29:172 Assignment 3 - Due Wednesday, Feb. 7

1.) Use the Rodrigues formula for the Laguerre polynomials to derive the constants k_n, k'_n on page 212 of the text.

2.) Let

$$g(x,t) = \frac{1}{\sqrt{1-2xt+t^2}}. \tag{1}$$

Show that

$$g(x,t) = \sum_{n=0} \infty t^n P_n(x) \tag{2}$$

where $P_n(x)$ is the $n-th$ degree Legendre polynomial. The function $g(x,t)$ is called a generating function.

3.) The Schrödinger equation for a harmonic oscillator has the form

$$(-\frac{d^2}{dx^2} + x^2)F_n(x) = 2nF_n(x). \tag{3}$$

Assume that

$$F_n(x) = C_n(x)e^{-\frac{1}{2}x^2}. \tag{4}$$

Show that C_n satisfies the same differential equation as one of the classical orthogonal polynomials.

4.) Find the Gauss Legendre points and weights for a two point Gauss Legendre quadrature on $[-1,1]$. Show that

$$\int_{-1}^{1} x^n dx = \sum_{i=1}^{2} x_i^n w_i \tag{5}$$

is exact or $n = 0, 1, 2, 3$. How good is the integral for x^6?

5. Use the recursion relations to generate the first four Laguerre Polynomials with $\nu = 0$.

6. Show that the Legendre Polynomials satisfy the following recursion relations involving first derivatives

$$(n+1)P_{n+1}(x) = (n+1)xP_n - (1-x^2)\frac{dP_n}{x}. \tag{6}$$