The Light Front Vacuum and Dynamics

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Light-Front Vacuua

Collaborator:

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Background Material:

F. Coester and W. N. Polyzou, Foundations of Physics 24, 387(1994).

W. N. Polyzou, Few-Body Systems 27,57(1999).

Summary

Setting:
Algebra of free scalar fields restricted to a light front.

Summary of Results

1. There is a one-to-one correspondence between kinematically invariant $(3+1)$ vacuum states and Euclidean-invariant path measures $D[\phi]$ in two dimensions.
2. For every positive P^- that annihilates the Fock vacuum there is a corresponding positive P^- on the Hilbert space with non-trivial vacuum that annihilates the non-trivial vacuum.

Summary of Results

3. For each non-trivial vacuum there is a unitary transformation that maps to a model with a different light-front orientation. The condition that the scattering operator is independent of the orientation of the light front is a necessary and sufficient condition for Poincaré invariance.

Kinematic Conventions

$$x_{\hat{n}}^{\pm} = x^0 \pm \hat{n} \cdot \vec{x} \qquad \vec{x}_{\hat{n}\perp} = \hat{n} \times \vec{x}$$

$$p_{\hat{n}}^{\pm} = p^0 \pm \hat{n} \cdot \vec{p} \qquad \vec{p}_{\hat{n}\perp} = \hat{n} \times \vec{p}$$

$$\tilde{x}_{\hat{n}} := (x_{\hat{n}}^{-}, \vec{x}_{\hat{n}\perp}) \qquad \tilde{p}_{\hat{n}} := (p_{\hat{n}}^{+}, \vec{p}_{\hat{n}\perp})$$

$$\tilde{p}_{\hat{n}} \cdot \tilde{x}_{\hat{n}} = -\frac{1}{2} p_{\hat{n}}^{+} x_{\hat{n}}^{-} + \vec{p}_{\hat{n}\perp} \cdot \vec{x}_{\hat{n}\perp}$$

Kinematic Conventions

$$f_{\hat{n}}(x) = \delta(x^{+}_{\hat{n}}) f(\tilde{x}_{\hat{n}})$$

$$\hat{f}(\tilde{p}_{\hat{n}}) = (2\pi)^{-3/2} \int d\tilde{x}_{\hat{n}} e^{-i\tilde{p}_{\hat{n}}\cdot\tilde{x}_{\hat{n}}} f(\tilde{x}_{\hat{n}})$$

$$f(\tilde{x}_{\hat{n}}) = (2\pi)^{-3/2} \int d\tilde{p}_{\hat{n}} e^{i\tilde{p}_{\hat{n}}\cdot\tilde{x}_{\hat{n}}} \hat{f}(\tilde{p}_{\hat{n}})$$

Light-Front Fields

$$\phi(f_{\hat{n}}) = \int d\tilde{x}\phi(\tilde{x})f(\tilde{x})$$

$$[\phi(f_{\hat{n}}), \phi(g_{\hat{n}})]_- = \frac{1}{2}[(f,g)_{\hat{n}} - (g,f)_{\hat{n}}]$$

$$(f,g)_{\hat{n}} := \int \frac{d\tilde{p}\theta(p^+)}{p^+}\hat{f}^*(\tilde{p})\hat{g}(\tilde{p})$$

Key Observation

$(f,g)_{\hat{n}}$ has a logarithmic singularity at $p^+_{\hat{n}} = 0$.

A non-singular $[\phi(f),\phi(g)]_-$ requires test functions that vanish at $p^+_{\hat{n}} = 0$

$$\hat{f}(\tilde{p}_{\hat{n}}) = p^+_{\hat{n}}\hat{h}(\tilde{p}_{\hat{n}}) \Rightarrow [\phi(f_{\hat{n}}),\phi(g_{\hat{n}})] \text{ defined}$$

Light-Front Free Field Algebra $\mathcal{A}_{\hat{n}}$

$$A := \sum_{k=1}^{n} c_k e^{i\phi(f_{\hat{n}k})} \qquad f_k \in \mathcal{S}_{\hat{n}}^{+}, c_k \in \mathbb{C}$$

$$AA' := \sum_{k=1}^{m} \sum_{l=1}^{n} c_k d_l e^{-\frac{1}{2}[\phi(f_{\hat{n}k}),\phi(g_{\hat{n}l})]} e^{i\phi(f_{\hat{n}k}+g_{\hat{n}l})}$$

$$\|A\| := [\sum c_l c_k^* e^{-\frac{1}{4}((f_l,f_l)+(f_k,f_k)-2(f_l,f_k)}]^{1/2}$$

Light-Front Free Field Algebra

$$A, A' \in \mathcal{A}_{\hat{n}}, \quad c \in \mathbb{C}$$

$$\Downarrow$$

$$A + cA', A^{\dagger}, AA', I \in \mathcal{A}_{\hat{n}}$$

$\mathcal{A}_{\hat{n}}$ = C^* algebra

Kinematic Symmetries

$$f'_k(\tilde{x}_{\hat{n}}) := f_k(\tilde{\Lambda}^{-1}(\tilde{x}_{\hat{n}} - \tilde{a}))$$

$$A' := \sum_{k=1}^{m} c_k e^{i\phi(f'_k)}$$

$$\alpha_{(\tilde{\Lambda},\tilde{a})}(A) = A'$$

- Leads to a unitary representation of the kinematic subgroup

Irreducibility (Weyl algebra)

$$U(f) = e^{i\phi(f)}$$

$$\Downarrow$$

$$U(f)U(g) = U(f+g)e^{\frac{i}{2}(Im(f,g)_{\hat{n}})}.$$

- Ensures Hilbert space operators are elements of $\mathcal{A}_{\hat{n}}$

Algebraic Isomorphisms

$$x \longrightarrow x' = Rx \qquad R \in SO(3)$$

$$f_{\hat{n}}(x) \longrightarrow f_{\hat{n}}(Rx) = f'_{R\hat{n}}(x')$$

$$\Downarrow$$

$$\mathcal{A}_{\hat{n}} \longrightarrow \mathcal{R}\mathcal{A}_{\hat{n}}\mathcal{R}^{-1} = \mathcal{A}_{\hat{n}'} \qquad \hat{n}' = R\hat{n}$$

- Relates light-front fi eld algebras with different light fronts.

Fock Vacuum - Fock Space

$$_{\hat{n}}\langle 0|e^{i\phi(f_{\hat{n}})}|0\rangle_{\hat{n}} = \mathcal{V}_{\hat{n}0}[e^{i\phi(f_{\hat{n}})}] := e^{-\frac{1}{4}(f,f)_{\hat{n}}}$$

$$|\Psi\rangle = A|0\rangle \qquad |\Phi\rangle = B|0\rangle$$

$$\Downarrow$$

$$\langle\Phi|\Psi\rangle =_{\hat{n}} \langle 0|B^{\dagger}A|0\rangle_{\hat{n}} = \mathcal{V}_{\hat{n}0}(B^{\dagger}A) =$$

$$\sum_{k=1}^{m}\sum_{l=1}^{n} c_k^* d_l \mathcal{V}_{\hat{n}0}[e^{i\phi(g_{\hat{n}l}-f_{\hat{n}k})}]e^{\frac{1}{2}[\phi(f_k),\phi(g_l)]_-}$$

General Vacuum States

$$\langle 0|A|0\rangle := \mathcal{V}_{\hat{n}}[A]$$

$$\mathcal{V}_{\hat{n}}[A + aA'] = \mathcal{V}_{\hat{n}}[A] + a\mathcal{V}_{\hat{n}}[A'] \qquad \mathcal{V}_{\hat{n}}[A^\dagger] = \mathcal{V}^*_{\hat{n}}[A]$$

$$\mathcal{V}_{\hat{n}}[A^\dagger A] \geq 0 \qquad \mathcal{V}_{\hat{n}}[\alpha_{(\tilde{\Lambda},\tilde{a})}(A)] = \mathcal{V}_{\hat{n}}[A]$$

$$\lim_{m\to\infty} \mathcal{V}_{\hat{n}}[e^{i\phi(f_m)}] = \mathcal{V}_{\hat{n}}[e^{i\phi(\lim f_m)}]$$

$$\mathcal{V}_{\hat{n}}[1] = 1 \qquad \lim_{\eta\to\infty} \mathcal{V}_{\hat{n}}[e^{i\phi(f+h_\eta)}] = \mathcal{V}_{\hat{n}}[e^{i\phi(f)}]\mathcal{V}_{\hat{n}}[e^{i\phi(h)}]$$

where $h_\eta(\tilde{x}) = h(\tilde{x} - \eta\tilde{d})$

Structure of Light-Front Vacuua

$$\mathcal{V}_{\hat{n}}[e^{i\phi(f)}] = e^{-\frac{1}{4}(f,f)_{\hat{n}}} e^{\sum i^n S^t_n(f)} = \mathcal{V}_{\hat{n}0}[e^{i\phi(f_{\hat{n}})}] v_{2\hat{n}}[f]$$

Kinematic Poincaré covariance

$$\Downarrow$$

$$S^t_n(f) = \frac{1}{n!} \int d\tilde{p}_1 \cdots d\tilde{p}_n s^t_n(p_{1\perp}, \cdots, p_{n\perp}) \times$$

$$\delta^2(p_{1\perp} + \cdots + p_{n\perp}) \prod \delta(p_i^+) \frac{\tilde{f}(\tilde{p}_i)}{p_i^+}$$

Structure of $v_{\hat{n}}[f_s]$

$$\tilde{f}_s(p_\perp) := \lim_{p^+\to 0} \frac{\tilde{f}(\tilde{p})}{p^+}$$

$$v_{\hat{n}2}[0] = 1 \qquad v_{\hat{n}2}[f_s]^* = v_{\hat{n}2}[-f_s^*] \qquad v_{\hat{n}2}[f_{s(R_z,a)}] = v_{\hat{n}2}[f_s]$$

$$\lim_{m\to\infty} v_{\hat{n}2}(f_{sm}) = v_{\hat{n}2}(\lim_{m\to\infty} f_{sm}) \qquad \sum_{ij} a_i a_j^* v_{\hat{n}2}[f_{si} - f_{sj}] \geq 0$$

$$\lim_{\eta\to\infty} v_{\hat{n}2}[f_s + h_{s\eta}] = v_{\hat{n}2}[f_s] v_{\hat{n}2}[h_s]$$

where

$$f_{s(R_z,a)}(\vec{x}_\perp) = f_s(R_z^{-1}(\vec{x}_\perp - \vec{a}_\perp)) \qquad h_{s\eta}(\vec{x}_\perp) := h_s(\vec{x}_\perp + \eta\hat{n}_\perp)$$

Classification of Vacuum Functionals

Theorem (Bochner-Minlos)

$$v_{\hat{n}2}[f_s] = \int D[\phi_2] e^{i\phi_2(f_s)}$$

where $\phi_2(\vec{x}_\perp)$ is a two-dimensional Euclidean invariant field.

Inequivalent Representations

Theorem

$$\mathcal{V}_1[A] \neq \mathcal{V}_2[A]$$

$$\Downarrow$$

$$|0\rangle_2 \neq U|0\rangle_1$$

Structure of P^-

- A Fock space P^- satisfies:

$$(P^-) = (P^-)^\dagger \geq 0 \qquad \mathcal{V}_{\hat{n}0}[P^-] = 0$$

$$\Downarrow$$

$$P^-|0\rangle = 0$$

- Commutation relations with kinematic generators

Structure of P^-

- P^- is a positive operator in $\mathcal{A}_{\hat{n}}$.
- QP^-Q is a positive operator on the Hilbert space with non-trivial vacuum where $Q := (I - |0\rangle_{vv}\langle 0|)$,
- Q commutes with all of the kinematic generators
- $P_v := QP^-Q$ defines a suitable "P^-" on the non-trivial Hilbert space.

Changing Light Fronts

$$\mathcal{A}_{\hat{n}'} = \mathcal{R}\mathcal{A}_{\hat{n}}\mathcal{R}^{-1}$$

$$\mathcal{V}_{\hat{n}'}[A'] = \mathcal{V}_{\hat{n}}[\mathcal{R}^{-1}A'\mathcal{R}]$$

Poincaré Invariance

$$P^-_{\hat{n}'} = \mathcal{R} P^-_{\hat{n}} \mathcal{R}^{-1}$$

Theorem: If S is independent of $\hat{n}$ then

$$S = \Omega^\dagger_{+\hat{n}} \Omega_{-\hat{n}} = \Omega^\dagger_{+\hat{n}'} \Omega_{-\hat{n}'}$$

$$\Downarrow$$

Poincaré Invariance

$$A[R] := \Omega_{+\hat{n}}\Omega^{\dagger}_{+\hat{n}'} = \Omega_{-\hat{n}}\Omega^{\dagger}_{-\hat{n}'}$$

$$\Downarrow$$

$$U_0(R)\Omega_{\pm\hat{n}} = \Omega_{\pm R\hat{n}}U_f(R)$$

$$U_{\hat{n}}(R) = A[R]U_0(R) = \Omega_{\pm\hat{n}}U_f(R)\Omega^{\dagger}_{\pm\hat{n}}$$

is the dynamical rotation subgroup of the Poincaré group

(note: the $\hat{n}$ independence is essential and non-trivial)

Concluding Remarks

- Candidates for non-trivial vacuua are in 1-1 correspondence with 2-d Euclidean measures.
- Each vacuum constrains the structure of the interaction.
- Scattering equivalence of dynamics on different light fronts is equivalent to Poincaré invariance - it relates the vacuum and dynamics.

Concluding Remarks

- Non-trivial vacuua may lead to useful framework for generating phenomenological models.