

Three-body scattering in Poincaré Invariant Quantum Mechanics

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Outline

- Why use quantum mechanics?
- Properties
- Structure of models
- Technical considerations
- Preliminary results
- Outlook

Why (I) use quantum theory

- QCD - more theory needed to distinguish approximation from truncation (Clay Millennium problem).
- Poincaré invariant quantum theory - satisfies all axioms of QFT except locality.
- Uniformly convergent solution algorithms. Limited scope.

Ideal Goals

- Quantitative understanding of hardonic physics in transition region
- Quantum theory
- Poincaré invariant probabilities
- Spectral condition
- Cluster properties
- Locality

Poincaré invariant quantum mechanics

Isolated particles

$$P = |\langle \Psi_f | \Psi_i \rangle|^2 = |\langle \Psi'_f | \Psi'_i \rangle|^2 = P'$$



$$U_1(\Lambda, a)$$



$$P_1^\mu, J_1^{\mu\nu}$$

(Wigner - 1939)

One-particle Hilbert space

II

(M, J) irreducible representation

$$M, J, h_i, \Delta h_i$$

$$\mathcal{H}_1 = L^2(\sigma(h)) \quad |(m, j) \mathbf{h}\rangle$$

$$U_1(\Lambda, a)| (m, j) \mathbf{h}\rangle = \sum' \int' |(m, j) \mathbf{h}'\rangle d\mathbf{h}' D_{\mathbf{h}'\mathbf{h}}^{jm}(\Lambda, a)$$

eg. $\mathbf{h} = (p^+, \mathbf{p}_\perp, \hat{\mathbf{z}} \cdot \mathbf{j}_f)$

N-particle Hilbert space

$$\mathcal{H} = \bigotimes_{i=1}^N \mathcal{H}_i$$

Tensor product basis

$$|(m_1, j_1), \mathbf{h}_1, \dots, (m_N, j_N), \mathbf{h}_N\rangle$$

Kinematic Poincaré group

$$U_0(\Lambda, a) := \bigotimes U_i(\Lambda, a)$$

$$U_0(\Lambda, a) |(m_1, j_1), \mathbf{h}_1, \dots, (m_N, j_N), \mathbf{h}_N\rangle =$$

$$\sum' \int' |(m_1, j_1), \mathbf{h}'_1, \dots, (m_N, j_N), \mathbf{h}'_N\rangle d\mathbf{h}' \prod_{i=1}^N D_{\mathbf{h}'_i \mathbf{h}_i}^{j_i m_i}(\Lambda, a)$$

This is reducible!

N free particle irreducible representations

Poincaré Clebsch-Gordan coefficients

$$\bigotimes_{i=1}^N |(m_i, j_i), \mathbf{h}_i\rangle \langle \text{CG} \rangle = \langle \text{CG} \rangle \bigoplus_{j, m, \mathbf{d}} |(m, j), \mathbf{h}; \mathbf{d}\rangle$$

$$\bigotimes_{i=1}^N D_{\mathbf{h}'_i \mathbf{h}_i}^{j_i m_i}(\Lambda, a) \langle \text{CG} \rangle = \langle \text{CG} \rangle \bigoplus_{j, m, \mathbf{d}} D_{\mathbf{h}' \mathbf{h}}^{j m}(\Lambda, a)$$

⇓

$$U_0(\Lambda, a) |(m, j), \mathbf{h}; \mathbf{d}\rangle = \sum' \int' |(m, j), \mathbf{h}'; \mathbf{d}\rangle d\mathbf{h}' D_{\mathbf{h}' \mathbf{h}}^{j m}(\Lambda, a)$$

Dynamics

$$M^2 = M_0^2 + V \geq 0$$

$$[V, j^2] = [V, \mathbf{h}] = [V, \Delta \mathbf{h}] = 0$$

Diagonalize M^2 in free particle irreducible basis



Simultaneous eigenstates of M^2, j^2, \mathbf{h}

$$|(\lambda, j), \mathbf{h}, d_I\rangle$$

(Bakamjian & Thomas - 1953)

Dynamical representation of Poincaré group

$$\{ |(\lambda, j), \mathbf{h}, \mathbf{d}_I \rangle \}$$

complete and

$$U(\Lambda, a) |(\lambda, j), \mathbf{h}, \mathbf{d}_I \rangle = \sum' \int' |(\lambda, j), \mathbf{h}'; \mathbf{d}_I \rangle d\mathbf{h}' D_{\mathbf{h}'\mathbf{h}}^{j\lambda}(\Lambda, a)$$

$U(\Lambda, a)$ = solution of dynamical problem

Ekstein's Theorem

$$\bar{M} := WMW^\dagger \quad \bar{M}_0 := M_0 \quad WW^\dagger = I$$

$$S = \bar{S} \Leftrightarrow \lim_{t \rightarrow \pm\infty} \|(W - I)e^{-iM_0 t}|\psi\rangle\| = 0$$

$$W = \bar{\Omega}_\pm(\bar{M}, M_0)\Omega_\pm^\dagger(M, M_0) \oplus |\bar{B}\rangle\langle B|$$

(Ekstein - 1960)

$$N = 2$$

$$V = 4mV_{nr} \quad M_0^2 = 4(\mathbf{k}^2 + m^2) \quad M^2 = 4m(H_{nr} + m)$$

$$\begin{aligned} \frac{d\sigma}{d\Omega}(k) &= \frac{(2\pi)^4}{v_{nr}} |\langle f | T_{H_{nr}} | i \rangle|^2 k^2 \frac{dk}{dE_{nr}} \\ &= \frac{(2\pi)^4}{v_r} |\langle f | T_M | i \rangle|^2 k^2 \frac{dk}{dE_r} \end{aligned}$$

No relativistic corrections for $N = 2$!

Ekstein \Rightarrow no loss of generality in choosing $V = 4mV_{nr}$

(Coester, Pieper & Serduke - 1975)

$$N = 3$$

$$M = M_0 + V_{12} + V_{23} + V_{31}$$

$$V_{12} = \sqrt{M_{12}^2 + q_{12}^2} - \sqrt{M_{012}^2 + q_{12}^2} \quad \dots$$

$$M_{12}^2 = 4(\mathbf{k}^2 + m^2) + 4mV_{NN} \quad M_{012}^2 = 4(\mathbf{k}^2 + m^2)$$

(Coester - 1965)

Structure dictated by S -matrix cluster properties

Faddeev Equations

$$T^{ab}(z) = \bar{\delta}^{ab}(z - M_0) + \sum_{c \neq a} V_c(z - M_c)^{-1} T^{cb}(z)$$

Input

$$\int' \langle a|c' \rangle dc' \langle c'|V_c(z - M_c)^{-1}|c \rangle$$

Technical considerations

$\langle a|c\rangle =$ Poincaré Group Racah coefficient

$$\langle a|c\rangle = \delta[\mathbf{h}, \mathbf{h}']\delta(M - M')\delta_{JJ'}R^{MJ}(d_a, d_c)$$

Technical considerations

$$z = z_c$$

$$\langle c' | V_c (z_c - M_c)^{-1} | c \rangle = \langle c' | T_c(z_c) (z_c - M_0)^{-1} | c \rangle =$$

$$\langle c' | V_c | c^- \rangle (z_c - M_0)^{-1} = \langle c' | \frac{M_c^2 - M_0^2}{m'_c + m_c} | c^- \rangle (z_c - M_0)^{-1} =$$

$$\langle c' | \frac{4m V_{NN}}{m'_c + m_c} | c^- \rangle (z_c - M_0)^{-1} = \frac{4m}{m'_c + m_c} \langle c' | t_{NN}(z_c) (z_c - M_0)^{-1} | c \rangle$$

Technical considerations

$$z \neq z_c$$

First resolvent equation

$$T_c(z') = T_c(z_c) + T_c(z') \frac{(z' - z_c)}{(z' - M_0)(z_c - M_0)} T_c(z_c)$$

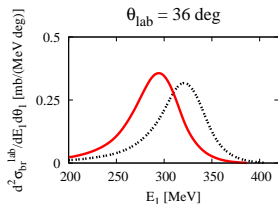
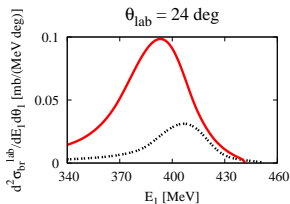
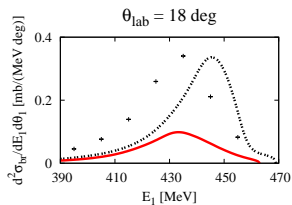
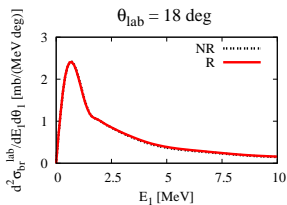
$$\langle c' | T_c(z_c) | c \rangle = \frac{4m}{m_c + m'_c} \langle c' | t_{NN}(z_c) | c \rangle$$

Input to Faddeev kernel

Calculations by T. Lin and Ch. Elster

(Lin et. al. nucl-th/0702005)

$$E_{lab} = 495 \text{ MeV}$$



Cluster properties

||

Poincaré invariance of isolated subsystems

Formulation of cluster properties

$a_i = \text{subsystem}$

$$\mathcal{H} = \mathcal{H}_{a_1} \otimes \cdots \otimes \mathcal{H}_{a_m}$$

$$T_{a_i}(x_i) := U_{a_i}(I, x_i)$$

$$\| [U(\Lambda, a) - \bigotimes_{i=1}^m U_{a_i}(\Lambda, a)] \bigotimes_{i=1}^m T_{a_i}(x_i) |\psi\rangle \| \rightarrow 0$$

as $(x_i - x_j)^2 \rightarrow +\infty$

Clustering in the 2 + 1 body problem

$$\begin{array}{ccc}
 U_{(12)_0}(\Lambda, a) \otimes U_3(\Lambda, a) & \xrightarrow{\langle AB|C \rangle_0} & U_{(12)_0(3)}(\Lambda, a) \\
 \downarrow V_{(12)(3)} & & \downarrow \bar{V}_{(12)(3)} \\
 U_{(12)_i}(\Lambda, a) \otimes U_3(\Lambda, a) & \xrightarrow{\langle AB|C \rangle_i} > & U_{(12)_i(3)}(\Lambda, a) \underbrace{\sim}_{W_p} \bar{U}_{(12)_i(3)}(\Lambda, a)
 \end{array}$$

Scattering equivalence does not preserve cluster properties of $U(\Lambda, a)$!

W_p Ekstein operator

$N = 3$ cluster properties of S follows from construction,

\exists Ekstein transformations W

$$\bar{U}(\Lambda, a) = WU(\Lambda, a)W^\dagger$$

restore cluster properties of $U(\Lambda, a)$

Ekstein transformations reduce cluster properties of N -particle systems to a **solvable** combinatorial problem.

(Sokolov - 1977)

Tensor and spinor densities

$$\langle (\lambda, s)\mathbf{h} | J^\mu(x) | (\lambda', s')\mathbf{h}' \rangle = \sum \int \langle (\lambda, s)\bar{\mathbf{h}} | J^\nu(\Lambda x + a) | (\lambda', s')\bar{\mathbf{h}}' \rangle D_{\bar{\mathbf{h}}\mathbf{h}}^{*j\lambda}(\Lambda, a) D_{\bar{\mathbf{h}}'\mathbf{h}'}^{j'\lambda'}(\Lambda, a) \Lambda_\nu^\mu$$

$$\langle (\lambda, s)\mathbf{h} | J^\mu(x) | (\lambda', s')\mathbf{h}' \rangle = \sum \langle GC \rangle \times \langle \text{invariant matrix elements} \rangle$$

invariant matrix elements = independent form factors

$\langle CG \rangle$ depend on dynamics through λ, λ'

Cluster Properties and currents

$$U(\Lambda, a) J^\nu(x) U^\dagger(\Lambda, a) = J^\nu(\Lambda x + a) \Lambda_\nu^\mu$$

↓

$$\sum_i U_{a_i}(\Lambda, a) J_{a_i}^\nu(x) U_{a_i}^\dagger(\Lambda, a) = \sum_i J_{a_i}^\nu(\Lambda x + a) \Lambda_\nu^\mu$$

Requires that $U(\Lambda, a)$ clusters

$$\langle f | J^\mu(x) | i \rangle =$$

$$\underbrace{\langle f | W}_{\langle \bar{f} |} \underbrace{W^\dagger J^\mu(x) W}_{\bar{J}(x)} \underbrace{W^\dagger | i \rangle}_{| \bar{i} \rangle}$$

Summary and Outlook

- Poincaré invariant quantum theory is a framework for making mathematical models of limited scope that satisfy all of the axioms of quantum field theory except microscopic locality.
- Uniformly convergent solution algorithms relate experiment directly to Hamiltonian.
- The structure of the dynamical models is only determined up to a (large) group of unitary scattering equivalences. This group has a subgroup that preserves cluster properties.
- High precision NN interactions can be used as input without modification.

- Preliminary results (first term in the multiple scattering series) for three-body breakup using a Malfliet-Tjon interaction were presented.
- Dynamically covariant currents or fields can be calculated using the Poincaré Wigner-Eckart theorem. The Clebsch-Gordan coefficients generate exchange currents and dynamical correlations.
- Open problems - cluster properties with infinite number of degrees of freedom. Coupling to electromagnetic interaction beyond one photon exchange.

Thanks!

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