Particles, fields, dynamics and the light-front vacuum

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Outline

- **Light-front representations of particles.**
- **Light-front dynamics of particles - finite number of DOF.**
- **Particles $\rightarrow$ fields.**
- **Light-front operator algebra.**
- **Noether's theorem and light-front dynamics of fields.**
- **Zero modes and the light-front vacuum.**

Notation

Poincaré group - inhomogeneous $SL(2,\mathbb{C})$

$$X = x^\mu \sigma_\mu = \begin{pmatrix} x^+ & x_\perp^* \\ x_\perp & x^- \end{pmatrix} \qquad x^\mu = \frac{1}{2}\mathrm{Tr}(\sigma_\mu X)$$

$$\det(X) = -(x^0)^2 + \mathbf{x}^2$$

$$X' = AXA^\dagger + B \qquad B = B^\dagger \qquad \det(A) = 1$$

Light-front:

$$X \to \begin{pmatrix} 0 & x_\perp^* \\ x_\perp & x^- \end{pmatrix} \qquad \tilde{\mathbf{x}} = (x^-, \mathbf{x}_\perp)$$

Kinematic subgroup, preserves $x^+ = 0$, (7 parameters):

$$A = \begin{pmatrix} \alpha e^{i\phi} & 0 \\ (\beta_1 + i\beta_2) & e^{-i\phi}/\alpha \end{pmatrix} \qquad B = \begin{pmatrix} 0 & b_\perp^* \\ b_\perp & b^- \end{pmatrix}$$

Relativistic particles

$$U(\Lambda, a)$$

$$\Downarrow$$

$$\{P^\mu, M^{\mu\nu}\}$$

$$\Downarrow$$

Commuting observables:

$$|(m,j)\tilde{\mathbf{p}},\mu\rangle_f \qquad \tilde{\mathbf{p}} := (p^+, \mathbf{p}_\perp) \qquad p^- = \frac{\mathbf{p}_\perp^2 + m^2}{p^+}$$

$$j_{fi} = \frac{1}{2}\epsilon_{ijk} B_f^{-1}(P/M)^i{}_\mu B_f^{-1}(P/M)^j{}_\nu M^{\mu\nu}$$

Mass m spin j irreducible unitary representation of P.G.:

$$\Downarrow$$

$$U(\Lambda, a)|(m,j)\tilde{\mathbf{p}},\mu\rangle_f =$$

$$\sum e^{i\Lambda p\cdot a}|(m,j)\tilde{\mathbf{\Lambda}}p,\nu\rangle_f D^j_{\nu\mu}[B_f^{-1}(\Lambda p/m)\Lambda B_f(p/m)]\sqrt{\frac{(\Lambda p)^+}{p^+}}$$

Free-particle dynamics: $U(I, x^+)$.

$(\Lambda, b) \in$ **kinematic subgroup**

$$\Downarrow$$

$$\{e^{i\Lambda p\cdot b}, R_{wf}(\Lambda, p) := B_f^{-1}(\Lambda p/m)\Lambda B_f(p/m)], \sqrt{\frac{(\Lambda p)^+}{p^+}}\}$$

independent of mass (m)**.**

Special light-front properties:

$$D^j_{\mu\nu}[R_{wf}(B_f(p'/m), p)] = \delta_{\mu\nu} \qquad D^j_{\mu\nu}[R_{wf}(R, p)] \neq D^j_{\mu\nu}[R]$$

$P^+ \geq 0$ **kinematic**

$P^+ = P^- = M$ **rest frame dynamical**

Systems of free particles:

$$|(m_1, j_1)\tilde{\mathbf{p}}_1, \mu_1\rangle_f \cdots |(m_n, j_n)\tilde{\mathbf{p}}_n, \mu_n\rangle_f$$

Infinitesimal generators: $G_0 = \sum_n G_n \otimes I_{!n}$.

Commuting observables (system):

$$\{M_0, \mathbf{j}^2, P_0^+, \mathbf{P}_{0\perp}, \mathbf{j}_{0f} \cdot \hat{\mathbf{z}}, \mathbf{d}\}$$

Irreducible basis states:

$$|(m_0, j_0)\tilde{\mathbf{p}}_0, \mu_0; \mathbf{d}\rangle_{0f}$$

$\mathbf{d}$ = **invariant degeneracy parameters**

$$U(\Lambda, b)|(m_0, j_0)\tilde{\mathbf{p}}_0, \mu; \mathbf{d}\rangle_f =$$

$$\sum e^{i\Lambda P_0 \cdot b}|(m_0, j_0)\tilde{\boldsymbol{\Lambda}} p_0, \nu; \mathbf{d}\rangle_f D^j_{\nu\mu}[R_{wf}(\Lambda, p_0)]\sqrt{\frac{(\Lambda p_0)^+}{p_0^+}}$$

Dynamics - Finite DOF

Problem: Add non-trivial interactions to M and $\mathbf{j}^2$ that satisfy $[M, \mathbf{j}^2] = 0$ and commute with all kinematic generators.

Bakamjian-Thomas solution (finite DOF)
set $\mathbf{j} = \mathbf{j}_0$, add interactions to M.

$$M^2 = M_0^2 + V$$

$$[V, \mathbf{j}_0] = [V, \mathbf{E}_{\perp 0}] = [V, K_0^3] = [V, P_0^+] = [V, \mathbf{P}_{\perp 0}] = 0$$

$$M > 0 \qquad M = M^\dagger$$

Diagonalize $M^2 = M_0^2 + V$ in non-interacting irreducible basis:

$$|(m_0, j_0)\tilde{\mathbf{p}}_0, \mu_0; l_0, s_0\rangle \qquad \mathbf{d} = \{l_0, s_0\}$$

Dynamics: This gives a complete set of irreducible eigenstates with a light-front kinematic subgroup that is consistent with the dynamics of M^2.

$$U(\Lambda, b)|(m_I, j_0)\tilde{\mathbf{p}}_0, \mu; \mathbf{d}_I\rangle_f =$$

$$\sum e^{i\Lambda p\cdot b}|(m_I, j_0)\tilde{\boldsymbol{\Lambda}}p, \nu; \mathbf{d}_I\rangle_f D^j_{\nu\mu}[R_{wf}(\Lambda, p)]\sqrt{\frac{(\Lambda p)^+}{p^+}}$$

Interacting angular momentum:

$$\mathbf{J}_\perp = \frac{1}{P_0^+}[\frac{1}{2}(P_0^+ - P^-)(\hat{\mathbf{z}} \times \mathbf{E}_0) - (\hat{\mathbf{z}} \times \mathbf{P}_0)K_0^3 + \mathbf{P}_{\perp 0} j_{f0}^3 + M\mathbf{j}_{f\perp 0}]$$

$N \geq 3$

- **The construction can be generalized to any number of particles.**
- **Cluster properties require a dynamical spin:**

$$U_{ij}(\Lambda, a) \otimes U_k(\Lambda, a) \quad \to \quad \mathbf{j}_{(ij)\otimes k} \neq \mathbf{j}_0$$

- **Key tool: Tensor product representations are related to representations with non-interacting spin by S-matrix-preserving unitary transformations.**
- **Cluster properties require all dynamical operators are many-body operators.**
- **Dirac's other forms of dynamics - same construction; start with different sets of commuting observables.**
- **Dynamics in all of Dirac's other forms of dynamics related by an S-matrix preserving unitary transformation.**

Particles $\to$ fields:

Lorentz covariant particle states:

$$U(\Lambda,0)|(m,j)\tilde{\mathbf{p}},\mu\rangle_f =$$

$$\sum |(m,j)\tilde{\mathbf{\Lambda}}p,\nu\rangle_f D^j_{\nu\mu}[B_f^{-1}(\Lambda p)\Lambda B_f(p)]\sqrt{\frac{(\Lambda p)^+}{p^+}}$$

$$\Downarrow$$

$$U(\Lambda,0)|(m,j)\tilde{\mathbf{p}},\mu\rangle_f D^j_{\nu\mu}[B_f^{-1}(p)]\sqrt{p^+} =$$

$$\sum |(m,j)\tilde{\mathbf{\Lambda}}p,\nu\rangle_f D^j_{\nu\alpha}[B_f^{-1}(\Lambda p)]\sqrt{(\Lambda p)^+}D^j_{\alpha\mu}[\Lambda]$$

$$U(\Lambda,0)|(m,j)\tilde{\mathbf{p}},\mu\rangle_f D^j_{\nu\mu}[B_f^{\dagger}(p)]\sqrt{p^+} =$$

$$\sum |(m,j)\tilde{\mathbf{\Lambda}}p,\nu\rangle_f D^j_{\nu\alpha}[B_f^{\dagger}(\Lambda p)]\sqrt{(\Lambda p)^+}D^j_{\alpha\mu}[(\Lambda^{\dagger})^{-1}]$$

Particles $\rightarrow$ free fields

Local right or left handed, mass m spin j light-front free fields can be constructed from the occupation number representation using the Lorentz-covariant single-particle basis:

$$\Psi_{rf\mu}(x) = \frac{Z}{(2\pi)^{3/2}} \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{\sqrt{p^+}} (e^{ip\cdot x} D^j_{\mu\nu}(B_f(p/m)) a_f(\tilde{\mathbf{p}}, \nu)$$

$$+ e^{-ip\cdot x} D^j_{\mu\nu}(B_f(p/m)\sigma_2) b^\dagger_f(\tilde{\mathbf{p}}, \mu))$$

$$\Psi_{lf\mu}(x) = \frac{Z}{(2\pi)^{3/2}} \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{\sqrt{p^+}} (e^{ip\cdot x} D^j_{\mu\nu}(B_f^{\dagger -1}(p/m)) a_f(\tilde{\mathbf{p}}, \nu)$$

$$+ e^{-ip\cdot x} D^j_{\mu\nu}(B_f^{\dagger -1}(p/m)\sigma_2) b^\dagger_f(\tilde{\mathbf{p}}, \mu))$$

Free field dynamics:

$$U(\Lambda, a)\Psi_{rf\mu}(x)U^{\dagger}(\Lambda, a) = \sum D^{j}_{\mu\nu}[\Lambda^{-1}]\Psi_{rf\nu}(\Lambda x + a)$$

$$U(\Lambda, a)\Psi_{lf\mu}(x)U^{\dagger}(\Lambda, a) = \sum D^{j}_{\mu\nu}[\Lambda^{\dagger}]\Psi_{lf\nu}(\Lambda x + a)$$

- **Free fields constructed using different Lorentz covariant irreducible bases (instant form, point front) are identical to these.**

Interacting fields

Light-front quantization:

- **Free field operators restricted to the light-front hyperplane are irreducible (i.e. they can be used to represent any operator).**

- **The spectral condition on P^+ suggests that the physical vacuum is identical to the free-field vacuum.**

- **Noether's theorem gives formal expressions for the Poincaré generators as operators integrated over the light-front hyperplane.**

The conventional wisdom about the spectral condition on P^+ in interacting theories:

$$[V, P^+] = 0$$

$$P^+ V|0\rangle = VP^+|0\rangle = 0$$

$$V|0\rangle = 0 \qquad \textbf{or}$$

$$V|0\rangle \neq 0$$

is an eigenstate of P^+ with eigenvalue 0

This is the origin of the assertion that the light-front vacuum is the free Fock vacuum.

Irreducibility - free scalar field case

$$\phi(x) = \frac{1}{(2\pi)^{3/2}} \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{\sqrt{p^+}} (e^{ip\cdot x} a(\tilde{\mathbf{p}}) + e^{-ip\cdot x} a^\dagger(\tilde{\mathbf{p}}))$$

Restriction to light front irreducible:

$$\phi(\tilde{\mathbf{p}}) = \frac{1}{2(2\pi)^{3/2}} \int_{x^+=0} e^{-i\tilde{\mathbf{x}}\cdot\tilde{\mathbf{p}}} \phi(\tilde{\mathbf{x}}, 0) d\tilde{\mathbf{x}}$$

$$a(\tilde{\mathbf{p}}) = \sqrt{p^+}\theta(p^+)\phi(\tilde{\mathbf{p}})$$

$$a^\dagger(\tilde{\mathbf{p}}) = \sqrt{p^+}\theta(p^+)\phi(-\tilde{\mathbf{p}})$$

$[O, \phi(\tilde{\mathbf{x}}, 0)] = 0 \; \forall \, \tilde{\mathbf{x}}$ **implies** O **is proportional to the identity (exception** $p^+ = 0$ **contributions).**
$p^+ = 0$ **is strongly suppressed for operators constructed out of 4d smeared fields.**

Noether's theorem - kinematic generators

$$P^+ = 4\int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-}\frac{\partial\phi(x)}{\partial x^-} :$$

$$P^i = 2\int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-}\frac{\partial\phi(x)}{\partial x^i} :$$

$$J^3 = 2\int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-}(x^j\frac{\partial\phi(x)}{\partial x^i} - x^i\frac{\partial\phi(x)}{\partial x^j}) :$$

$$E^i = 2\int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-}(2x^i\frac{\partial\phi(x)}{\partial x^-} - x^+\frac{\partial\phi(x)}{\partial x^i}) :$$

$$K^3 = 4\int_{x^+=0} d\tilde{\mathbf{x}} : \frac{\partial\phi(x)}{\partial x^-}\frac{\partial\phi(x)}{\partial x^-} : x^-$$

Noether's theorem - dynamical generators

$$P^- = 4\int_{x^+=0} d\tilde{\mathbf{x}} : \phi(x)(-\nabla_\perp^2\phi(x) + m^2\phi(x) + \frac{d\mathcal{V}}{d\phi}(x)) :$$

$$F^i =$$

$$2\int_{x^+=0} d\tilde{\mathbf{x}} : \phi(x)(2x^i(-\nabla_\perp^2\phi(x) + m^2\phi(x) + \frac{d\mathcal{V}}{d\phi}(x))) - x^-\frac{\partial^2\phi(x)}{\partial x^-\partial x^i}) :$$

- Dynamical generators formally defined as operators on the irreducible light-front algebra.

Free field case

- Free field

$$P^{-} = \int d\tilde{\mathbf{p}} a^{\dagger}(\tilde{\mathbf{p}}) \frac{\mathbf{p}_{\perp}^{2} + m^{2}}{p^{+}} a(\tilde{\mathbf{p}})$$

- Light-front Heisenberg equations generate x^{+} dependence:

$$\frac{d\phi(x)}{dx^{+}} = i[P^{-}, \phi(x)]$$

- The solution can be expressed in terms of the light-front creation and annihilation operators which can be expressed in terms of the irreducible algebra of fields on the light front.

$$\phi(x) = \int F_m(x, \tilde{\mathbf{y}})\phi(\tilde{\mathbf{y}}, 0)d\tilde{\mathbf{y}}$$

$$F_m(x, \tilde{\mathbf{y}}) = \frac{1}{2(2\pi)^3}\int d\tilde{\mathbf{p}} e^{-\frac{i}{2}\frac{\mathbf{p}_\perp^2 + m^2}{p^+}x^+ + i\tilde{\mathbf{p}}\cdot(\tilde{\mathbf{x}} - \tilde{\mathbf{y}})}$$

F_m : **local 4d field algebra** $\rightarrow$ **light-front field algebra.**

$$\frac{1}{2(2\pi)^3}\int d^4x g(x^+, \tilde{\mathbf{x}}) e^{-\frac{i}{2}\frac{\mathbf{p}_\perp^2 + m^2}{p^+}x^+ + i\tilde{\mathbf{p}}\cdot\tilde{\mathbf{x}}} = g_{ft}\left(\frac{\mathbf{p}_\perp^2 + m^2}{p^+}, \tilde{\mathbf{p}}\right)$$

- $g_{ft}(\tilde{\mathbf{p}}, \frac{\mathbf{p}_\perp^2 + m^2}{p^+})$ **vanishes strongly near** $p^+ = 0$.
  All spectral information is lost for $x^+ = 0$.

$$\langle 0|\phi(x)\phi(y)|0\rangle =$$

$$\int F_m(x, \tilde{\mathbf{z}})F_m(y, \tilde{\mathbf{w}})\langle 0|\phi(\tilde{\mathbf{z}}, 0)\phi(\tilde{\mathbf{w}}, 0)|0\rangle d\tilde{\mathbf{z}}d\tilde{\mathbf{w}} = W_{20}(s) =$$

$$-i\frac{\epsilon(s^0)\delta(s_0^2 - \mathbf{s}^2)}{4\pi}$$

$$+\frac{im\theta(s_0^2 - \mathbf{s}^2)}{8\pi\sqrt{(s_0^2 - \mathbf{s}^2)}}(\epsilon(s^0)J_1(m\sqrt{(s_0^2 - \mathbf{s}^2)})$$

$$-iN_1(m\sqrt{(s_0^2 - \mathbf{s}^2)}))$$

$$-\frac{m\theta(\mathbf{s}^2 - s_0^2)}{4\pi^2\sqrt{\mathbf{s}^2 - s_0^2}}K_1(m\sqrt{\mathbf{s}^2 - s_0^2})$$

Gives correct 2-point Wightman function using light-front Fock vacuum.

Limit $z^+ \to 0$:

$$\langle 0|\phi(x)\phi(y)|0\rangle \to -i\frac{\epsilon(z^-)\delta(\mathbf{z}_\perp^2)}{4\pi} - \frac{m}{4\pi^2\sqrt{\mathbf{z}_\perp^2}}K_1(m\sqrt{\mathbf{z}_\perp^2}).$$

(Lorentz invariant - remembers mass)

Direct calculation on light front

$$\langle 0|\phi(\tilde{\mathbf{x}})\phi(\tilde{\mathbf{y}})|0\rangle = \frac{1}{(2\pi)^3}\int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{2p^+}e^{i\tilde{\mathbf{p}}\cdot\tilde{\mathbf{z}}}.$$

(not Lorentz invariant - does not know about mass)

(all information about spectrum lost)

limit of integral $\neq$ integral of limit

Remark on Haag's theorem

Different mass free fields restricted to the light front are unitarily equivalent (Schlieder and Seiler).

Extensions to local $4d$ algebra with different F_m are inequivalent.

$$f \sim_{m_1} g \qquad \phi_{m_1}(f) = \phi_{m_1}(g)$$

$$f \not\sim_{m_2} g \qquad \phi_{m_2}(f) \neq \phi_{m_2}(g)$$

Representations of Heisenberg fields in the light-front algebra ($F_m(x, \tilde{\mathbf{y}})$ for interacting fields)

Hagg expansion - irreducibility of in(out) fields:

$$\phi(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n L_n(x; x_1, \cdots, x_n) : \phi_{in}(x_1) \cdots \phi_{in}(x_n) : .$$

$$\phi(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots d^4x_n L_n(x; x_1, \cdots, x_n) \times$$
$$F_{m_1}(x_1^+; \tilde{\mathbf{x}}_1 - \tilde{\mathbf{y}}_1) \cdots F_{m_n}(x_n^+; \tilde{\mathbf{x}}_n - \tilde{\mathbf{y}}_n) \times$$
$$d\tilde{\mathbf{y}}_1 \cdots d\tilde{\mathbf{y}}_n : \tilde{\phi}_{10}(\tilde{\mathbf{y}}_1) \cdots \tilde{\phi}_{n0}(\tilde{\mathbf{y}}_n) :$$

- **Maps local Heisenberg field algebra into light-front Fock algebra.**

Problem 2: Interacting fields, zero modes and renormalization

Observations:

- **Changing the orientation of the light front should not change the physics.**
- **Invariance under change of light-front orientation = rotational invariance.**
- $p^+ \to 0$ **with one light front corresponds to** $p_i \to -\infty$ **in a theory with a different light front.**
- **Dynamical generators in local field theories involve products of fields at the same point, which are not well defined.**
- **Ultraviolet and infrared renormalization are constrained by rotational covariance.**

Is the light front vacuum really the Fock vacuum?

Example $: \phi^4(\tilde{\mathbf{x}}) :$

$$V|0\rangle = \int d\tilde{\mathbf{x}} : \phi^4(\tilde{\mathbf{x}}) : |0\rangle =$$

$$2(2\pi)^2 \int p^+ \theta(p^+) \delta(p^+) dp^+ \prod d\mathbf{p}_{i\perp} \frac{d\xi_1 d\xi_2 d\xi_3 d\xi_4 (1 - \sum_n \xi_n)}{\sqrt{\xi_1 \xi_2 \xi_3 \xi_4}} \times$$

$$\delta(\sum \mathbf{p}_{i\perp}) \prod a^\dagger(\xi_i \mathbf{p}^+, \mathbf{p}_{i\perp})|0\rangle + \cdots$$

which looks like the interaction should annihilate the vacuum, however

$$\| V|0\rangle \|^2 \to \int \frac{\delta(p^+)^2 \theta(p^+)}{p^+} \cdots$$

is not defined of the free field Fock vacuum!

- **The assertion that V annihilates the vacuum is misleading. Quantities like $\delta(p^+)^2$, $\delta(p^+)\theta(p^+)$ are not even distributions. The singular terms ocurr at $p^+ = 0$.**

- **The light front Hamiltonian should be a well-defined operator of the physical Hilbert space.**

$$[V, P^+] = 0 \qquad \rightarrow \qquad V = V(P^+)$$

  it could still have a complicated kernel in transverse relative momenta and momentum fractions.

- **$P^+ \geq 0$ implies that any modifications to the vacuum are associated with $p^+ = 0$.**

The light-front vacuum generating functional

$$\phi(\tilde{f}) = \phi_+(\tilde{f}) + \phi_-(\tilde{f})$$

$\phi_+(\tilde{f})$ **increases** p^+ $\phi_-(\tilde{f})$ **decreases** p^+

Algebraic normal ordering on the free field light-front algebra (independent of mass spectrum):

$$: e^{i\phi(\tilde{f})} := e^{i\phi_+(\tilde{f})} e^{i\phi_-(\tilde{f})}$$

$$e^{i\phi(\tilde{f})} =: e^{i\phi(\tilde{f})} : e^{\frac{1}{2}(\tilde{f},\tilde{f})}$$

$$(\tilde{f},\tilde{g}) = \int \frac{d\tilde{\mathbf{p}}\theta(p^+)}{p^+} \tilde{f}(-\tilde{\mathbf{p}})\tilde{g}(\tilde{\mathbf{p}})$$

Light-front Fock vacuum

$$\langle 0| : e^{i\phi(\tilde{f})} : |0\rangle = 1$$

$$E_{Fock}[\tilde{f}] := \langle 0|e^{i\phi(\tilde{f})}|0\rangle = e^{\frac{1}{2}(\tilde{f},\tilde{f})}$$

Non-Fock vacuum - free light front algebra

Spectral condition: non-zero contributions from $p^+ = 0$:

$$\langle 0| : e^{i\phi(\tilde{f})} : |0\rangle = 1 + \sum \langle 0| : \phi(f)^n : |0\rangle$$

Cluster properties imply:

$$\langle 0| : e^{i\phi(\tilde{f})} : |0\rangle = e^{\sum \frac{i^n}{n!} w_{cn}(f)}$$

$$E[\tilde{f}] = E_{Fock}[\tilde{f}] e^{\sum \frac{i^n}{n!} w_{cn}(f)}$$

- w_{cn} **multilinear connected functionals. They can only depend on $\mathbf{p}_{i\perp}$ and test functions and their derivatives at $p^+ = 0$.**

- **Translational invariance of vacuum** $\rightarrow P^-$ **densely defined self-adjoint operator satisfying:**

$$P^-|0\rangle = 0$$

$$:: \phi^n(\tilde{x}) ::\rightarrow \lim_{y_i \rightarrow 0}(\prod \phi(\tilde{x} + y_i) - \sum B_k(\tilde{x}) C_k(y_1 \cdots y_n))$$

- **The requirement** $:: \phi^n(\tilde{x}) :: |0\rangle = 0$ **relates the coefficients** $C_k(y_1 \cdots y_n)$ **to the vacuum.**

- **Non-trivial contributions are due to** $p^+ = 0$

- **The structure of vacuum, the definition of** P^-**, and rotational covariance must be treated simultaneously.**

Summary

- There are no issues with ill-posed initial value problems with light-front treatments of interacting particles. All forms of dynamics are equivalent.
- Free fields are independent of Lorentz covariant basis.
- Both particle and field dynamics require constructing self-adjoint dynamical generators on the Hilbert space.

  For fields on the light front this requires (1) extending the Noether generators to operators in the light-front algebra, and (2) a consistent treatment of the vacuum, renormalization of local operator products on the light front in the dynamical generators and rotational covariance.
- Extension should lead to same theory as canonical quantization.

Open questions
(for the students in the audience)

- Can light-front field theories be formulated in terms of the free field algebra (with a non-Fock vacuum)?
- Role of rotational covariance in relating infrared and ultraviolet divergences?
- Is there a systematic way to simultaneously construct a self-adjoint P^- and compatible vacuum?
- More information (answers?): See next four talks by Collins, Brodsky, Beane and Hiller

Thanks to the conference organizers and sponsors!