

29:206 HW# 3 due 2/26ca

1 Consider the Hamiltonian

$$H = \frac{1}{2}p^2 + \frac{1}{2}q^2.$$

Let $D_F G = \{F, G\}$ and calculate

$$e^{-tD_H} p = p(t) \quad e^{-tD_H} q = q(t).$$

Show that $p(t)$ and $q(t)$ are harmonic oscillator solutions to Hamiltonian's equations with initial conditions p and q .

2 Show that

$$\{A, \{B, C\}\} + \{B, \{C, A\}\} + \{C, \{A, B\}\} = 0$$

and

$$\{A, BC\} = \{A, B\}C + C\{A, B\}.$$

3 Consider the Hamiltonian for central force motion where r and θ are polar coordinates and p_r and p_θ are conjugate momenta:

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + V(r).$$

Show that this Hamiltonian is completely integrable, find the action variables for this system.

4 Let $p' = p(t)$ and $q' = q(t)$ in problem 1. Show that the transformation $(p, q) \rightarrow (p', q')$ preserves the canonical two form, $\omega = dp \wedge dq$.

5. Show that the product of two symplectic matrices is symplectic. Show that the inverse of a symplectic matrix is symplectic. (Recall M is symplectic if $MSM^t = S$.)