29:171 - Homework Assignment #8

1. By integrating
$$\int \frac{z dz}{a - e^{-iz}}$$
over a rectangular curve with corners at $-\pi$, π, $\pi + in$ $-\pi + in$ and letting $n \to \infty$ show
$$\int_0^{\pi} \frac{x \sin(x) dx}{1 + a^2 - 2a\cos(x)} = \frac{\pi}{a} \log(1 + a) \qquad (0 < a < 1)$$

2. Evaluate
$$\int_0^{\infty} \frac{\ln^2(z)}{z^2 + 1} dz$$

3. Express the integral
$$\int_0^{\infty} e^{-\alpha x^2} x^{\beta} dx$$
where α and β are real and positive.

4. Prove that if $a > 0, -\frac{1}{2}\pi < a\lambda < \frac{1}{2}\pi$
$$\int_0^{\infty} e^{-r^a \cos(a\lambda)} \cos(r^a sin(a\lambda)) dr = \cos(\lambda) \frac{1}{a} \Gamma(\frac{1}{a})$$

5. Calculate
$$\int_0^{\pi/2} \sin^{\alpha}(\theta) \cos^{\beta}(\theta) d\theta$$
for $\alpha, \beta > 0$.

6. Evaluate $\beta(m, n)$ and relate it to the binomial coefficients.