

29:171 - Homework Assignment #12

Consider the matrix

$$M := \begin{pmatrix} 1 & -i & 0 \\ i & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1. Find the characteristic polynomial of M
2. Find the eigenvalues of M
3. Find the polynomials $\phi_i(\lambda)$
4. Calculate $\phi_i(M)$.
5. Let $|v\rangle$ be any vector such that $\phi_i(M)|v\rangle \neq 0$. Show that

$$M\phi_i(M)|v\rangle = \lambda_i\phi_i(M)|v\rangle$$

where λ_i is the i -th eigenvalue.

6. Calculate $\sin(M)$
7. Show that $\sum_i \phi_i(M) = I$
8. Find a similarity transform that diagonalizes M
9. Find M^{-1}