29:171 - Homework Assignment #3

1. Let $f(z) = e^z$. Let C be the curve in the complex plane that starts at the origin, $z = 0$, goes along the positive real axis to the point $z = 2$, and then proceeds in a straight line in positive imaginary direction from $z = 2$ to $z = 2 + 3i$.

   Calculate the contour integral

   $$\int_C f(z)dz$$

   directly. Check your answer by noting that $\frac{df}{dz}(z) = f(z)$

2. Show that the conformal mapping

   $$z \to z' = \frac{1}{z}$$

   maps the circle

   $$|z - a| = r$$

   into another circle. Find the origin and radius of the transformed circle. Determine the condition for the transformed circle to become a line (i.e. circle of infinite radius).

3. Show that any real valued analytic function is a constant.

4. If C is the circle $|z| = 1$, calculate the line integral (in the counterclockwise direction) of

   $$\int_C \frac{dz}{z}$$

5. Consider the homographic transformation

   $$z' = \frac{az + b}{cz + d} \qquad bc - ad \neq 0$$

   Calculate the inverse transformation. Is it homographic?

6. Use Darboux's theorem to put a bound on the integral

   $$|\int_C \sin(z)dz|$$

   where C is the circle $|z| = 5$.