Exchange currents in null-plane Poincaré invariant quantum mechancis

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Elastic electron-deuteron scattering observables

$$\frac{d\sigma}{d\Omega}(Q^2,\theta) = \frac{\alpha^2 \cos^2(\theta/2)}{4E_i^2 \sin^4(\theta/2)} \frac{E_f}{E_i} [A(Q^2) + B(Q^2) \tan^2(\theta/2)]$$

$$T_{20}(Q^2,\theta) = \sqrt{2} \frac{\frac{d\sigma}{d\Omega}_1(Q^2,\theta) - \frac{d\sigma}{d\Omega}_0(Q^2,\theta)}{\frac{d\sigma}{d\Omega}(Q^2,\theta)}$$

Electron-deuteron scattering observables ($A(Q^2), B(Q^2), T_{20}(Q^2)$) are functions of the deuteron current matrix elements.

$$\langle \mathbf{P}', \mu', d | I^{\nu}(0) | \mathbf{P}, \mu, d \rangle$$

Theoretical problem: compute current matrix elements.

Features - null plane kinematic subgroup

- **All current matrix element are linear functions of matrix elements of $I^+(0)$ ($Q^+ = 0$).**
- **All current matrix element of $I^+(0)$ are invariant with respect to kinematic Lorentz transformations.**
- **Matrix elements of the one-body part of $I^+(0)$ factor out of current matrix elements with physical momentum transfer.**
- **Boosts are kinematic and form a subgroup.**

Observables and current matrix elements

$$A(Q^2) = G_0^2(Q^2) + \frac{2}{3}\eta G_1^2(Q^2) + G_2^2(Q^2)$$

$$B(Q^2) = \frac{4}{3}\eta(1+\eta) G_1^2(Q^2)$$

$$T_{20}(Q^2,\theta) =$$

$$-\frac{G_2^2(Q^2) + \sqrt{8} G_0(Q^2) G_2(Q^2) + \frac{1}{3}\eta G_1^2(Q^2)[1 + 2(1+\eta)\tan^2(\theta/2)]}{\sqrt{2}[A(Q^2) + B(Q^2)\tan^2(\theta/2)]}$$

$$I^{\mu}_{\nu\nu'} :=_c \langle \frac{\mathbf{Q}}{2}, \nu | I^{\mu}(0) | -\frac{\mathbf{Q}}{2}, \nu', d \rangle_c$$

$$G_0(0) = \frac{1}{3}(2I^0_{00} + I^0_{11})$$

$$G_1(Q^2) = \sqrt{\frac{2}{\eta}} I^1_{-10}$$

$$G_2(Q^2) = \frac{\sqrt{2}}{3}(I^0_{00} - I^0_{11})$$

Symmetry considerations

State covariance

$$U(\Lambda, a)|\mathbf{P}, \mu, d\rangle = |\mathbf{\Lambda P}, \mu', d\rangle J(\Lambda, P) e^{i\Lambda P \cdot a} D^{1}_{\mu' \mu}[R_w(\Lambda, P)]$$

Current covariance

$$U(\Lambda, a) I^{\mu}(x) U^{\dagger}(\Lambda, a) = (\Lambda^{-1})^{\mu}{}_{\nu} I^{\nu}(\Lambda x + a)$$

Current conservation

$$g_{\mu\nu}[P^{\mu}, I^{\nu}(0)]_{-} = 0$$

State covariance, current covariance, current conservation

$$\Downarrow$$

All current matrix elements linear functions of 3 independent matrix elements

$$\Downarrow$$

Null plane kinematics, $Q^+ = 0 \quad \Rightarrow$ **independent matrix elements can be chosen as matrix elements of** $I^+(0)$ **with light-front spin**

Model construction

Single nucleon Hilbert space - null plane basis

$$\psi(\tilde{\mathbf{p}}, \mu) = \langle \tilde{\mathbf{p}}, \mu | \psi \rangle \qquad \tilde{\mathbf{p}} := (p^1, p^2, p^+ = p^0 + p^3)$$

$$\int_0^\infty dp^+ \int_{\mathbb{R}^2} d\mathbf{p}_\perp \sum_{\mu=-j}^{j} |\psi(\tilde{\mathbf{p}}, \mu)|^2 < \infty.$$

Covariance of nucleon states - null plane basis

$$\langle \tilde{\mathbf{p}}, \mu | U_1(\Lambda, a) | \psi \rangle =$$

$$\int_0^\infty dp^{+\prime} \int_{\mathbb{R}^2} d\mathbf{p}'_\perp \sum_{\mu'=-j}^{j} \mathcal{D}^{m,j}_{\tilde{\mathbf{p}},\mu;\tilde{\mathbf{p}}',\mu'}[\Lambda, a] \langle \tilde{\mathbf{p}}', \mu' | \psi \rangle,$$

$$\mathcal{D}^{m,j}_{\tilde{\mathbf{p}},\mu;\tilde{\mathbf{p}}',\mu'}[\Lambda, a] := \langle \tilde{\mathbf{p}}, \mu | U(\Lambda, a) | \tilde{\mathbf{p}}', \mu' \rangle =$$

$$\delta(\tilde{\mathbf{p}} - \tilde{\mathbf{\Lambda}}(p')) \sqrt{\frac{p^+}{p^{+\prime}}} D^j_{\mu\mu'}[\Lambda_f^{-1}(\tilde{\mathbf{p}}/m) \Lambda \Lambda_f(\tilde{\mathbf{p}}'/m)] e^{ip\cdot a}$$

Two-nucleon Hilbert space

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_1$$

Kinematic representation of Poincaré group

$$U_0(\Lambda, A) := U_1(\Lambda, A) \otimes U_1(\Lambda, A).$$

Poincaré Clebsch-Gordan coefficients - null plane basis

$$\langle m_1, j_1, \tilde{\mathbf{p}}_1, \mu_1; m_2, j_2, \tilde{\mathbf{p}}_2, \mu_2 | k, j(l,s)\tilde{\mathbf{P}}, \mu \rangle =$$

$$\delta(\tilde{\mathbf{P}} - \tilde{\mathbf{p}}_1 - \tilde{\mathbf{p}}_2) \frac{\delta(k - k(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2))}{k^2} | \sqrt{\frac{P^+ \omega_1(\mathbf{k}) \omega_2(\mathbf{k})}{(\omega_1(\mathbf{k}) + \omega_2(\mathbf{k})) p_1^+ p_2^+}} \times$$

$$\sum D^{1/2}_{\mu_1,\mu_1'}[\Lambda_f^{-1}(\mathbf{k}/m_1)\Lambda_c(\mathbf{k}/m_1)] D^{1/2}_{\mu_2,\mu_2'}[\Lambda_f^{-1}(-\mathbf{k}/m_2)\Lambda_c(-\mathbf{k}/m_2)] \times$$

$$Y_{lm}(\hat{\mathbf{k}}(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2)) \langle \frac{1}{2}, \mu_1', \frac{1}{2}, \mu_2' | s, \mu_s \rangle \langle l, m, s, \mu_s | j, \mu \rangle$$

Irreducible representations - null plane basis

$$\langle k, j(l,s)\tilde{\mathbf{P}}, \mu | U_0(\Lambda, a) | \psi\rangle =$$

$$\int_0^\infty dP^{+\prime} \int_{\mathbb{R}^2} d\mathbf{P}'_\perp \sum_{\mu'=-j}^{j} \mathcal{D}^{M_0(k),j}_{\tilde{\mathbf{P}},\mu;\tilde{\mathbf{P}}',\mu'}[\Lambda, a] \langle k, j(l,s)\tilde{\mathbf{P}}', \mu' | \psi\rangle$$

Model input

Dynamical 2 nucleon model

$$M^2 = M_0^2 + 4mv_{nn}$$

$$\langle k', j', (l', s')\tilde{\mathbf{P}}', \mu' | v_{nn} | k, j, (l, s)\tilde{\mathbf{P}}, \mu\rangle =$$

$$\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})\delta_{j'j}\delta_{\mu'\mu}\langle k', l', s' \| v^j \| k, l, s\rangle$$

Mass eigenvalue problem

$$(4k^2 + 4m^2 - \lambda^2)\phi_{\lambda,j}(k,l,s) =$$

$$-\sum_{s=0}^{1}\sum_{l=|j-s|}^{|j+s|}\int_0^\infty 4m\langle k,l,s\|v_{nn}^j\|k',l',s'\rangle k'^2 dk' \phi_{\lambda,j}(k',l',s')$$

Dynamical representation of Poincaré group

$$\langle k', j', (l', s')\tilde{\mathbf{P}}', \mu' | U(\Lambda, a) | \lambda, j, \tilde{\mathbf{P}}, \mu\rangle =$$

$$\int \sum_{\mu''=-j}^{j} \langle k', j', (l', s')\tilde{\mathbf{P}}', \mu' | \lambda, j, \tilde{\mathbf{P}}'', \mu''\rangle \times$$

$$d\tilde{\mathbf{P}}'' \langle \lambda, j, \tilde{\mathbf{P}}'', \mu'' | U(\Lambda, A) | \lambda, j, \tilde{\mathbf{P}}, \mu\rangle =$$

$$\phi_{\lambda,j}(k', l', s') \mathcal{D}^{\lambda, j'}_{\tilde{\mathbf{P}}', \mu'; \tilde{\mathbf{P}}, \mu}[\Lambda, a]$$

Realistic two-body model

$$S(H_{nr}, H_{0nr}) = \Omega_+^\dagger(H_{nr}, H_{0nr})\Omega_-(H_{nr}, H_{0nr}) =$$

$$\Omega_+^\dagger(H_r, H_{0r})\Omega_-(H_r, H_{0r}) = S(H_r, H_{0r}).$$

Nucleon currents

$$\langle \tilde{\mathbf{p}}', \nu' | I_1^\mu(0) | \tilde{\mathbf{p}}, \nu \rangle =$$

$$\sqrt{\frac{m}{p^{+\prime}}} \bar{u}(p') \Gamma^\mu u(p) \sqrt{\frac{m}{p^+}}$$

where

$$\Gamma^\mu = \gamma^\mu F_1(Q^2) + \frac{1}{2}[\gamma^\mu, \frac{1}{2m}\gamma \cdot Q] F_2(Q^2)$$

Exchange current

$$\langle \tilde{\mathbf{P}}', \nu', d | I^{+}_{ex}(0) | \tilde{\mathbf{P}}, \nu, d \rangle :=$$

$$\int \langle \tilde{\mathbf{P}}', \nu', d | \tilde{\mathbf{p}}'_1, \nu'_1, \tilde{\mathbf{p}}'_2, \nu'_2 \rangle \times$$

$$(-\frac{1}{2m}) \sqrt{\frac{m}{p_1^+}} \bar{u}_f(p'_1) \Gamma^+ \gamma_5 (\frac{P \cdot \gamma}{M_d}) u_f(p''_1) \sqrt{\frac{m}{p_1^{+\prime\prime}}} d\tilde{\mathbf{p}}'_1 d\tilde{\mathbf{p}}''_1 d\tilde{\mathbf{p}}'_2 \times$$

$$\langle \tilde{\mathbf{p}}''_1, \nu''_1, \tilde{\mathbf{p}}'_2, \nu'_2 | U(\Lambda_f(P/M_d)) \tilde{v}_{ope} | \tilde{\mathbf{P}}_0, \nu, d \rangle + (1 \leftrightarrow 2) + hc$$

Independent matrix element of $I^{+}(0)$

9 matrix elements - 3 independent matrix elements

$I^{+}_{ex}(0)$ **constructed to be kinematically covariant**

kinematic symmetries eliminate 5 matrix elements

$I^{+}_{11}(0), I^{+}_{10}(0), I^{+}_{1-1}(0), I^{+}_{00}(0)$ **kinematically independent**

(dynamical) rotational symmetry eliminates 1 matrix element.

Choosing independent matrix elements

I. Count spin flips.

II. (FFS) Use linear combinations of canonical spin matrix elements that are largest in the infinite momentum frame.

III. (FC) Use linear combinations null plane matrix elements constructed to minimize dependence of matrix elements on mass.

Rotational covariance - Poincaré Wigner Eckart theorem

Independent matrix element - choice I:

$$(1+\eta)G_0(Q^2) = (\frac{1}{2} - \frac{\eta}{3})(I_{11} + I_{00}) + \frac{5\sqrt{2\eta}}{3} I_{10} + (\frac{2\eta}{3} - \frac{1}{6}) I_{1-1}$$

$$(1+\eta)G_1(Q^2) = I_{11} + I_{00} - I_{1-1} - (1-\eta)\sqrt{\frac{2}{\eta}} I_{10}$$

$$(1+\eta)G_2(Q^2) = -\frac{\sqrt{2}\eta}{3}(I_{11} + I_{00}) + \frac{4\sqrt{\eta}}{3} I_{10} - \frac{\sqrt{2}}{3}(2+\eta) I_{1-1}$$

Rotational covariance - Poincaré Wigner Eckart theorem

Independent matrix element - choice II ($G_0(Q^2)$ of choice I replaced by:

$$(1+\eta)G_0(Q^2) = (\frac{2\eta}{3}+1)I^+_{1,1} - \frac{\eta}{3}I^+_{00} + \frac{2\sqrt{2\eta}}{3}I^+_{1,0} + (\frac{2\eta+1}{3})I^+_{1,-1}]$$

Independent matrix element - choice III ($G_0(Q^2)$ and $G_2(Q^2)$ of choice I replaced by

$$G_0(Q^2) = (1+\frac{2\eta}{3})I^+_{11} + \frac{1}{3}I^+_{1,-1} + -\frac{2\eta}{3}G_1(Q^2)$$

$$G_2(Q^2) = \frac{2\sqrt{2}}{3}(\eta I^+_{1,1} - I^+_{1,-1} - \eta G_1(Q^2))$$

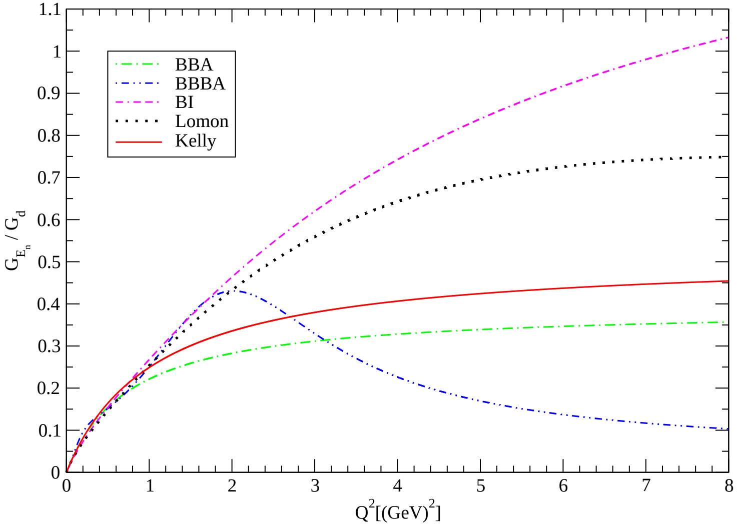
The model input includes the realisitic nucleon-nucleon interaction, nucleon form factors, the choice of independent matrix elements, and the model exchange current.

Without the model exchange current the results are not very sensitive to the choice of independent current matrix elements.

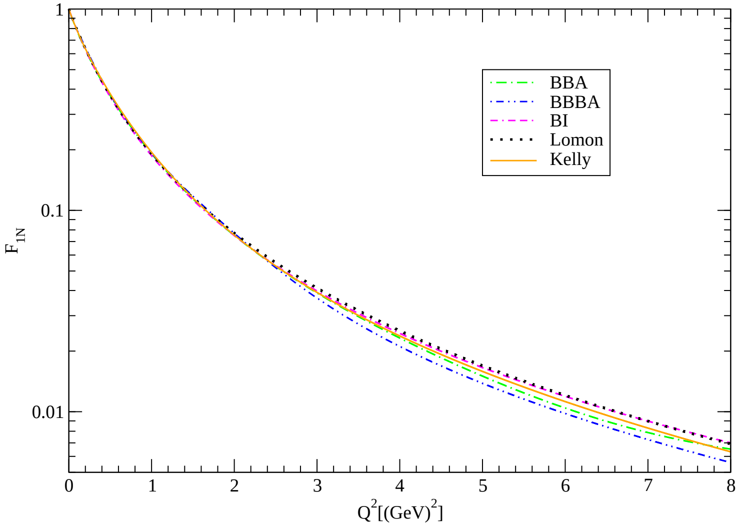
The model exchange current increases the sensitivity to the choice of independent current matrix elements.

Summary of results

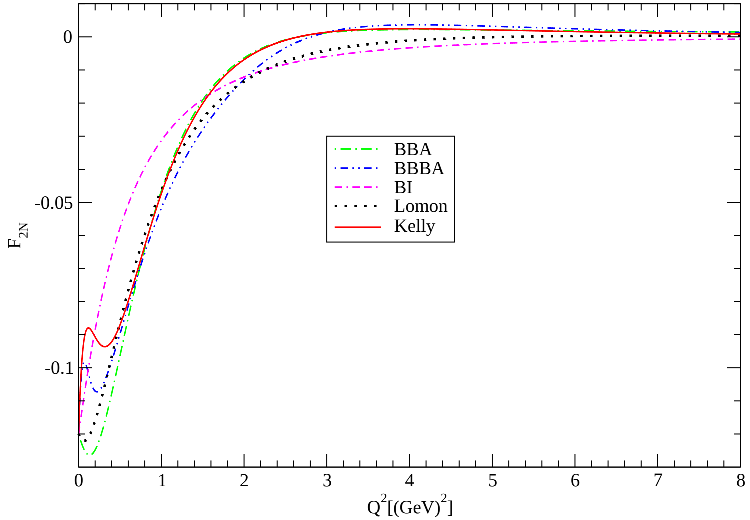
Neutron Electric Form Factors



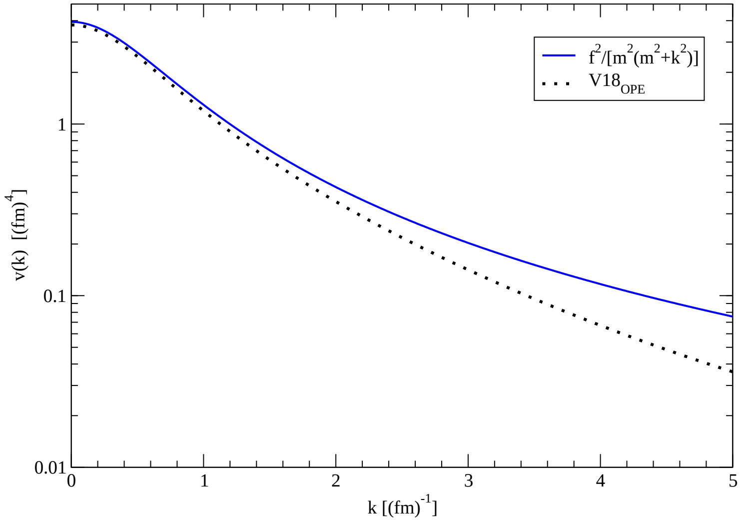
Isoscalar Nucleon Form Factor: F_{1N}



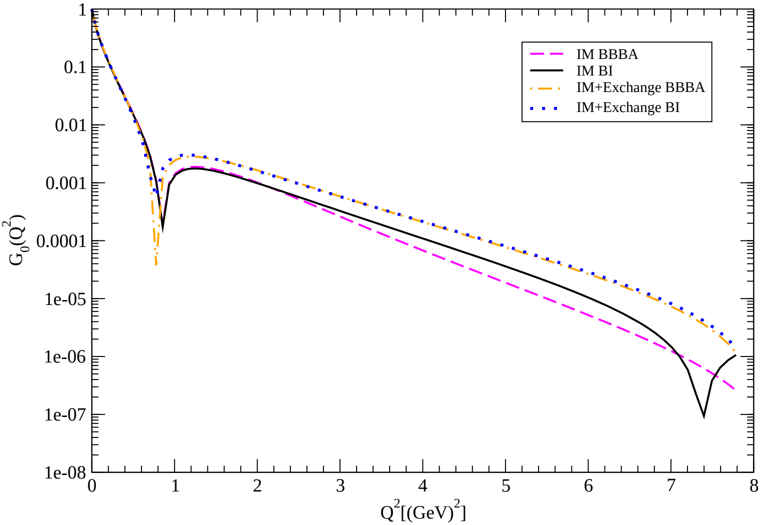
Isoscalar Nucleon Form Factor: F_{2N}



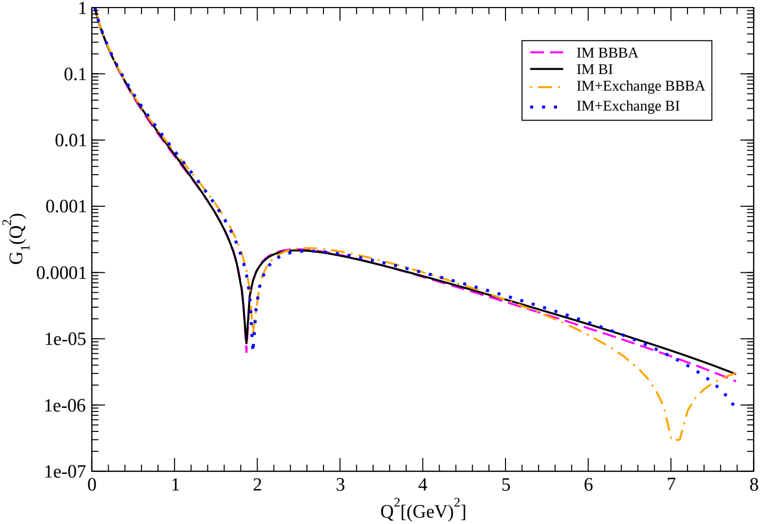
One-Pion Exchange Potential



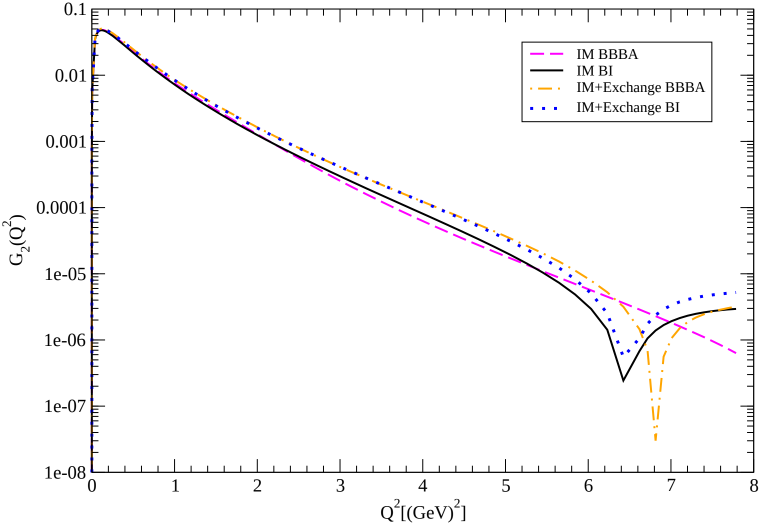
Deuteron Form Factor: $G_0(Q^2)$

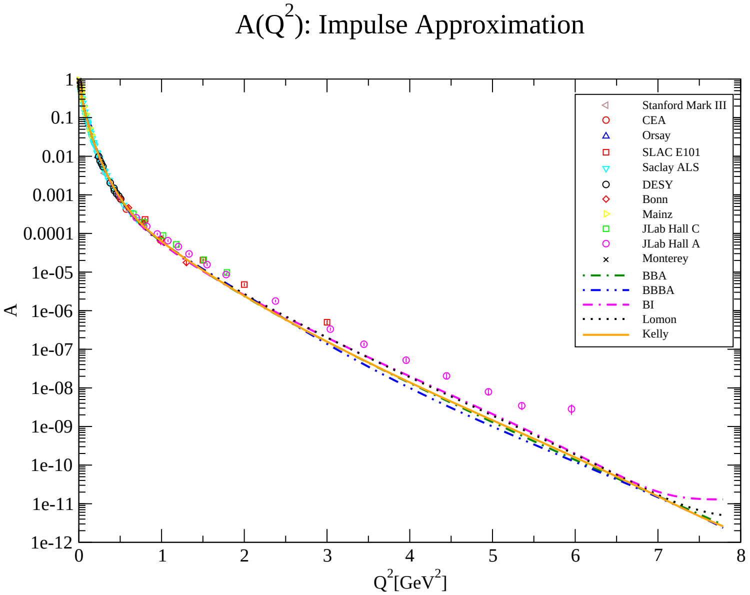


Deuteron Form Factor: $G_1(Q^2)$

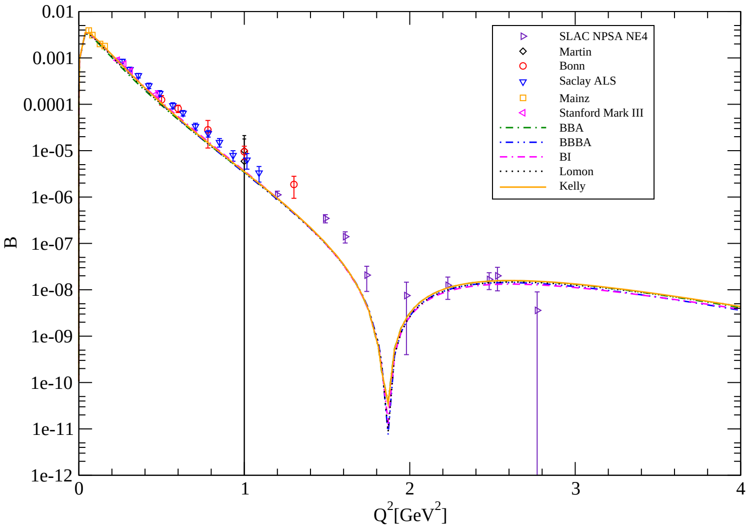


Deuteron Form Factor: $G_2(Q^2)$

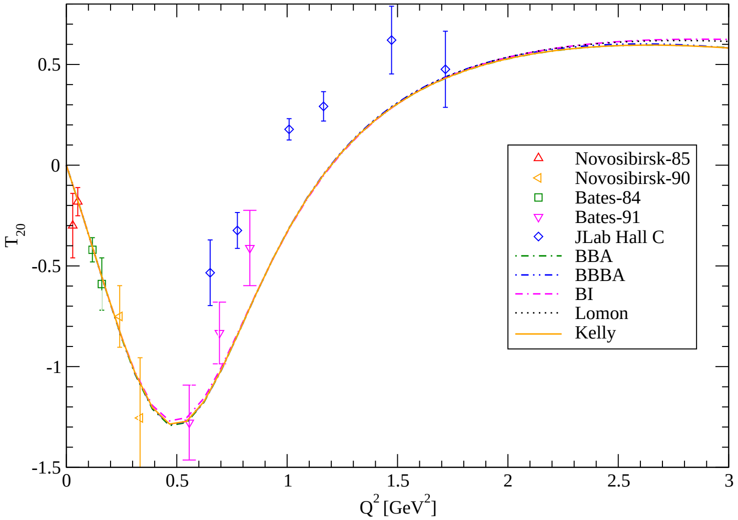



A(Q^2): Impulse Approximation
A
Q^2[GeV^2]
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
1e-12
0
1
2
3
4
5
6
7
8
Stanford Mark III
CEA
Orsay
SLAC E101
Saclay ALS
DESY
Bonn
Mainz
JLab Hall C
JLab Hall A
Monterey
BBA
BBBA
BI
Lomon
Kelly

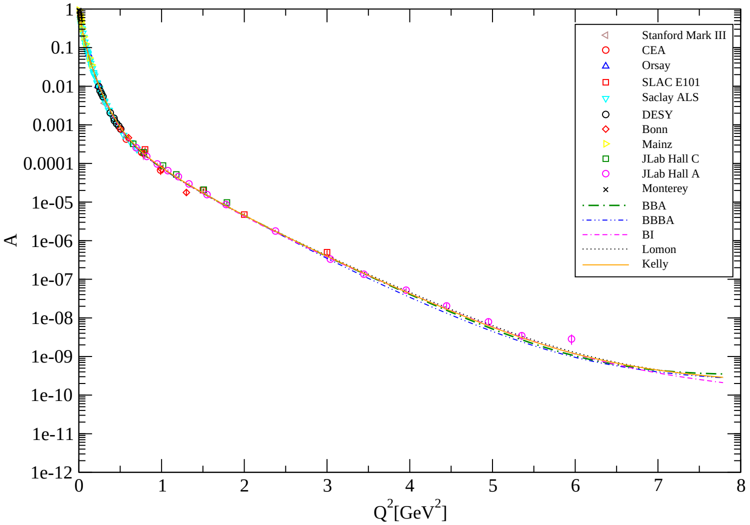
$B(Q^2)$: Impulse Appoximation



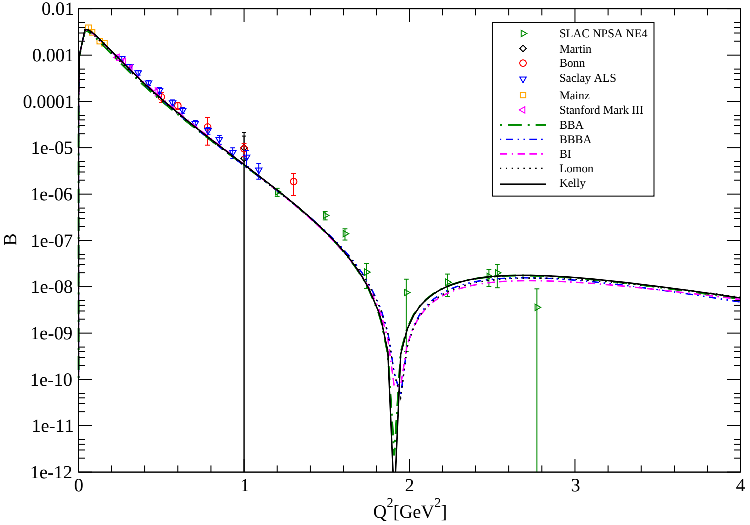
Deuteron Structure Function T_{20}



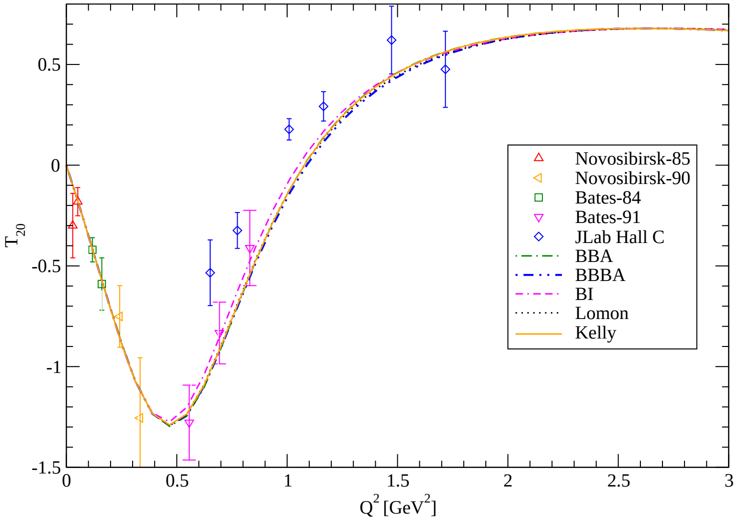
$A(Q^2)$: Impulse + Exchange Current



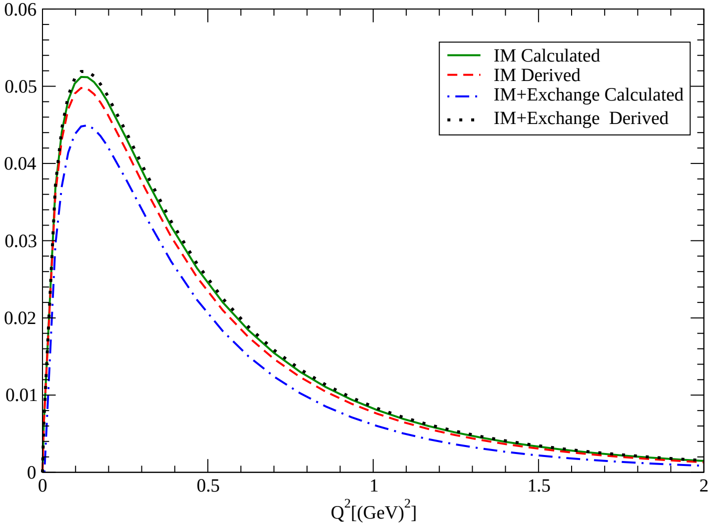
$B(Q^2)$: Impulse + Exchange Current



$T_{20}(Q^2, 70^o)$: Impulse + Exchange Current

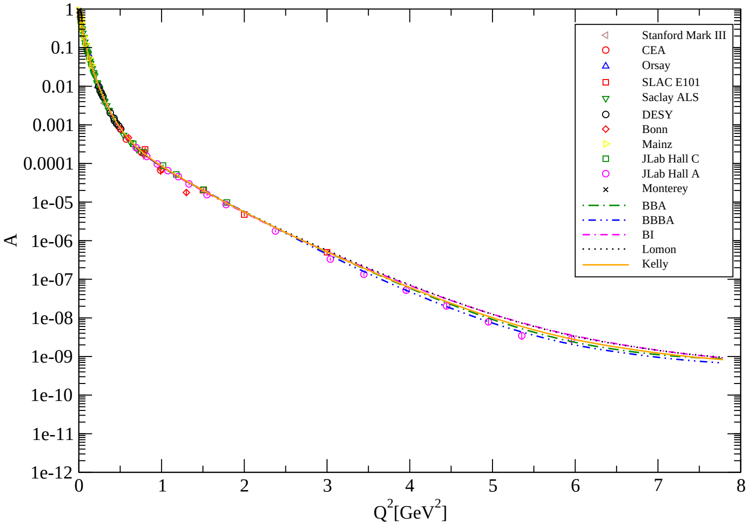


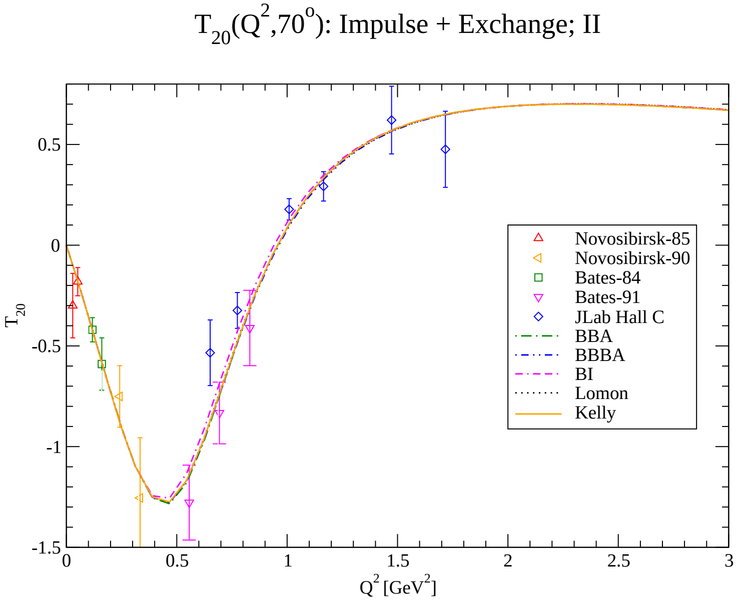
$I^{+}_{++}(Q^2) - I^{+}_{00}(Q^2)$



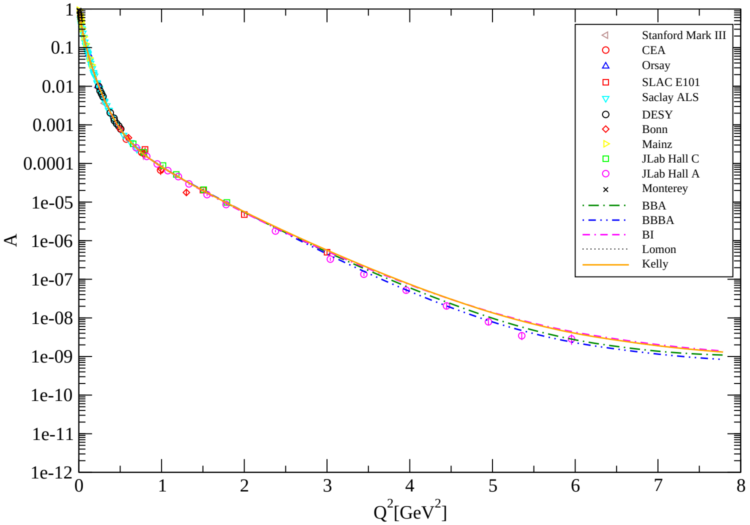
0.06
0.05
0.04
0.03
0.02
0.01
0
0.5
1
1.5
2
$Q^2[(GeV)^2]$
IM Calculated
IM Derived
IM+Exchange Calculated
IM+Exchange Derived

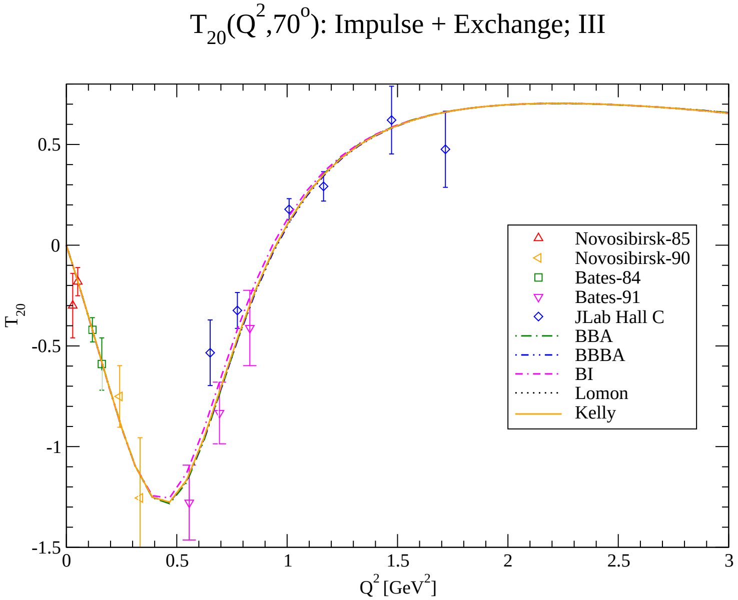
$A(Q^2)$: Impulse + Exchange; II



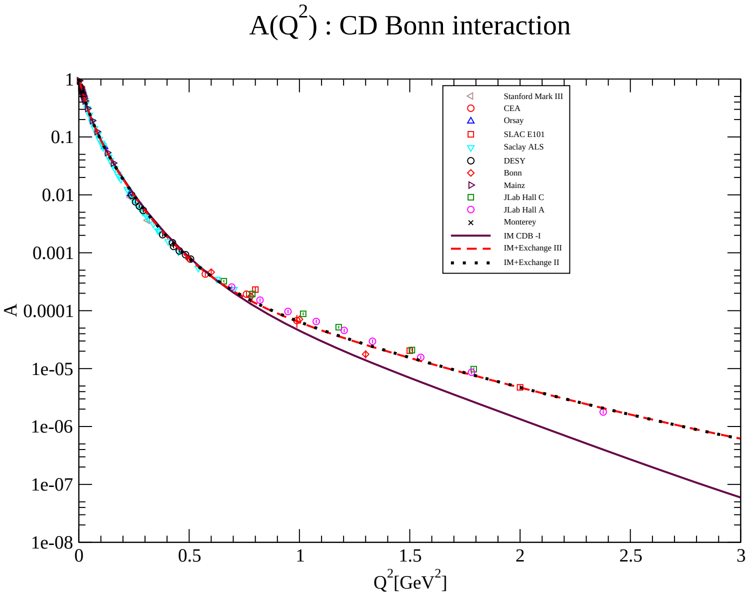

$T_{20}(Q^2,70^o)$: Impulse + Exchange; II
T_{20}
Q^2 [GeV^2]
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5
3
Novosibirsk-85
Novosibirsk-90
Bates-84
Bates-91
JLab Hall C
BBA
BBBA
BI
Lomon
Kelly

A(Q^2): Impulse + Exchange; III

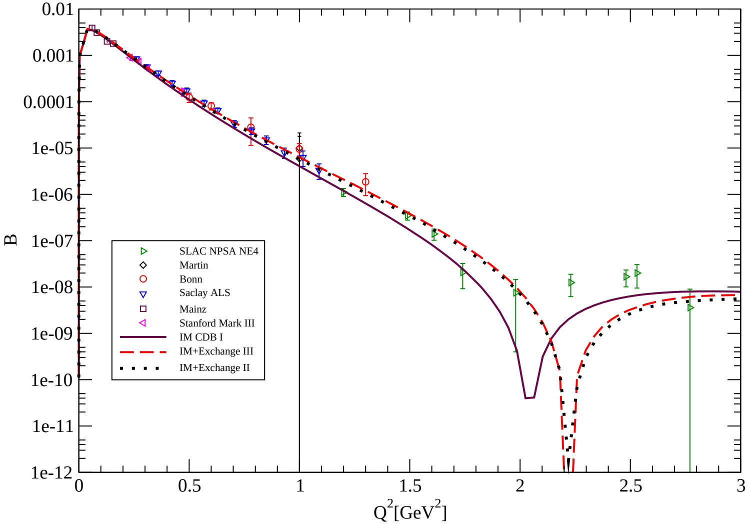




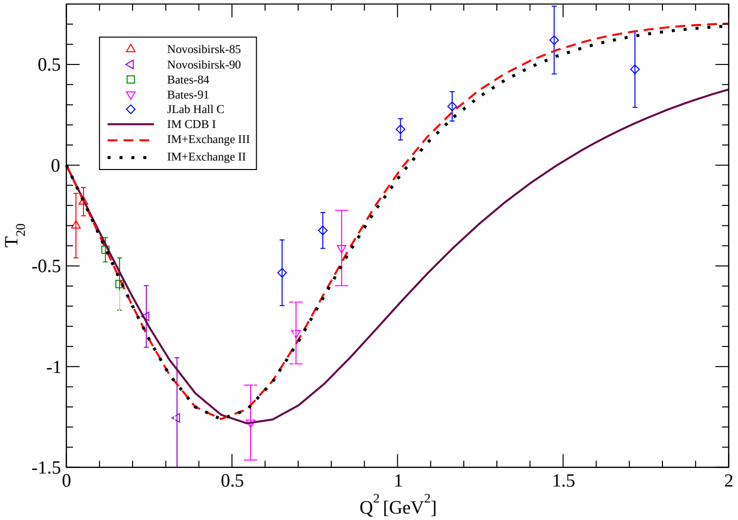
T20(Q2,70o): Impulse + Exchange; III
T20
Q2 [GeV2]
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5
3
Novosibirsk-85
Novosibirsk-90
Bates-84
Bates-91
JLab Hall C
BBA
BBBA
BI
Lomon
Kelly

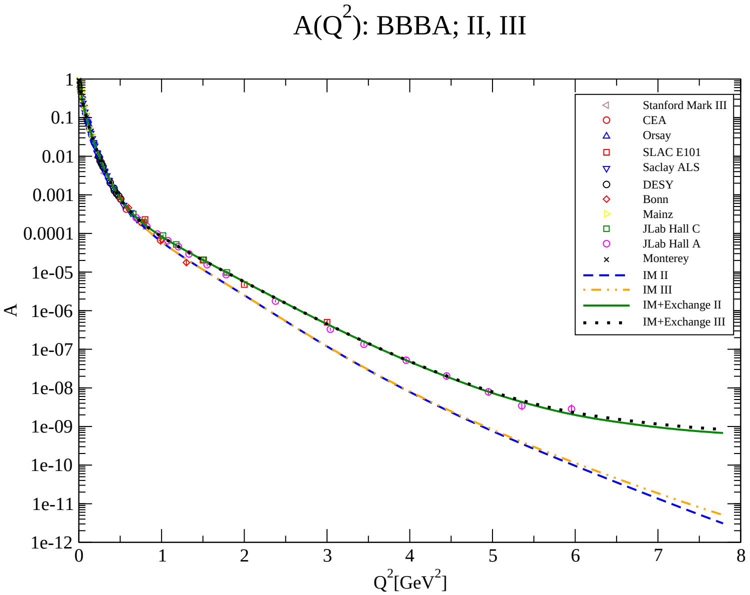

A(Q²) : CD Bonn interaction
A
Q²[GeV²]
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
0
0.5
1
1.5
2
2.5
3
Stanford Mark III
CEA
Orsay
SLAC E101
Saclay ALS
DESY
Bonn
Mainz
JLab Hall C
JLab Hall A
Monterey
IM CDB -I
IM+Exchange III
IM+Exchange II

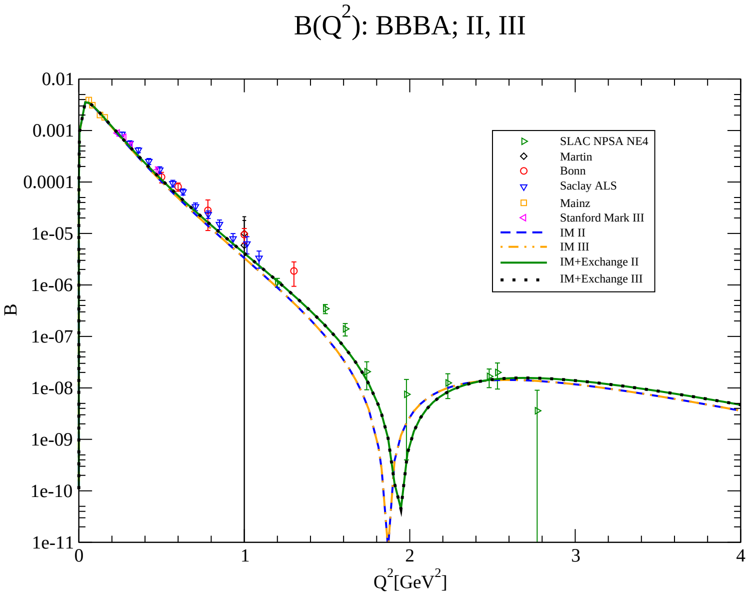
B(Q^2): CD Bonn Interaction



$T_{20}(Q^2,70^o)$: CD Bonn Interaction

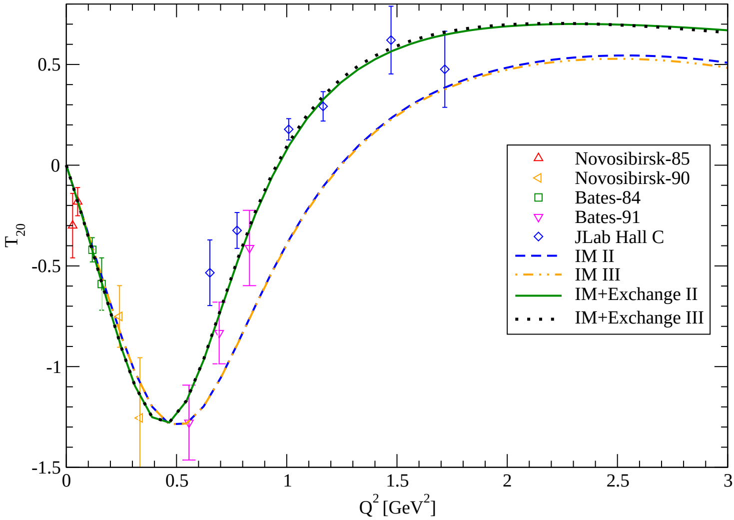



A(Q^2): BBBA; II, III
A
Q^2[GeV^2]
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
1e-12
0
1
2
3
4
5
6
7
8
Stanford Mark III
CEA
Orsay
SLAC E101
Saclay ALS
DESY
Bonn
Mainz
JLab Hall C
JLab Hall A
Monterey
IM II
IM III
IM+Exchange II
IM+Exchange III



B(Q^2): BBBA; II, III
0.01
0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
B
0
1
2
3
4
Q^2[GeV^2]
SLAC NPSA NE4
Martin
Bonn
Saclay ALS
Mainz
Stanford Mark III
IM II
IM III
IM+Exchange II
IM+Exchange III

$T_{20}(Q^2, 70^o)$: BBBA; II, III



Conclusion

I Model exchange current explains difference between measurement and impulse approximation.

II Sensitivity to implementation of the Wigner-Eckart theorem

III Model can be generalized to treat more complex systems