Few Body Currents Generated by Cluster Separability Constraints

Mark Tucker[1] Brad Keister[2] Wayne Polyzou[1]

[1]Department of Physics and Astronomy
The University of Iowa

[2]Division of Nuclear Physics
National Science Foundation

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Outline

Definitions

Poincaré invariant quantum theory:

- Unitary representation, $U(\Lambda, a)$, of the Poincaré group.
- Dynamics given by $U(\Lambda, a)$.
- Infinitesimal generators H, P, J, and K. Specifically,

Definition

$$H := i \left(\frac{\partial}{\partial a^0} U(I, a) \right) U^\dagger(I, a)$$

Definition

$$P := -i \left(\frac{\partial}{\partial \mathbf{a}} U(I, a) \right) U^\dagger(I, a)$$

Current Conservation and Current Covariance

Dynamical constraints on current operators:

- Depend on representation of the Poincaré group, $U(\Lambda, a)$:

Definition

Current conservation: $[H, J^0(0)] - \sum_i [P^i, J^i(0)] = 0$

Definition

Current covariance: $U(\Lambda, a) J^\mu(x) U^\dagger(\Lambda, a) = (\Lambda^{-1})^\mu_\nu J^\nu(\Lambda x + a)$

Clustering

Definition

$U(\Lambda, a)$ is **cluster separable** if,

$$\lim_{(a_i - a_j)^2 \to +\infty} \left\| \left[U(\Lambda, a) - \otimes U_i(\Lambda, a) \right] \prod U_i(I, a_i) |\psi\rangle \right\| = 0$$

This ensures that the Poincaré invariance of the system also holds for isolated subsystems (large space-like separation).

What Happen to Constraints in the Clustering Limit?

In the clustering limit, the current should break up into a sum:

Current Conservation

$$\left[H, J^0(0)\right] - \sum_i \left[P^i, J^i(0)\right] = 0$$
$$\downarrow$$
$$\sum_a \left([H_a, J^0_a(0)] - \sum_i [P^i_a, J^i_a(0)]\right) = 0$$
$$\Downarrow$$
$$\left[H_a, J^0_a(0)\right] - \sum_i \left[P^i_a, J^i_a(0)\right] = 0 \quad \text{for each } a.$$

Poincaré invariance, current covariance, and current conservation should *all* hold for isolated subsystems.

Construction of BT, TP Representations

Bakamjian-Thomas Construction

Bakamjian-Thomas, Coester, Sokolov:

- Add interactions to mass operator to construct $U(\Lambda, a)$.
- Kinematic spin; Does not satisfy cluster separability.

Tensor Product Construction

Derivative of BT constructed to satisfy cluster separability.

- Interaction-dependent spin.
- Difficult to add more than one two body interaction.

BT, $N > 2$: Clustering Fails, but Restored by $\mathcal{A}$

In the Bakamjian-Thomas construction,

- Systems with $N > 2$ fail to satisfy cluster properties.
- For $N = 3$, scattering equivalent to one that clusters.

  $\Longrightarrow$ there is a unitary operator $\mathcal{A}$ such that

  $$\mathcal{A}U(\Lambda, a)\mathcal{A}^\dagger = U'(\Lambda, a)$$

  where $U'(\Lambda, a)$ satisfies cluster properties.
- The operator, $\mathcal{A}$, is called a **packing operator**.
- Essentially, $\mathcal{A}$ restores cluster properties for $N = 3$.

Relation to Electromagnetic Observables

Electromagnetic observables can be calculated from current matrix elements ($|\psi_i\rangle$ and $|\psi_f\rangle$ are eigenstates of H):

$$\langle\psi_f|J^\mu(0)|\psi_i\rangle$$

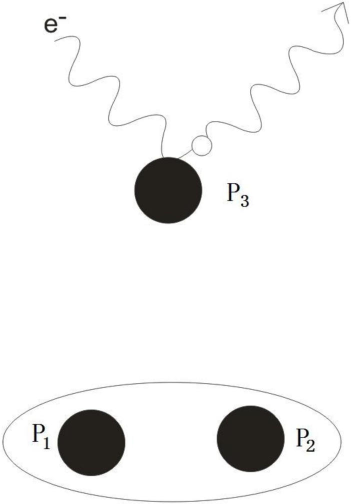
Using $\mathcal{A}$,

- $H \rightarrow H' = \mathcal{A}H\mathcal{A}^\dagger$
- $|\psi_i\rangle \rightarrow |\psi_i'\rangle = \mathcal{A}|\psi_i\rangle$
- $|\psi_f\rangle \rightarrow |\psi_f'\rangle = \mathcal{A}|\psi_f\rangle$
- $\langle\psi_f|J^\mu(0)|\psi_i\rangle \rightarrow \langle\psi_f'|J^\mu(0)|\psi_i'\rangle = \langle\psi_f|\mathcal{A}^\dagger J^\mu(0)\mathcal{A}|\psi_i\rangle$

How can we Test the Effects of $\mathcal{A}$?

Model:

- First assume 3 particles interacting in Bakamjian-Thomas representation.
- Turn off 13- and 23- pair interactions.
- $\mathcal{A}$ connects resulting and TP representations.
- Scatter an electron off of particle 3.
- Can construct both BT and TP exactly.



Spectator Deuteron

Illustrate the Two Calculations: Details

$\mathcal{A}$ changes a delta function in p_3 to a delta function in relative momentum, q_3, obtained by boosting p_3 to rest frame of non-interacting 2+1 body system, using

$$q_3 = B^{-1}(P/M_0)p_3.$$

Realistic three body calculations are now being performed using this formalism and the issues being discussed are very relevant if these solutions are used to calculate electromagnetic observables!

$$\langle \psi_f | \mathcal{A}^\dagger J^\mu(0) \mathcal{A} | \psi_i \rangle \quad \text{– good}$$
$$\langle \psi_f | J^\mu(0) | \psi_i \rangle \quad \text{– bad}$$

$N = 4$ Scattering Experiment, with and without $\mathcal{A}$

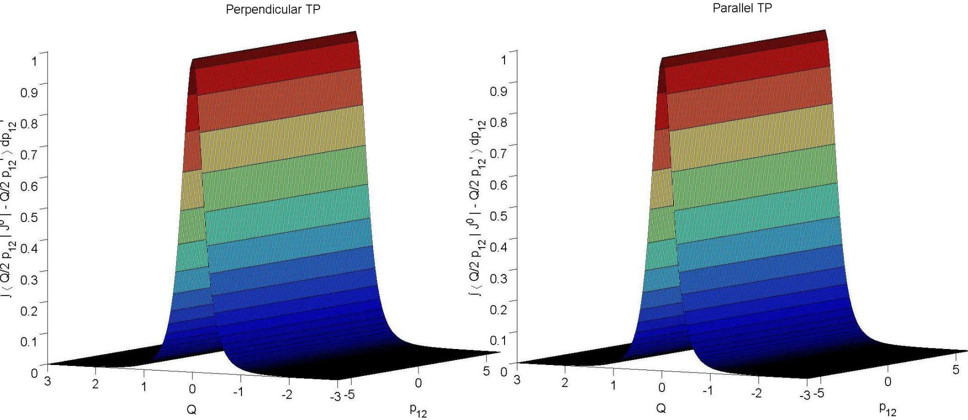
Recap:

- Will calculate $\langle \psi_f | J^0(0) | \psi_i \rangle$ and $\langle \psi_f | \mathcal{A}^\dagger J^0(0) \mathcal{A} | \psi_i \rangle$.
- Here there is a difference since $N = 4$.

Recall

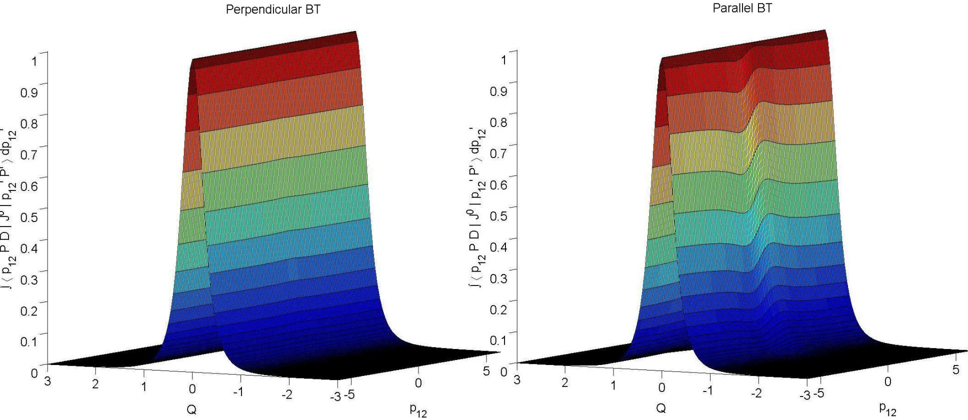
Current matrix elements...
TP: $\langle \psi_f | \mathcal{A}^\dagger J^\mu(0) \mathcal{A} | \psi_i \rangle$
BT: $\langle \psi_f | J^\mu(0) | \psi_i \rangle$

TP: Q and p_{12} vs $\langle \psi_f | \mathcal{A}^\dagger J^0(0) \mathcal{A} | \psi_i \rangle$ (good)



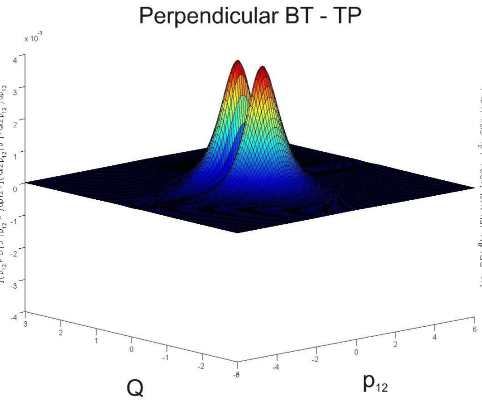
$$\int \langle Q/2\, p_{12} | J^0 | -Q/2\, p'_{12} \rangle dp'_{12}$$

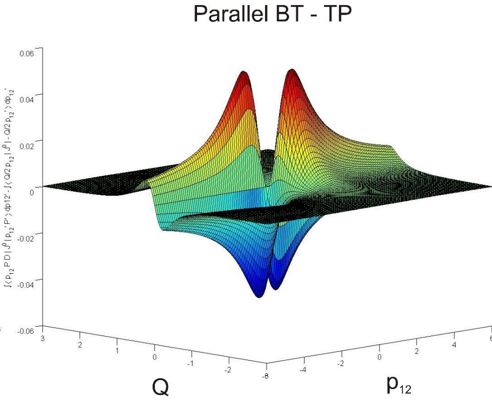
BT: Q and p_{12} vs $\langle \psi_f | J^0(0) | \psi_i \rangle$ (bad)



$$\int \langle p_{12}\ P\ D | J^0 | p'_{12}\ P' \rangle dp'_{12}$$

Difference: BT - TP





Up to 6% difference.

Conclusion

- Two constructions of $U(\Lambda, a)$ – BT, TP.
  - BT does not cluster for $N > 2$.
  - $N = 3$: scattering equivalent (special case).
  - $N > 3$: $\exists$ measurable differences.
- EM observables from current matrix elements.
- Clustering should hold for Poincaré invariance, current conservation, and current covariance.
- In BT representation, clustering should be restored using $\mathcal{A}$.
- Electromagnetic calculations being done without clustering.
- Calculations with and without $\mathcal{A}$ differ by up to 6%.

Goals

- Repeat with Light Front and Point forms.
- Use more complicated models.

Here the Minkowski metric is taken to have signature (-,+,+,+):

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

so that $\lim_{(a_i - a_j)^2 \to +\infty}$ means space-like separation.

What Happen to Constraints in the Clustering Limit?

Current Covariance

$$U(\Lambda, a)J^{\mu}(x)U^{\dagger}(\Lambda, a) = (\Lambda^{-1})^{\mu}_{\nu}J^{\nu}(\Lambda x + a)$$
$$\downarrow$$
$$\sum_a U_a(\Lambda, a)J^{\mu}_a(x)U^{\dagger}_a(\Lambda, a) = \sum_a (\Lambda^{-1})^{\mu}_{\nu}J^{\nu}_a(\Lambda x + a)$$
$$\Downarrow$$
$$U_a(\Lambda, a)J^{\mu}_a(x)U^{\dagger}_a(\Lambda, a) = (\Lambda^{-1})^{\mu}_{\nu}J^{\nu}_a(\Lambda x + a) \quad \text{for each } a.$$

Illustration of BT and TP Constructions

$$\begin{array}{ccccc} |(12)\otimes(3)\rangle & & \xrightarrow{\qquad\qquad\langle AB|C\rangle_0\qquad\qquad} & & |((12)(3))\rangle \\ V_{(12)(3)}\Big\downarrow & & & & \Big\downarrow V_{((12)(3))} \\ |(12)_I\otimes(3)\rangle & \xrightarrow[\langle AB|C\rangle_I]{} & |((12)_I(3))\rangle & \underbrace{\sim}_{A_{(12)(3)}} & \overline{|((12)(3))_I\rangle} \end{array}$$

Methods

Wave Functions

Gaussian

$$\left|\phi(k^2)\right|^2 = \left|\exp\left(-k^2/{k_0}^2\right)\right|^2$$

Malfliet-Tjon

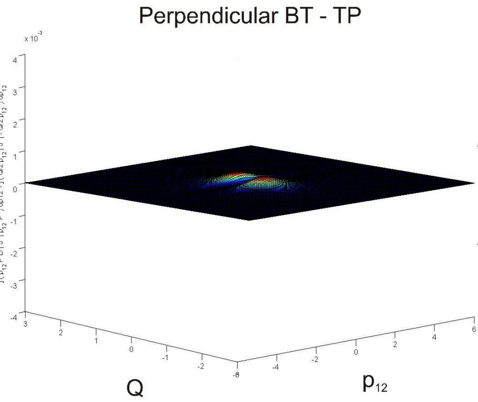
$$\begin{cases} a = 1438.4812 \\ b = -626.893 \\ c_1 = 3.11 \\ c_2 = 1.55 \end{cases}$$

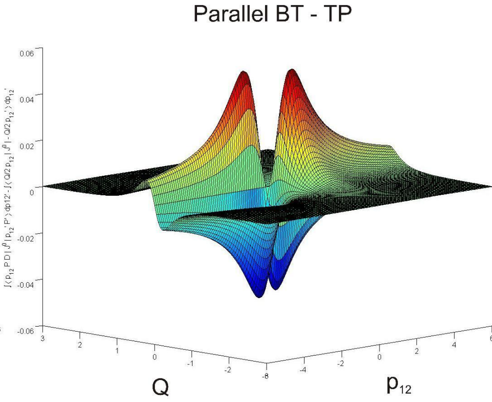
Form Factor

Monopole:

$$F(q^2) = \left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2$$

Difference: BT - TP





Up to 6% difference.