

Cluster Properties in Relativistic Quantum Mechanical Models with Particle Production

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Outline

- **Motivation.**
- **Poincaré invariant quantum mechanics.**
- **Failure of Cluster properties.**
- **Solution fixed N.**
- **Production.**

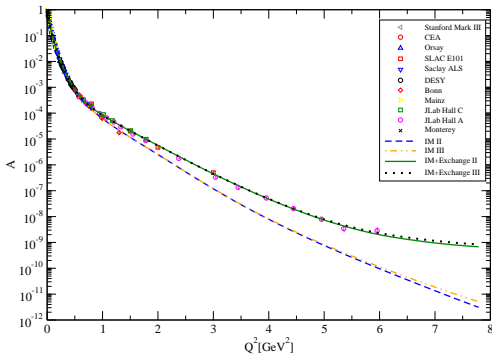
Motivation

- Why use Poincaré invariant quantum mechanics?
 - To formulate realistic quantum mechanical modes of few-hadron systems at the few GeV scale.
 - Consistent with QM, exact Poincaré invariance, spectral condition, and **cluster properties (fixed N)**.
 - Predictive power comes from existence of few-body problems directly related to experiment and cluster properties.

Examples of realistic calculations based on RQM

Elastic electron-deuteron scattering: A 0-8 GeV²

$A(Q^2)$: BBBA; II, III

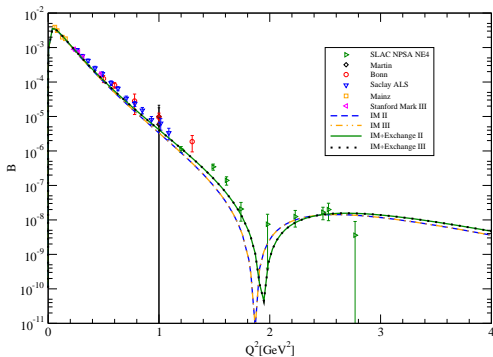


Y. Huang and W.P. PRC 80,025503(2009)

Examples of realistic calculations based on RQM

Elastic electron-deuteron scattering: B 0-8 GeV^2

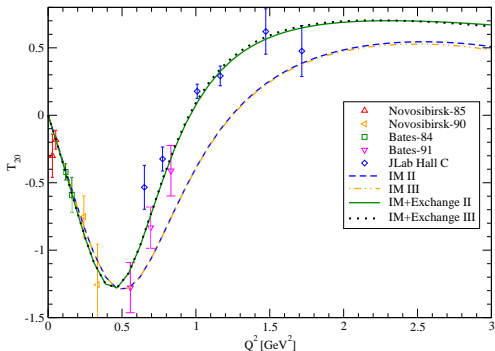
$B(Q^2)$: BBBA; II, III



Examples of realistic calculations based on RQM

Elastic electron-deuteron scattering: T_{20} 0-3 GeV^2

$T_{20}(Q^2, 70^\circ)$: BBBA; II, III



What is Poincaré invariant quantum mechanics?

- Quantum theory of a bounded number of degrees of freedom:

$$\mathcal{H} = \bigoplus_{j=1}^N \left(\bigotimes_i^{n_j} \mathcal{H}_i \right)$$

- The Poincaré group is a symmetry group of the theory:

$$U(\Lambda, a) : \mathcal{H} \rightarrow \mathcal{H}$$

Construction of dynamical $U(\Lambda, a)$ (summary)

- Model Hilbert space = direct sum of tensor products of irreducible representation spaces of the Poincaré group.
- Use Poincaré group Clebsch-Gordan coefficients to decompose the tensor products into direct integrals of Poincaré group irreducible representations.
- Add interactions to the non-interacting mass Casimir operator, M_0 , that commute with the non-interacting spin and commute with and are independent of the operators that label vectors in irreducible subspaces.
- Diagonalize M in the non-interacting irreducible basis - **eigenstates are complete and transform irreducibly.**

$N = 2$ example

$$M = M_0 + V$$

$$|(m_1, j_1), \mathbf{p}_1, \mu_1\rangle \otimes |(m_2, j_2), \mathbf{p}_2, \mu_2\rangle \rightarrow |(m, j), \mathbf{p}, \mu, l, s\rangle$$

$$\langle (j, m), \mathbf{p}, \mu, l, s | V | (j', m'), \mathbf{p}', \mu', l', s' \rangle =$$

$$\delta_{jj'} \delta(\mathbf{p} - \mathbf{p}') \delta_{\mu\mu'} \langle k, l, s | v^j | k', l', s' \rangle$$

$$M_0 = m(k) = \sqrt{m_1^2 + \mathbf{k}^2} + \sqrt{m_2^2 + \mathbf{k}^2}$$

- Diagonalize M in non-interacting irreducible basis.
- Simultaneous eigenstates of M, j, \mathbf{p}, j_z are **complete** and transform irreducibly.

Dynamical $U(\Lambda, a)$

$$|(j', \lambda), \mathbf{p}', \mu'\rangle$$

$$\langle (j, m), \mathbf{p}, \mu, l, s, |(j', \lambda), \mathbf{p}', \mu'\rangle = \delta_{jj'} \delta_{\mu\mu'} \delta(\mathbf{p} - \mathbf{p}') \phi_{j,\lambda}(k, l, s)$$

$$\begin{aligned} & (\lambda - \sqrt{m_1^2 + k^2} + \sqrt{m_2^2 + k^2}) \phi_{j,\lambda}(k, l, s) = \\ & \sum_{s', l'} \int_0^\infty \langle k, l, s | v^j | k', l', s' \rangle k^2 dk \phi_{j,\lambda}(k', l', s') \end{aligned}$$

$$U(\Lambda, a) |(j, \lambda), \mathbf{p}, \mu\rangle = \int \mathcal{D}^{\mathbf{p}' \mu'} |(j, \lambda), \mathbf{p}', \mu'\rangle d\mathbf{p}' \mathcal{D}_{\mathbf{p}', \mu'; \mathbf{p}, \mu}^{j\lambda}[\Lambda, a]$$

$$\mathcal{D}_{\mathbf{p}', \mu'; \mathbf{p}, \mu}^{j\lambda}[\Lambda, a] := \langle (j, \lambda), \mathbf{p}', \mu' | U(\Lambda, a) |(j, \lambda), \mathbf{p}, \mu \rangle$$

Failure of cluster properties (2 + 1 system)

$$\mathbf{q} := \Lambda^{-1}(\mathbf{P}/M_0(k))p_3$$

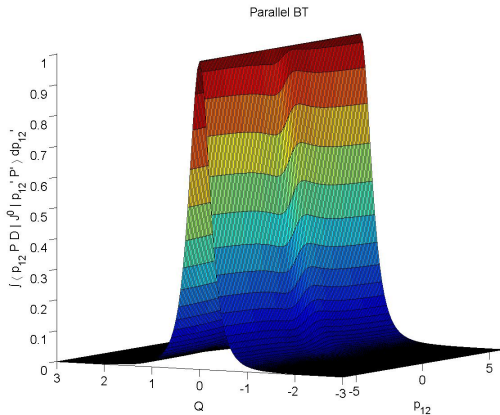
$$p_3 = \mathbf{q} + \mathbf{P}F(k, \mathbf{q}, \mathbf{P})$$

$$\langle \mathbf{P}, \mathbf{q}, \mathbf{k} | V_{12} | \mathbf{P}', \mathbf{q}', \mathbf{k}' \rangle = \\ \delta(\mathbf{P} - \mathbf{P}') \delta(\mathbf{q} - \mathbf{q}') \langle \mathbf{k} | v_{12} | \mathbf{k}' \rangle$$

$$\langle \mathbf{P}, \mathbf{q}, \mathbf{k} | e^{i\mathbf{p}_3 \cdot \mathbf{a}} V_{12} e^{-i\mathbf{p}_3 \cdot \mathbf{a}} | \mathbf{P}', \mathbf{q}', \mathbf{k}' \rangle = \\ \delta(\mathbf{P} - \mathbf{P}') \delta(\mathbf{q} - \mathbf{q}') \underbrace{e^{i\mathbf{P} \cdot \mathbf{a} [F(k, \mathbf{q}, \mathbf{P}) - F(k', \mathbf{q}, \mathbf{P})]}}_{\rightarrow 0 \text{ as } |\mathbf{a}| \rightarrow \infty \text{ for } \vec{P} \neq \vec{0}} \langle \mathbf{k} | v_{12} | \mathbf{k}' \rangle .$$

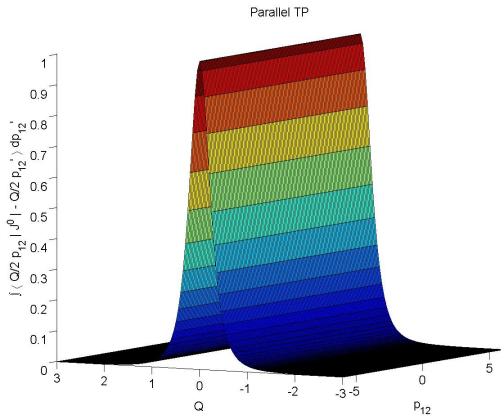
Does it matter?

e-p scattering - deuteron spectator



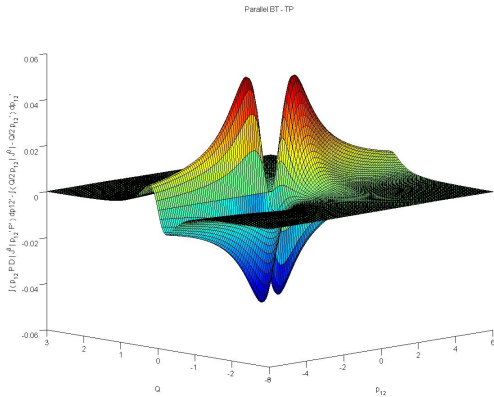
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Does it matter?

e-p scattering - deuteron spectator?



Fixed N solution:

**S. N. Sokolov, Dokl. Akad. Nauk. SSSR 233,575(1977)
S. N. Sokolov, Sov. Phys. Dokl. 22,4(1977) (translation).**

**Few Gev dynamics requires an extension that can treat
non-trivial particle production.**

The Fix

Ekstein's Theorem

$$M = M_0 + V \quad WW^\dagger = I \quad \bar{M} := WMW^\dagger = M_0 + \bar{V}$$

$$S = \Omega_+^\dagger(M, M_0)\Omega_-(M, M_0) \quad \bar{S} = \Omega_+^\dagger(\bar{M}, M_0)\Omega_-(\bar{M}, M_0)$$

$$S = \bar{S} \Leftrightarrow \lim_{t \rightarrow \pm\infty} \|(W - I)e^{-iM_0 t}|\psi\rangle\| = 0$$

$$W = \bar{\Omega}_\pm(\bar{M}, M_0)\Omega_\pm^\dagger(M, M_0) + |\bar{B}\rangle\langle B|$$

$W =$ “scattering equivalence”

(H. Ekstein - Phys. Rev. 117,1590(1960).)

Ekstein Theorem

$$\langle \mathbf{P}, \mathbf{q}, \mathbf{k} | \bar{V}_{12} | \mathbf{P}', \mathbf{q}', \mathbf{k}' \rangle = \delta(\mathbf{P} - \mathbf{P}') \delta(\mathbf{q} - \mathbf{q}') \langle \mathbf{k} | v_{12} | \mathbf{k}' \rangle$$

$$\langle \mathbf{P}, \mathbf{p}_3, \mathbf{k} | V_{12}^{\otimes} | \mathbf{P}', \mathbf{p}_3', \mathbf{k}' \rangle = \delta(\mathbf{P} - \mathbf{P}') \delta(\mathbf{p}_3 - \mathbf{p}_3') \langle \mathbf{k} | v_{12} | \mathbf{k}' \rangle$$

⇓

$$S = \bar{S}$$

⇓

∀ ij ∃ scattering equivalence W_{ij}

Sokolov's solution (2 + 1)

$$\begin{aligned}\bar{M} &= \bar{M}_{(12)(3)} + \bar{M}_{(23)(31)} + \bar{M}_{(31)(2)} - 2M_0 + \bar{V}_{(123)} \\ &= \sum_a C_a \bar{M}_a + \bar{V}_{(123)} \quad C_a = (-)^{n_a} (n_a - 1)!\end{aligned}$$

$$[\bar{M}, \mathbf{j}_0]_- = 0$$

(Allows Poincaré invariant addition of interactions)

$$M =$$

$$W_{12}^\dagger M_{(12)(3)}^\otimes W_{12} + W_{23}^\dagger M_{(23)(1)}^\otimes W_{23} + W_{31}^\dagger M_{(31)(2)}^\otimes W_{31} - 2M_0 + \bar{V}_{(123)}$$

$$M = W \bar{M} W^\dagger \quad W := \prod_s W_{ij} = e^{\sum \ln W_{ij}}$$

$$U(\Lambda, a) = W \bar{U}(\Lambda, a) W^\dagger$$

Production

- **No few-body problem directly related to experiment → use physical particles with restriction on invariant energy. Use effective interaction that work on a fixed scale.**
- **Implications - no bound states, no vertices → replaced by $2 \leftrightarrow 3$ production interaction and scattering probes of structure.**
- **Sokolov's construction can be carried over with minimal change if the particles are initially treated as distinguishible.**
- **Separate solutions for each invariant energy scale; lower energy systems determine part of the dynamics of higher energy systems. Construction is inductive**

$N = 2$ - below π -production threshold

$$\mathcal{H}_{2NN} = (\mathcal{H}_N \otimes \mathcal{H}_{N'}) \oplus (\mathcal{H}_D \otimes \mathcal{H}_\pi)$$

$$\mathcal{H}_{2N\pi} = (\mathcal{H}_N \otimes \mathcal{H}_\pi)$$

$$\mathcal{H}_{2\pi\pi} = (\mathcal{H}_\pi \otimes \mathcal{H}_{\pi'})$$

$N = 3$ - below 2 π -production threshold

$$\begin{aligned} \mathcal{H}_{3NN} = & (\mathcal{H}_N \otimes \mathcal{H}_{N'}) \oplus (\mathcal{H}_D \otimes \mathcal{H}_\pi) \oplus (\mathcal{H}_D \otimes \mathcal{H}_{\pi'}) \oplus \\ & (\mathcal{H}_N \otimes \mathcal{H}_{N'} \otimes \mathcal{H}_\pi) \oplus (\mathcal{H}_N \otimes \mathcal{H}_{N'} \otimes \mathcal{H}_{\pi'}) \oplus (\mathcal{H}_D \otimes \mathcal{H}_\pi \otimes \mathcal{H}_{\pi'}) \end{aligned}$$

The cluster condition

$$\lim_{(x_k - x_j)^2 \rightarrow \infty} \| [U(\Lambda, b) - \otimes U_{a_i}(\Lambda, b)] \prod U_{a_j}(I, x_j) \Pi_a |\Psi\rangle \| = 0$$

$$\mathcal{H} = \left(\bigotimes_i \mathcal{H}_{a_i} \right) \oplus \mathcal{H}^a \quad \Pi_a : \mathcal{H} \rightarrow \bigotimes_i \mathcal{H}_{a_i}$$

The cluster condition separates

- Kinetic energy.
- Tensor products of lower-energy subsystems.
- New many-body interactions.
- New interactions responsible for higher energy reactions.

NN scattering below 2π production threshold

	NN'	$D\pi$	$D\pi'$	$NN'\pi$	$NN'\pi'$	$D\pi\pi'$
NN'	●●	●	●	●	●	●
$D\pi$	●	●●	●	●	●	●
$D\pi'$	●	●	●●	●	●	●
$NN'\pi$	●	●	●	●●●●●	●	●●
$NN'\pi'$	●	●	●	●	●●●●●	●●
$D\pi\pi'$	●	●	●	●●	●●	●●●●●

- Kinetic energy
- Two-cluster subsystems
- “three-body” or high-energy interactions

Representative example of a • contribution

$$M_a^\otimes = W_a \bar{M}_a W_a^\dagger =$$

	NN'	$D\pi$	$D\pi'$	$NN'\pi$	$NN'\pi'$	$D\pi\pi'$
NN'	0	0	0	0	0	0
$D\pi$	0	0	0	0	0	0
$D\pi'$	0	0	0	0	0	0
$NN'\pi$	0	0	0	$M_0 + V_{NN';NN'}$	0	$V_{NN';D\pi'}$
$NN'\pi'$	0	0	0	0	0	0
$D\pi\pi'$	0	0	0	$V_{D\pi';NN'}$	0	$M_0 + V_{D\pi';D\pi'}$

Representative example of Ekstein operator for a •
contribution

$$W_a = \left(\begin{array}{c|cccccc} & NN' & D\pi & D\pi' & NN'\pi & NN'\pi' & D\pi\pi' \\ \hline NN' & I & 0 & 0 & 0 & 0 & 0 \\ D\pi & 0 & I & 0 & 0 & 0 & 0 \\ D\pi' & 0 & 0 & I & 0 & 0 & 0 \\ NN'\pi & 0 & 0 & 0 & W_{NN';NN'} & 0 & W_{NN';D\pi'} \\ NN'\pi' & 0 & 0 & 0 & 0 & I & 0 \\ D\pi\pi' & 0 & 0 & 0 & W_{D\pi';NN'} & 0 & W_{D\pi';D\pi'} \end{array} \right)$$

Structure of solution - NN scattering below 2π production threshold

$$C_a := (-)^{n_a} (n_a - 1)!$$

$$\bar{M} := \sum_s C_a W_a^\dagger M_a^\otimes W_a + \bar{V}_R$$

$$W = e^{\sum_a C_a \ln(W_a)}$$

$$M = W \bar{M} W^\dagger$$

$$U(\Lambda, b) = W \bar{U}(\Lambda, b) W^\dagger$$

Conclusion

- New model required for each energy range.
- Poincaré invariance exact for each model.
- Cluster properties relate higher energy models to lower energy models.
- Few-degree-of-freedom model directly related to experiment for each energy range.
- A recursive construction that preserves these properties follows the method used in F. Coester, W.P., in Phys. Rev. D 26,1348(1982).

Thanks! FB19 Organizers

Thanks! U.S. D.O.E Office of Science