Kinematic subgroups and forms of dynamics in non-perturbative quantum field theory

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Motivation/Questions:

Motivation: Recent talks by John Collins, Matthias Burkhart and Philip Mannheim on the light-front vacuum and the relation between instant and front-form dynamics.

• $U_0(\Lambda, a)$ and $U(\Lambda, a)$ act on inequivalent representations of the field theory Hilbert space. What is a kinematic subgroup?

• What do we mean by forms of the dynamics in the absence of a kinematic representation of the Poincare group?

• Hard to separate issues related to renormalization, rotational covariance and triviality of the vacuum.

• "Axiomatic treatment": Use accepted properties of field theories to bypass difficult questions about renormalization and rotational covariance.

• Can we understand equivalence (or not) of instant and front-form theories, the relation of the vacua, triviality of the light-front vacuum and the different realizations of spontaneous symmetry breaking?

Summary of results:

• Equivalence: It is possible to construct S-matrix equivalent representations of the field theory that have the same vacuum where transformations in the light-front resp. instant-form kinematic subgroups can be computed by acting on the basis vectors. The basis vectors are not from a kinematic representation, and the dynamical generators are not diagonal in this representation.

• Triviality of the vacuum: The vacuum is a linear functional on an algebra of smeared fields, $\langle 0|\phi(f_1)\cdots\phi(f_N)|0\rangle$. Vectors are represented by equivalence classes of test functions in 4 variables. Different vacuum functionals can agree for test functions restricted to a light front. Dynamics is needed to extend the definition of the vacuum to equivalence classes with test functions in $S(R^4)$.

• Spontaneous symmetry breaking: If there is spontaneous symmetry breaking there is a Goldstone boson in both representations, but it can only be seen using a cutoff instant form charge operator that exists in both representations.

Forms of Dynamics: (Dirac 1949)

Generators $\{H, \mathrm{P}, \mathrm{J}, \mathrm{K}\}$ of the Poincaré group satisfy the commutation relations

$$[K^i, P^j] = i\delta_{ij} H \qquad H = H_0 + V$$

Three independent commutators with H on the right - require at least three generators with interactions.

Forms of dynamics classified by kinematic subgroups:

$$U(K)U_0^\dagger(K) = I \qquad K = \text{Kinematic subgroup}$$

The three largest kinematic subgroups, K, are the 3-dimensional Euclidean group (6 parameters), the Lorentz group (6 parameters), and the group that leaves a light-front hyperplane invariant (7 parameters).

The problem:

The separation $H = H_0 + V$ does not make sense in quantum field theory.

$U(\Lambda, a)$ and $U_0(\Lambda, a)$ live on inequivalent representations of the Hilbert space.

What is a kinematic subgroup if there is no kinematics?

Assumptions: (generally accepted properties of field theory)

1) Hilbert space, $\mathcal{H}$

2) Unitary representation, $U(\Lambda, a)$, of the Poincaré group on $\mathcal{H}$

3) Poincaré invariant vacuum vector, $|0\rangle$

4) One-particle states

5) Cluster properties

6) Locality

7) Asymptotically complete Haag-Ruelle scattering theory

Hilbert space: (comments on inequivalent representations)

Dense set of vectors in the Hilbert space $\mathcal{H}$ (GNS construction):

$$|\psi\rangle = P(\phi(f))|0\rangle \qquad f_i(x) \in S(R^4)$$

The vacuum determines the Hilbert space inner product. $\langle\psi_f|\psi_i\rangle$ can be expressed in terms of

$$L[\phi(f_1)\cdots\phi(f_N)] = \langle 0|\phi(f_1)\cdots\phi(f_N)|0\rangle$$

Vectors are represented by equivalence classes of test functions. The classes are determined by the vacuum functional. For example for free fields test functions whose Fourier transforms agree on the mass shell are in the same class. These functions will not generally be equal on a different mass shell.

Test functions in different equivalence classes can agree on a light-front.

Relativity:

If the vacuum is invariant and the fields transform covariantly there is a unitary representation of the Poincaré group on $\mathcal{H}$. It decomposes $\mathcal{H}$ into a direct integral of Poincaré irreducible subspaces:

$$I = \sum \int_{\oplus} |(m,s,d)p,\mu\rangle \delta(p^2+m^2)\theta(p^0)d^4p\langle(m,s,d)p,\mu|$$

Each Poincaré irreducible subspace is characterized by a mass, spin and Poincaré invariant degeneracy parameters d.

$U(\Lambda,a)$ acts irreducibly on each of these subspaces

$$U(\Lambda,a)|(m,s,d)p,\mu\rangle = \sum_{\nu=-s}^{s} e^{i\Lambda p\cdot a}|(m,s,d)\Lambda p,\mu\rangle D^s_{\nu\mu}[B^{-1}(\Lambda p)\Lambda B(p)]$$

(covariant normalization)

Particles:

One-particle subspaces are associated with point spectrum eigenvalues, m, of the mass Casimir operator of the representation of the Poincaré group.

A basis on each one-particle subspace can be constructed out of simultaneous eigenstates of four mutually commuting functions of the Poincaré generators. Relevant choices are

$$|(m,s,d)\,\mathrm{p},\mu_c\rangle \qquad |(m,s,d)\,p^+,\mathrm{p}_\perp,\mu_f\rangle \qquad |(m,s,d)\,\mathrm{v},\mu_c\rangle$$

I call these instant, front and point-form bases.

The three one-particle bases are related by variable changes. The variable changes depend on m, so the transformations are different for each m in the spectrum of the mass Casimir operator. For example:

$$|(m,s,d)\mathrm{p},\mu_c\rangle = \sum_{\nu_f} |(m,s,d)p^+,\mathrm{p}_\perp,\nu_f\rangle \sqrt{\frac{p^+}{e_m(\mathrm{p})}} D^s_{\nu_f\nu_c}[B_f^{-1}(p/m)B_c(p/m)]$$

S-matrix equivalent quantum theories

In order to study the equivalence of different forms of dynamics it is useful to study the group of "S-matrix equivalent" quantum theories.

This requires a formulation scattering theory.

Because we want to construct a group of operators is it useful to use a formulation of scattering theory that uses strong limits.

Haag-Ruelle scattering is the field-theoretic generalization of non-relativistic time-dependent scattering based on strong limits.

Unitary equivalence $\neq$ S-matrix equivalence. Example: Two Hamiltonians with different short-range repulsive potentials have the same spectrum (are unitarily equivalent) but do not have the S-matrices.

Haag-Ruelle scattering - two Hilbert space treatment

Single-particle states can be expressed as functions of the field operators applied to the vacuum

$$|(m,s,d)\mathrm{p},\mu_c\rangle = A^\dagger_{m,s,I}(\mathrm{p},\mu)|0\rangle \qquad \langle 0|A^\dagger_{m,s,I}(\mathrm{p},\mu_c) = 0$$

or

$$|(m,s,d)p^+,\mathrm{p}_\perp,\mu_f\rangle = A^\dagger_{m,s,F}(p^+,\mathrm{p}_\perp,\mu_f)|0\rangle \qquad \langle 0|A^\dagger_{m,s,F}(p^+,\mathrm{p}_\perp,\mu_f) = 0$$

or

$$|(m,s,d)\mathrm{v},\mu_c\rangle = A^\dagger_{m,s,P}(\mathrm{v},\mu)|0\rangle \qquad \langle 0|A^\dagger_{m,s,P}(\mathrm{v},\mu_c) = 0$$

where normalizable single particle states have the form

$$|\psi\rangle = \sum \int d\mathrm{p} A^\dagger_{m,s,I}(\mathrm{p},\mu)|0\rangle f(\mathrm{p},\mu) \qquad etc.$$

The $A^\dagger$s create (only) one-particle states out of the vacuum however products of these operators applied to the vacuum are not mass eigenstates, and cannot be identified with N-particle states.

The labels I, F, P correspond to instant, front, point form. The operators $A^\dagger_{m,s,X}$ can be constructed to transform covariantly with the X kinematic subgroup.

Two Hilbert space scattering theory: (asymptotic channel spaces - $\mathcal{H}_\alpha$)

For each N- particle scattering channel α construct a channel Hilbert space $\mathcal{H}_\alpha$:

$$\mathcal{H}_\alpha = \prod_{i\in\alpha} L^2(\mathbf{p}_i,\mu_i), \qquad \mathcal{H}_\alpha = \prod_{i\in\alpha} L^2(p_i^+,\mathbf{p}_{i\perp},\mu_i) \qquad \cdots$$

and a mapping from $\mathcal{H}_\alpha$ in the instant form basis to $\mathcal{H}$:

$$\Phi_{I\alpha}|f_1\cdots f_N\rangle_\alpha = \int d^{3N}p \sum A^\dagger_{m_N,s_N,I}(\mathbf{p}_N,\mu_N)\cdots A^\dagger_{m_1,s_1,I}(\mathbf{p}_1,\mu_1)|0\rangle \times$$

$$f_1(\mathbf{p}_1,\mu_1)\cdots f_1(\mathbf{p}_N,\mu_N)$$

and

$$\Phi_{F\alpha}|f_1\cdots f_N\rangle_\alpha = \int d^{3N}\tilde{p} \sum A^\dagger_{m_N,s_N,F}(p_N^+,\mathbf{p}_{N\perp},\mu_N)\cdots A^\dagger_{m_1,s_1,F}(p_1^+,\mathbf{p}_{1\perp},\mu_1)|0\rangle \times$$

$$f_1(p_1^+,\mathbf{p}_{1\perp},\mu_1)\cdots f_1(p_N^+,\mathbf{p}_{N\perp},\mu_N)$$

from $\mathcal{H}_\alpha$ in the front-form basis to $\mathcal{H}$.
There is a N-free particle unitary representation of the Poincaré group on $\mathcal{H}_\alpha$:

$$U_\alpha(\Lambda,a) = \otimes_i U_{m_i s_i}(\Lambda,a)$$

The $\Phi_{X\alpha}$ (X=F,I,P) are constructed to satisfy

$$U(K_X)\Phi_{X\alpha} = \Phi_{X\alpha}U_\alpha(K_X) \qquad K_X = \text{ X - kinematic subgroup}$$

Denote the set of all channel, including the one-body and vacuum channels, by $\mathcal{A}$. Define the asymptotic Hilbert space

$$\mathcal{H}_{\mathcal{A}} = \oplus_{\alpha \in \mathcal{A}} \mathcal{H}_{\alpha}$$

and the mapping $\Phi_{X\mathcal{A}} : \mathcal{H}_{\mathcal{A}} \to \mathcal{H}$ by

$$\Phi_{X\mathcal{A}} = \sum_{\alpha \in \mathcal{A}} \Phi_{X\alpha}$$

Channel wave operators are defined by the strong limits

$$\Omega_{X\alpha\pm} := \lim_{t \to \pm\infty} U(I,t)\Phi_{X\alpha} U_{\alpha}(I,-t) \qquad S_{\alpha\beta} = \Omega^{\dagger}_{X\alpha+} \Omega_{X\beta-}$$

These extend to all of $\mathcal{H}_{\mathcal{A}}$

$$\Omega_{X\pm} := \sum_{X\alpha \in \mathcal{A}} \Omega_{X\alpha\pm} \qquad S = \Omega^{\dagger}_{X+} \Omega_{X-}$$

Intertwining relations give relativistic invariance of S:

$$U(\Lambda, a)\Omega_{X\pm} = \Omega_{X\pm} U_{\mathcal{A}}(\Lambda, a)$$

but the symmetries of the mappings $\Phi_{X\mathcal{A}}$ are limited to the Kinematic subgroups

$$U(K_X)\Phi_{X\mathcal{A}} = \Phi_{X\mathcal{A}} U_{\mathcal{A}}(K_X)$$

Asymptotic completeness means that $\Omega_{\pm}$ are unitary mappings from $\mathcal{H}_{\mathcal{A}}$ to $\mathcal{H}$. S is the scattering operator, it is independent of the choice of $\Phi_{X\mathcal{A}}$.

Equivalences I: (general properties of scattering theory)

$$H = P^- + P^3 \qquad\qquad P^3\Phi_{I\mathcal{A}} = \Phi_{I\mathcal{A}} P^3_{\mathcal{A}}$$

$$e^{iHt}\Phi_{I\mathcal{A}}e^{-iH_{\mathcal{A}}} = e^{i(P^- + P^3)t}\Phi_{I\mathcal{A}}e^{-i(P^-_{\mathcal{A}} + P^3_{\mathcal{A}})t} = e^{iP^- t}\Phi_{I\mathcal{A}}e^{-iP^-_{\mathcal{A}} t}.$$

$$H = \frac{1}{2}(P^+ + P^-) \qquad\qquad P^+\Phi_{F\mathcal{A}} = \Phi_{F\mathcal{A}} P^+_{\mathcal{A}}$$

$$e^{iHt}\Phi_{F\mathcal{A}}e^{-iH_{\mathcal{A}}} = e^{i(P^- + P^+)t/2}\Phi_{F\mathcal{A}}e^{-i(P^-_{\mathcal{A}} + P^+_{\mathcal{A}})t/2} = e^{iP^- t/2}\Phi_{F\mathcal{A}}e^{-iP^-_{\mathcal{A}} t/2}.$$

$$\Omega_\pm(H, \Phi_{X\mathcal{A}}, H_{\mathcal{A}}) = \Omega_\pm(P^-, \Phi_{X\mathcal{A}}, P^-_{\mathcal{A}})$$

$\Phi_{I\mathcal{A}}$ and $\Phi_{F\mathcal{A}}$ result in the same scattering operator (follows by cluster properties - property of Haag-Ruelle scattering).

$$\Omega_\pm(H_F, \Phi_{F\mathcal{A}}, H_{\mathcal{A}}) = \Omega_\pm(H_I, \Phi_{I\mathcal{A}}, H_{\mathcal{A}})$$

For either $\Phi_{I\mathcal{A}}$. S can be calculated using H or P^-.

Equivalences II: (Ekstein's theorem - $W \in$ group of scattering equivalences)

$$S(H, \Phi_\mathcal{A}, H_\mathcal{A}) = S(H', \Phi'_\mathcal{A}, H_\mathcal{A})$$

$$\Downarrow$$

$$W = \Omega_+(H', \Phi'_\mathcal{A}, H_\mathcal{A})\Omega^\dagger_+(H, \Phi_\mathcal{A}, H_\mathcal{A}) = \Omega_-(H', \Phi'_\mathcal{A}, H_\mathcal{A})\Omega^\dagger_-(H, \Phi_\mathcal{A}, H_\mathcal{A})$$

$$W\Omega^\dagger_\pm(H, \Phi_\mathcal{A}, H_\mathcal{A}) = \Omega^\dagger_\pm(H', \Phi'_\mathcal{A}, H_\mathcal{A})$$

$$H' = WHW^\dagger \qquad \lim_{t\to\pm\infty} \|(\Phi'_\mathcal{A} - W\Phi_\mathcal{A})e^{-iH_\mathcal{A}t}|\psi\rangle\| = 0$$

Conversely, W unitary satisfying

$$H' := WHW^\dagger$$

$$\lim_{t\to\pm\infty} \|(\Phi'_\mathcal{A} - W\Phi_\mathcal{A})e^{-iH_\mathcal{A}t}|\psi\rangle\| = 0$$

$$\Downarrow$$

$$W\Omega^\dagger_\pm(H, \Phi_\mathcal{A}, H_\mathcal{A}) = \Omega^\dagger_\pm(H', \Phi'_\mathcal{A}, H_\mathcal{A})$$

and

$$S(H, \Phi A, H_\mathcal{A}) = S(H', \Phi' A, H_\mathcal{A})$$

Construction of Forms of Dynamics:

Choose V_I and V_F unitary operators on $\mathcal{H}$ satisfying

$$[V_F, U(K_F)] = 0 \qquad [V_I, U(K_I)] = 0$$

Define

$$\boxed{U_I(\Lambda, a) := V_I U(\Lambda, a) V_I^\dagger \qquad U_F(\Lambda, a) := V_F U(\Lambda, a) V_F^\dagger}$$

Assume in addition

$$\lim_{t\to\pm\infty} \|(\Phi_{VI\mathcal{A}} - V_I \Phi_{I\mathcal{A}}) e^{-iH_{\mathcal{A}} t} |\psi\rangle\| = 0$$

$$\lim_{t\to\pm\infty} \|(\Phi_{VF\mathcal{A}} - V_F \Phi_{F\mathcal{A}}) e^{-iP^-_{\mathcal{A}} t} |\psi\rangle\| = 0$$

By Ekstein's theorem the resulting unitary representations of the Poincaré group are scattering equivalent to the original theory and to each other.

Equivalence III

$$H_F = V_F H V_F^\dagger \qquad H_I = V_I H V_I^\dagger$$

$$P_F^- = V_F P^- V_F^\dagger \qquad P_I^- = V_I P^- V_I^\dagger .$$

for $(\Lambda, a) \in K$

$$\boxed{U_I(\Lambda_{K_I}, a_{K_I}) = U(\Lambda_{K_I}, a_{K_I}) \qquad U_F(\Lambda_{K_F}, a_{K_F}) := U(\Lambda_{K_F}, a_{K_F})}$$

Key relations:

$$V_I \Omega_\pm(H, \Phi_{I\mathcal{A}}, H_\mathcal{A}) = \Omega_\pm(H_I, V_I \Phi_{I\mathcal{A}}, H_\mathcal{A}) = \Omega_\pm(H_I, \Phi_{VI\mathcal{A}}, H_\mathcal{A})$$

$$V_F \Omega_\pm(H, \Phi_{F\mathcal{A}}, H_\mathcal{A}) = \Omega_\pm(H_F, V_F \Phi_{F\mathcal{A}}, H_\mathcal{A}) = \Omega_\pm(H_F, \Phi_{F\mathcal{A}}, H_\mathcal{A})$$

$$\Omega_\pm(H, \Phi_{I\mathcal{A}}, H_\mathcal{A}) = \Omega_\pm(H, \Phi_{I\mathcal{F}}, H_\mathcal{A})$$

Combining these gives

$$\Omega_\pm(H_I, \Phi_{VI\mathcal{A}}, H_\mathcal{A}) = V_I \Omega_\pm(H, \Phi_{I\mathcal{A}}, H_\mathcal{A}) =$$

$$V_I \Omega_\pm(H, \Phi_{F\mathcal{A}}, H_\mathcal{A}) = V_I V_F^\dagger \Omega_\pm(H, \Phi_{F\mathcal{A}}, H_\mathcal{A}) = V_I V_F^\dagger \Omega_\pm(P^-, \Phi_{F\mathcal{A}}, P_\mathcal{A}^-)$$

Since the scattering equivalences form a group it follows that

$$S(H_I, \Phi_{VI\mathcal{A}}, H_\mathcal{A}) = S(H, \Phi_\mathcal{A}, H_\mathcal{A}) = S(P^-, \Phi_{F\mathcal{A}}, P_\mathcal{A}^-)$$

Contrast with QM case:

Start with a direct integral of irreducible representations in covariant form, $U_{cov}(\Lambda, a)$

$$\underbrace{U_F(\Lambda, a) \leftrightarrow \left\{ \begin{array}{c} U_0(\Lambda, a) \\ U_{cov}(\Lambda, a) \end{array} \right\} \leftrightarrow U_I(\Lambda, a)}_{\leftarrow V_F V_I^\dagger \rightarrow}$$

$$QM: \qquad U_X(K_X)U_0^\dagger(K_X) = U_X(K_X)U_{cov}^\dagger(K_X) = I \qquad X = P, F, I$$

$$\underbrace{U_f(\Lambda, a) \leftrightarrow U_{cov}(\Lambda, a) \leftrightarrow U_I(\Lambda, a)}_{\leftarrow V_f V_I^\dagger \rightarrow}$$

$$QFT: \qquad U_X(K_X)U_{cov}^\dagger(K_X) = I \qquad X = P, F, I$$

Difference in field theory case: Absence of $U_0(\Lambda, a)$ or M_0.

Kinematic transformations?

Choose an irreducible basis associated with the representation $U_{cov}(\Lambda, a)$

$$\langle (m,s,d)\mathrm{p},\mu | U_I(K_I)|\Psi_I\rangle = \langle \Psi_I | U^\dagger_{cov}(K_I)|(m,s,d)\mathrm{p},\mu\rangle^*$$

$$\langle (m,s,d)p^+\mathrm{p}_\perp,\mu | U_F(K_F)|\Psi_F\rangle = \langle \Psi_F | U^\dagger_{cov}(K_F)|(m,s,d)p^+\mathrm{p}_\perp,\mu\rangle^*$$

Irreducible bases of $\mathcal{H}$ with instant or light-front variables are related by a variable change:

$$|(m,s,d)\mathrm{p},\mu_c\rangle \qquad |(m,s,d)p^+,\mathrm{p}_\perp,\mu_f\rangle \qquad |(m,s,d)\mathrm{v},\mu_c\rangle$$

The original $U(\Lambda, a)$ plays the role of $U_0(\Lambda, a)$

Remarks:

1. In this construction the unitary representation of the original field theory behaves like a kinematic representation - but it is not a free particle representation; it is scattering equivalent to the other representations.
2. The instant and front-form dynamical operators are not diagonal in the irreducible basis of the original field theory.
3. Instant and front-form dynamical operators commute with the generators of kinematic subgroups in the in the original representation of the field theory.
4. The theories have the same S matrix elements and are all related by unitary scattering equivalences.

Remarks:(continued)

5. The vacuum in the instant, light-front and original theory are unitarily equivalent and can be made identical depending on the choice of V_Xs.
6. There is a large class of scattering equivalences that preserve a given kinematic subgroup. This means that the "instant" and "front form" representations are not unique.
7. Some common elements in construction of Sokolov and Shatnyi (except no $U_0(\Lambda, a)$).

Spontaneous Symmetry Breaking 1:

The signal for spontaneous symmetry breaking is a 0 mass particle in the mass spectrum. Coleman (Goldstone) gives the condition:

$$\lim_{R\to\infty} \langle 0|[Q_R, \phi(y)]|0\rangle \neq 0$$

implies the existence of a 0 mass particle. Here

$$\langle 0|[Q_R, \phi(y)]|0\rangle := \langle 0|[\int d\mathrm{x}\chi_R(|\mathrm{x}|)j^0(\mathrm{x},t), \phi(y)]|0\rangle$$

where $\chi_R(|\mathrm{x}|)$ is a smooth function that is 1 for $|\mathrm{x}| < R$ and 0 for $|\mathrm{x}| > R + \epsilon$ for some finite positive ϵ. The cutoff function χ_R ensures that the integral converges for large $|\mathrm{x}|$. Locality implies that the commutator vanishes for $x - y$ space-like. In this case

$$\langle 0|[Q_R, \phi(y)]|0\rangle := \langle 0|[\int d\mathrm{x}\chi_R(|\mathrm{x}|)j^0(\mathrm{x},t), \phi(y)]|0\rangle = \int d\mathrm{x}\langle 0|[j^0(\mathrm{x},t), \phi(y)]|0\rangle.$$

This condition does not require that the charge operator to exist.

Spontaneous Symmetry Breaking 2:

Current conservation implies

$$\langle 0|[\int d\mathrm{x}\partial_\mu j^\mu(\mathrm{x},t),\phi(y)]|0\rangle = 0.$$

Inserting a complete set of intermediate states gives

$$0=\sum\int d\mathrm{x}\left(\langle 0|\partial_\mu j^\mu(\mathrm{x},t)|p,n\rangle\frac{d\mathrm{p}}{2p_n^0}\langle p,n|\phi(y)|0\rangle\right.$$

$$\left.-\langle 0|\phi(y)|p,n\rangle\frac{d\mathrm{p}}{2p_n^0}\langle p,n|\partial_\mu j^\mu(\mathrm{x},t)|0\rangle\right)=$$

$$\sum\int d\mathrm{x}\left(\langle 0|\partial_\mu j^\mu(0,0)|p,n\rangle\frac{d\mathrm{p}}{2p_n^0}\langle p_r,n|\phi(0)|0\rangle e^{ip\cdot(x-y)}\right.$$

$$\left.-\langle 0|\phi(0)|p,n\rangle\frac{d\mathrm{p}}{2p_n^0}\langle p,n|\partial_\mu j^\mu(0,0)|0\rangle e^{ip\cdot(y-x)}\right).$$

where Poincaré covariance has been used to remove the non-trivial x, y and p dependence from the matrix elements. p_r is the constant rest four momentum for massive states and a constant light-like vector for massless states.

Spontaneous Symmetry Breaking 3:

For a scalar field theory the vacuum expectation value of the current vanishes so the vacuum does not appear as an intermediate state. The Lehmann weights that appear in this matrix element

$$\sigma(m_n)m_n^2 = \int \frac{dp_{nr}}{2p_n^0} \langle 0|\partial_\mu j^\mu(0,0)|p_{nr}, n\rangle \langle p_n, n|\phi(0)|0\rangle$$

and

$$\sigma^*(m_n)m_n^2 = \int \frac{dp_{nr}}{2p_n^0} \langle 0|\phi(0)|p_n, n\rangle \langle p_n, n|\partial_\mu j^\mu(0,0)|0\rangle$$

must vanish when added. Using the same expansion in

$$\langle 0|[Q_R, \phi(y)]|0\rangle = 0$$

gives the same result, but without the factor m^2. This will vanish unless $\sigma(m)$ includes a $\delta(m_n)$. An operator like Q_R does not exist in the light front case because locality is not available to cutoff the integral. The operator Q_R exists in both representations, but it is dynamical in the light front case.

Conclusions:

1. It is possible to construct scattering equivalent theories with instant and light-front kinematic subgroups that are scattering equivalent to the covariant field theory (Wightman functions).
2. The covariant field theory unitary representation of the Poincaré group replaces the non-interacting representation.
3. The vacuum vectors can be chosen to be identical but generally are unitarily equivalent.
4. Restriction to a light front hyperplane does not completely define a vacuum functional.
5. Implications for perturbative calculations remain an open problem.

$$\langle 0|B^\dagger B|0\rangle$$

$$B(x) = U(x)BU^\dagger(x) \to \tilde{B}(p)$$

$$\tilde{B}(p) \to \tilde{B}(p)h(p) \to \hat{B}(x) =$$

$$U(t)\hat{B}(0,\mathrm{x})U^\dagger(t) = U(x^+)\hat{B}(0,x^-,\mathrm{x}_\perp)U^\dagger(x^+)$$

$$A_F = \tilde{\hat{B}}(x^+ = 0, p^+, \mathrm{p}_\perp)$$

$$A_I = \tilde{\hat{B}}(t = 0, \mathrm{p})$$

integrals of $\tilde{\phi}(p)h(p)$ over p^0 or p^-.