Cluster Properties and Particle Production in Relativistic Quantum Mechanics

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Relevance

- The cluster property is the justification for few-body physics

Few-body measurements/calculations

$$\Downarrow$$

Few-body operators

$$\Downarrow$$

Many-body Hamiltonian

Cluster properties and production

- Physics at sub-nucleon scales $\Rightarrow$ Poincaré invariant models
- Cluster properties are difficult to satisfy in Poincaré invariant models
- In models with particle production there is no few-body problem

Cluster Properties (fixed N)

$$\lim_{|r_i - r_j| \to \infty} [U(\Lambda, y) - U_a(\Lambda, y)] \, T_a(r_1, \cdots, r_n) \to 0$$

$$U_a(\Lambda, y) := \otimes_i U_{a_i}(\Lambda, y)$$

$$T_a(r_1, \cdots, r_n) := \otimes_i U_{a_i}(I, r_i)$$

Goal

- Construct Poincaré invariant quantum models of reactions which change particle number
- Construct few degree-of-freedom models which are (i) directly comparable to experiment (ii) decoupled from the many-body problem (iii) related to the many-body problem by cluster properties

Strategy - start with simple models

- Separate the difficulties due to infinite number of degrees of freedom from the difficulties due to particle production
- Limit the number of degrees of freedom using conservation laws which restrict the production channels

Models in this class include generalizations of the relativistic Lee model and relativistic isobar models

Conservation laws

- Introduce conserved positive "charges"

$$(n_1, n_2); \quad N = (1,0), \quad \pi = (0,1),$$
$$\Delta = (1,1), \quad \rho = (0,2), \quad \cdots$$

- Introduce an ordering on charge number

$$(n_1, n_2) \leq (n_1', n_2') \iff n_1 \leq n_1', n_2 \leq n_2'$$

- Use induction on charge number

Factorization into Tensor Products

$$\mathcal{H}_{(1,2)} = \mathcal{H}_{N\pi\pi'} \oplus \mathcal{H}_{\Delta\pi'} \oplus \mathcal{H}_{\Delta'\pi} \oplus \mathcal{H}_{N\rho}$$

$$\mathcal{H}_{(1,2)} = \underbrace{[(\mathcal{H}_{N\pi} \oplus \mathcal{H}_{\Delta}) \otimes \mathcal{H}_{\pi'}]}_{\mathcal{H}_a} \oplus \underbrace{[\mathcal{H}_{\Delta'\pi} \oplus \mathcal{H}_{N\rho}]}_{\mathcal{H}^a}$$

$$\mathcal{H} = \mathcal{H}_a \oplus \mathcal{H}^a = (\otimes_i \mathcal{H}_{a_i}) \oplus \mathcal{H}^a$$

$$H = \mathcal{H}_a \bigoplus \mathcal{H}^a$$

- Cluster property for separation a only makes sense on the subspace $\mathcal{H}_a$
- The residual spaces $\mathcal{H}^a$ cause technical difficulties
- General construction is treated in detail in

1. WP, nucl-th/0201013; JMP 43,6024(2002)
2. WP, nucl-th/0302023; to appear PRC

Cluster Condition on $\mathcal{H}_a \bigoplus \mathcal{H}^a$

$$\lim_{|r_i - r_j| \to \infty} [U(\Lambda, y) - U_a(\Lambda, y)] T_a(r_1, \cdots, r_n) \Pi_a = 0$$

"N-charge" interaction $V_N \quad \Rightarrow \quad \forall\, a$

$$\lim_{|r_i - r_j| \to \infty} V_N T_a(r_1, \cdots, r_n) \Pi_a = 0$$

- Cluster condition fixes $U(\Lambda, y)$, and generators, up to "N-charge" operators

Construction Summary

1. Use Wigner's form of dynamics for Poincaré invariant addition of interactions
2. Use Poincaré Clebsch-Gordan coefficients and Wigner's form of dynamics in different orders to generate scattering equivalences
3. Use algebraic properties of scattering equivalences to restore cluster properties

Wigner's form of dynamics

- Construct irreps of the Poincaré group using j, M, four commuting functions of generators , h_i, four complementary observables Δ_{h_i}

$$|(j,m),h\rangle$$

$$U(\Lambda,y)|(j,m)h,d\rangle = \sum_{h'} |(j,m)h',d\rangle D^{jm}_{h'h}(\Lambda,y)$$

Wigner's form of dynamics

- Use $(j,m)h$ Clebsch-Gordan coefficients

$$\langle (j,m)h,d|(j_1,m_1)h_1(j_2,m_2)h_2\rangle$$

to decompose $\mathcal{H}_1 \otimes \mathcal{H}_2$ into direct integral of free-particle irreducible representations

$$U_0(\Lambda,y)|(j,m)h,d\rangle = \sum_{h'} |(j,m)h',d\rangle D^{jm}_{h'h}(\Lambda,y)$$

Dynamical perturbations of irreps

$$M = M_0 + V \qquad [V, h_i] = [V, j^2] = [V, \Delta h_i] = 0$$

$$M|(j, m_n)h\rangle = m_n|(j, m_n)h\rangle$$

$$U(\Lambda, y)|(j, m_n)h\rangle = \sum_{h'} |(j, m_n)h'\rangle D^{j m_n}_{h'h}(\Lambda, y)$$

Cluster properties $\Leftrightarrow$ irreps

- Representations with $j = j_0$ do not cluster for more than two charges

$$
\begin{array}{ccc}
|(12)\otimes(3)\rangle & \xrightarrow{\langle AB|C\rangle_0} & |((12)(3))\rangle \\
V_{(12)(3)}\big\downarrow & & \big\downarrow V_{((12)(3))} \\
|(12)_I\otimes(3)\rangle & \xrightarrow[\langle AB|C\rangle_I]{} & |((12)_I(3))\rangle \underbrace{\sim}_{A_{(12)(3)}} \overline{|((12)(3))_I\rangle}
\end{array}
$$

- A_a scattering equivalence

Scattering Equivalences

- Scattering equivalences A_a are unitary elements of a C^* algebra of asymptotic constants
- The C^* algebra provides a functional calculus to construct functions of non-commuting scattering equivalences
- The operators in this C^* algebra relate Wigner's forms of dynamics to representations that satisfy the cluster condition

Example - three-charge problem

$$\mathcal{H} = \mathcal{H}_{(1,2)} = \mathcal{H}_{N\pi\pi'} \oplus \mathcal{H}_{\Delta\pi'} \oplus \mathcal{H}_{\Delta'\pi} \oplus \mathcal{H}_{N\rho}$$

$$H_{(N\pi)(\pi')} = \begin{pmatrix} K_{N\pi\pi'} + V_{N\pi} & V_{N\pi;\Delta} & 0 & 0 \\ V_{\Delta;N\pi} & K_{\Delta\pi'} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Example

$$H = A^{\dagger} \left(\sum_a \mathcal{C}_a \underbrace{A_a (\sum_i H_{a_i}) A_a^{\dagger}}_{\bar{H}_{wa}} \right) A$$

$$A = A(A_{a_1}, \cdots, A_{a_n}) \rightarrow A_a$$

$$\Downarrow$$

$$H \rightarrow H_a = \sum_i H_{a_i}$$

Observations

$$H = H_{(N\pi)(\pi')} + H_{(N\pi')(\pi)} + H_{(\pi\pi')(N)} - 2H_{(N)(\pi)(\pi')} + V_3$$

- The operators A generate many-body interactions and dynamical j^2, h_i, Δh_i
- The few-charge dynamics determines the many-charge dynamics up to "N-charge" operators

Summary

- Models have few-charge problems directly tied to experiment
- Models are Poincaré invariant
- Models satisfy cluster properties
- Many-body interactions and dynamical spin are unavoidable consequences of this construction
- Few-charge operators determine many-charge operators up to N-charge interactions

Outlook - Realistic Models

- Replace “conserved charges” by # of physical particles ?
- Construction requires threshold-by-threshold block diagonalizaton of $U(\Lambda, a)$
- Simplest production interaction is a short-range $2 \rightarrow 3$ interaction
- Allows clustering and a “few-body” problem directly tied to experiment