$ and in the null space of $\Pi_{>} \Theta S_0 \Pi_{>}$
- Leads to a **violation of the stability** condition.

Null Spaces

- If $\rho(m)$ has absolutely continuous spectrum then the instability proof breaks down:

$$\int \tilde{f}(\vec{p}, t) e^{-\omega_m(\vec{p})t} = 0.$$

$$\Downarrow \frac{\partial^n}{\partial m^n}$$

$$\int dt \tilde{f}(\vec{p}, t) t^n e^{-\omega_{m_2}(\vec{p})t} = 0$$

$$\int dt \tilde{f}(\vec{p}, t) L_n(\omega_{m_2}(\vec{p})t) e^{-\omega_{m_2}(\vec{p})t} = 0 \Rightarrow \tilde{f} = 0$$

Conclusion

- Euclidean BS with **free** S_0 is OS unstable
- Since $|K|_e \geq |K|_p$, \Rightarrow small Minkowski kernels lead to instability. **Problem extends to real-time BS equation.**
- The connection of the BS equation with a **free** S_0 (resp. G_0) as driving terms to a relativistic quantum theory is suspect.

Future

- What happens in realistic models?
- Case of Fermions (protons, neutrons, quarks)?