29:5742 Homework 11
Due 4/26

1. Dirac Equation: Consider an equation of the form

$$i\hbar\frac{\partial}{\partial x^0}\psi(x) = (-i\hbar\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \beta mc)\psi(x)$$

where $\boldsymbol{\alpha}$ and β are constant Hermitian matrices

a Find conditions on $\boldsymbol{\alpha}$ and β for the solution $\psi(x)$ to satisfy the Klein Gordon equation.

b. Show that the matrices that solve part a must be even dimensional.

c. Show that the conditions from part a cannot be satisfied using 2×2 matrices.

2. Show, using explicit Poincaré transformations, that time translation can be expressed as a finite combination of space translations and rotationless Lorentz transformations.

3. Show for

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \tilde{\sigma} & 0 \end{pmatrix}$$

where $\tilde{\sigma} = (I, -\boldsymbol{\sigma})$ satisfies

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = -2\eta_{\mu\nu}$$

4. Compute the anticommutator

$$\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu.$$

5. Show that

$$v(p) = S(B(p))v_+(0)$$

where

$$v_+(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

is a negative energy solution of the Dirac equation.

6. Show that

$$(\sum_\mu \gamma_\mu p^\mu - mcI)(\sum_\nu \gamma_\nu p^\nu + mcI) = 0.$$

This means that it is possible to find solutions to the soltions to the Dirac equation by applying $(\sum_\mu \gamma_\mu p^\mu + mcI)$ to any constant 4 component vector.