# Relativistic Three-Body Scattering in a First Order Faddeev Formulation

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**Abstract**

Relativistic Faddeev equations for three-body scattering at arbitrary energies are solved in first order in the two-body transition operator in terms of momentum vectors without employing a partial wave decomposition. Relativistic invariance is incorporated within the framework of Poincaré invariant quantum mechanics. Based on a Malfliet-Tjon type interaction, observables for elastic and breakup scattering are calculated and compared to the nonrelativistic ones.

*Key words:* Relativistic Quantum Mechanics, Three-body Problem, Faddeev Equations
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Traditionally three-nucleon calculations are carried out by solving Faddeev equations in a partial wave truncated basis, working either in momentum or in coordinate space. In Ref. [1] the Faddeev equations were solved directly as function of vector variables for scattering at intermediate energies. A key advantage of this formulation lies in its applicability at higher energies, where the number of partial waves proliferates. We investigate relativistic three-boson scattering in the framework of Poincaré invariant quantum mechanics. The main points are the construction of unitary irreducible representations of the Poincaré group, both for noninteracting and interacting particles. The dynamics is generated by a Hamiltonian, and the equations we use have the same operator form as their nonrelativistic counterparts, however the ingredients are quite different.

In the following we concentrate on the leading-order term of the Faddeev multiple scattering series within the framework of Poincaré invariant quantum mechanics. A detailed description of the formulation can be found in Ref. [2]. As a simplification we we consider three-body scattering with spin-independent interactions of Yukawa type, which is mathematically equivalent to three-boson scattering. The interaction employed is of Malfliet-Tjon type, i.e. consists of a short range repulsive and intermediate range attrac-

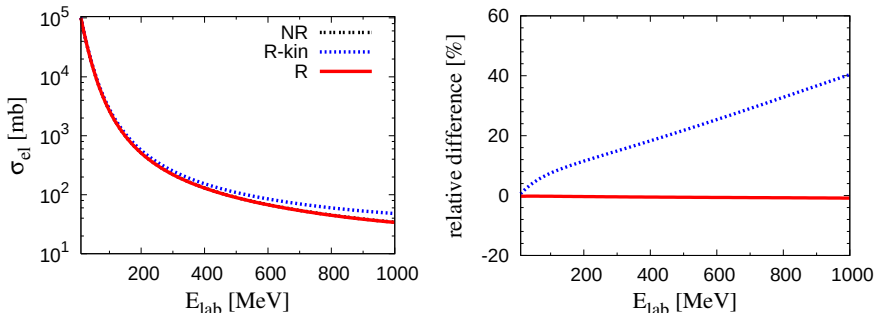


Fig. 1. The total c.m. cross section for elastic scattering calculated from a Malfliet-Tjon type potential (left panel). The nonrelativistic calculation is given by the dash-dotted line (NR), the relativistic one by the solid line (R). The dashed line (R-kin) represents a calculation which only contains the effects of relativistic kinematics. The right panel shows the relative difference of the two relativistic calculations with respect to the nonrelativistic one.

tive Yukawa force with parameters chosen such that a bound state at $E_d = -2.23$ MeV is supported [2]. To obtain a valid estimate of the size of relativistic effects, it is important that the interactions employed in the relativistic and nonrelativistic calculations are phase-shift equivalent. We follow here the suggestion by Coester, Piper, and Serduke (CPS) in constructing a phase equivalent interaction from a non-relativistic 2N interaction [3]. In the CPS method the relativistic interaction can not be analytically calculated from the non-relativistic one. However, there is a simple analytic connection between the relativistic and non-relativistic two-body t-matrices

$$t_{re}(\mathbf{p}, \mathbf{p}'; 2E_p^{rel}) = \frac{2m}{\sqrt{m^2+p^2}+\sqrt{m^2+p'^2}}\, t_{nr}(\mathbf{p}, \mathbf{p}'; 2E_p^{nr}), \tag{1}$$

where $2E_p^{rel} = 2\sqrt{m^2+p^2}$ and $2E_p^{nr} = \frac{p^2}{m} + 2m$. This relativistic two-body t-matrix $t_{re}(\mathbf{p}, \mathbf{p}'; 2E_p^{rel})$ is scattering equivalent to the non-relativistic one at the same relative momentum $\mathbf{p}$ [4]. This t-matrix serves then as input to obtain the Poincaré invariant transition amplitude of the 2N subsystem embedded in the three-particle Hilbert space via a first resolvent method as layed out in Ref. [2].

In Fig. 1 we consider the total cross section for elastic scattering, $\sigma_{el}$ for projectile ki-

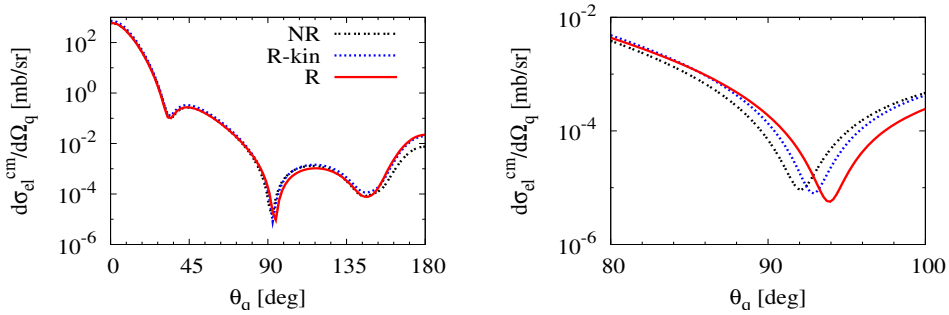


Fig. 2. The differential cross section for elastic scattering at 0.5 GeV projectile energy as function of the laboratory scattering angle. The definition of the lines is the same as in Fig. 1.

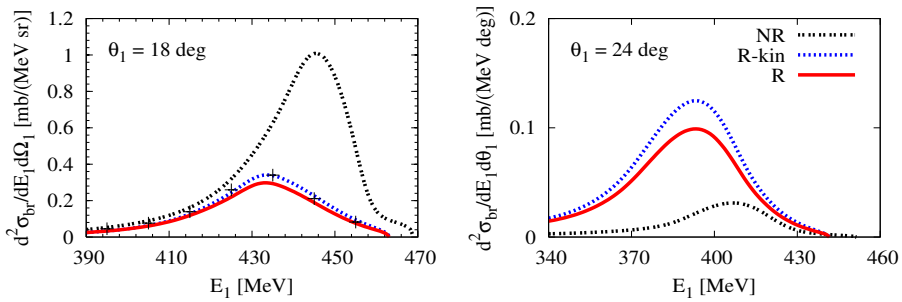


Fig. 3. The inclusive cross section at 0.495 GeV laboratory projectile kinetic energy as function of the energy of the emitted particle and fixed emission angles of $18^o$ and $24^o$ degrees. The definition of the lines is the same as in Fig. 1. The data are from Ref. [5]

netic energies from 10 MeV up to 1 GeV. Starting from the non-relativistic cross section, we successively implement relativistic features to study them in detail. The dashed line labeled *R-kin* shows the cross section when only the effects due to relativistic kinematics, like relativistic transformations from laboratory to c.m. frame, phase-space factor, and the relativistic kinematics due to the Poincaré-Jacobi coordinates are taken into consideration. Implementing in addition the effects due to the relativistic dynamics, i.e. solve the first order equation with the 2N amplitude embedded in the three-particle Hilbert space exactly, leads to a result close to the non-relativistic one. This can be taken as evidence that in a first order calculation the relativistic effects are mostly contained in the 2N amplitude, and if both, the relativistic and non-relativistic ones are equivalent, both 3N cross sections should be very close. Additional effects should be expected from the full solution of the Faddeev equation. The differential cross section for 0.5 GeV laboratory projectile energy is shown in Fig. 2. At the backward angle, the highest momentum transfer, the difference between the relativistic and non-relativistic calculations is clearly largest. A close-up of the second minimum shows that it is shifted by relativistic kinematic effects towards larger angles, and the relativistic dynamics pushes it even further out. As an example of breakup processes we consider inclusive breakup scattering at 0.495 GeV in Fig. 3. Very obvious is the shift of the quasi-free scattering (QFS) peak towards smaller ejectile energies, once relativistic kinematics is introduced into the calculation. The relativistic dynamics only influences the height of the peak, not its position. Since this is first order calculation, the peak height may be influenced in addition by multiple scattering contributions. A comparison of the two angles shown in Fig. 3 also illustrates, that relativistic effects can be quite different in different configurations: while the peak position always shifts towards smaller ejectile energies, the peak height can either be decreased ($18^o$) or increased ($24^o$).

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