DESIGN CONSIDERATIONS FOR A LABORATORY DUSTY PLASMA WITH MAGNETIZED DUST PARTICLES

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The design characteristics of a device in which electrons, ions and dust particles are magnetized are discussed. For dust particles to be magnetized, two criteria must be met: (i) the gyroradius of the dust particles must be considerably less than the lateral dimension of the vacuum chamber, and (ii) the dust cyclotron frequency must be much greater than the dust-neutral collision frequency. Both of these criteria involve the dust radius a, so that this is a crucial choice. Sub-micron grains are easier to magnetize, but may not be easily imaged with laser scattering. Micron size grains require magnetic field strengths in the Tesla range. The availability and affordability of multi-Tesla magnetic fields is then a consideration. The possibility of using a Q machine plasma source is considered.

I. INTRODUCTION

We consider the possibility of performing experiments in dusty plasmas in which the dust particles are magnetized. We are interested in a magnetized dusty plasma in which there is a sufficient amount of dust so that the dust forms a plasma component which interacts collectively with itself and with the other plasma components. This excludes consideration of a system containing a few isolated, magnetized dust particles. The availability of such a device allows for investigations of dusty plasma phenomena that so far have only been studied theoretically. The ubiquitous dusty plasmas in space and astrophysical settings are often immersed in magnetic fields, so that any realistic experimental investigations require magnetized dust grains. Although there have been a number of experimental studies of, e.g., waves in dusty plasmas, including the dust acoustic wave in which the particles participate in the wave motion, there have been no experiments conducted on the dusty plasma waves modes that are possible, e.g., the electrostatic dust-cyclotron wave, in a dusty plasma with magnetized dust.

The issues and challenges involved in a consideration of the feasibility of producing a magnetized dusty plasma are many. The complexities involved include not only the usual ones connected with producing a magnetized ‘normal’, i.e., electron—ion plasma, but the problems of dealing with particles having a mass $\sim 10^{12}$ proton masses and a charge $\sim 10^{3}$ elementary charges. Before calculating the values of the critical plasma parameters involved, e.g., dust gyro-frequency and gyro-radius, it might be reasonable to discuss some practical constraints that could help to reduce the range of the parameters that might be considered.

II. CRITERIA FOR MAGNETIZED DUST

There are two criteria that must be met for a dust particle to be magnetized:

- $r_{cd} << \ell_{\perp}$ (1)
- $\omega_{cd} >> \nu_{dn}$ (2)

where r_{cd} is the gyro-radius of the dust particle, $\ell_{\perp}$ is the plasma dimension transverse to the magnetic field B, ω_{cd} is the dust gyro-frequency, and ν_{dn} is the dust-neutral collision frequency. The first condition states that the plasma must contain many dust gyro-radii and the second condition states that the dust particles must complete many gyro-orbits before colliding with a neutral atom. An additional consideration is that the gyro-period, ω_{cd}^{-1} should be less than some characteristic timescale over which the plasma is stable. However, this condition is likely to be met if condition (2) is met.

A. Magnetic field strength

In terms of scaling with magnetic field, $r_{cd} \sim B^{-1}$ and $\omega_{cd} \sim B$. There are two ways to accommodate the first criteria—(i) use a high enough magnetic field so that r_{cd} is small, or (ii) use a moderate magnetic field strength in a plasma that is large enough to contain many gyro-radii of the dust particles. With scenario (i) the plasma transverse dimension is expected to be many tens of centimeters, whereas in (ii) the plasma transverse dimension would be ~ meters. This brings up considerations of trade-offs in terms of the cost of constructing a device. As B goes up so does the cost of the electromagnets (or possibly superconducting magnets), whereas with a moderate magnetic field, a large vacuum vessel and pumping system is required and the cost scales up with the size. There may be some optimal value of magnetic field which minimize the cost. Some limits on the size of the device and strength of the magnetic field should be established before proceeding further. A reasonable choice might be to limit consideration to a transverse plasma dimension of 2 meters and a maximum magnetic field strength of 2 T.

B. Dust size

Another important consideration in attempting to meet the criteria (1) and (2) is the dust size, characterized by the radius of the dust grains, a. From the scaling arguments that will be presented below, we find that

- $r_{cd} \sim a^{1/2}$ (3)
- $\omega_{cd} / \nu_{dn} \sim a^{-1}$ (4)

so that both criteria require small dust grains. Now there is a further point to consider. If it is desirable to image the dust grains with laser light scattering (LLS) or a PIV system, this places a lower limit on a. Grains with a diameter less than the wavelength of light used cannot be easily imaged. Since typically lasers with $\lambda \approx 500$ nm (0.5 μm) are used, the grain diameter should be $2a > 0.5$ μm, or $a > 0.25$ μm. Clearly, this restricts consideration to dust no bigger than 1 μm. If we remove the possibility of imaging the grains, then the size of the dust is limited only by desirability of having dust with several elementary charges.

The question of the ability to image the dust grains, however raises the issue of what diagnostics can be used on the magnetized dust. It is well-known that probes inserted into a dusty plasmas perturb the dust grains significantly, so this may eliminate the use of probes. On the other hand, even if the dust grains could be imaged, what information would this provide?

C. Plasma source

There are many ways in which a plasma can be made, and the choice impacts the neutral pressure, P, at which the device operates. Since $\nu_{dn} \sim P$, it will be difficult to satisfy the second criterion with a plasma source that operates at high neutral pressure. Just to point out the difficulties associated with the choice of plasma source, if we take $a = 0.5$ μm, $B = 2$ T, and $P = 1$

millitorr, calculations presented below (the other parameters are also given below) indicate that $\omega_{cd}/\nu_{dn} \simeq 0.6$.

This calculation indicates the magnitude of the difficulties one faces in attempting to magnetize dust, even with a 2 T magnetic field, and relatively low neutral pressure. Limiting the neutral pressure to values less than a few millitorr eliminates from consideration some possible plasma sources, e.g., DC glow discharge and rf discharges which usually operate at P ~ 1 Pa (7 millitorr). Hot filament discharges can operate at pressures ~ 10^{-5} torr, but it is nearly impossible (more optimistically—very difficult) to produce uniform plasmas in such discharges in high magnetic fields. A Q machine source can operate at much lower neutral pressures $<10^{-7}$ torr. The Q-3 device at Princeton, could operate in steady state up to 1.6 T. Most Q machines use hot plate sources with a diameter < 6cm. This could probably be pushed up to 10 cm, but any larger diameter might be technically quite difficult but not impossible.

Having touched upon some of the challenges of producing a magnetized dusty plasma and some of the critical design considerations, the next section will summarize the how these parameters are obtained.

III. FUNDAMENTAL PLASMA PARAMETERS FOR A MAGNETIZED DUSTY PLASMA

The three critical parameters that must be specified before all other parameters can be calculated are the dust radius (a), magnetic field strength (B), and the neutral gas pressure (P). These are the parameters that, to some extent, can be chosen by the experimentalist. Other important parameters that cannot be easily controlled are the electron, ion temperature, and dust temperature. The electron and ion temperature and the choice of ion species enter into the determination of the dust charge. The dust temperature is a critical parameter since it enters into the calculation of the dust gyro-radius. The dust mass depends on the dust radius and mass density. The choice of dust material has a relatively small influence on the mass density. The dust mass also depends on the structure of the individual grains, e.g., if hollow microspheres are used, the dust particle mass could be significantly reduced for a given dust radius. For example, a solid glass microsphere might have a mass density of 2000 kg/m^3, whereas a hollow glass microsphere of the same diameter could have an effective mass density of 350 kg/m^3. The availability of monodisperse hollow microspheres of radius 0.5 μm is however, unclear.

We first list all the parameters to be considered:

- elementary charge, permittivity of free space, Boltzmann's constant $= e, \varepsilon_o, k$, respectively
- dust radius = a
- dust mass density = ρ_d
- dust charge = Q_d
- dust surface potential = V_s
- dust charge number = $Q_d/e = Z_d$
- dust temperature = T_d
- dust mass = $m_d = 4\pi a^3 \rho_d / 3$
- electron, ion and neutral atom mass = m_e, m_i , m_n, respectively
- dust thermal speed = $v_{Td} = \sqrt{kT_d / m_d}$
- electron temperature = T_e
- ion temperature = T_i
- ion atomic mass = A
- neutral gas temperature = T_n
- thermal speed of neutral gas atoms = $v_{Tn} = \sqrt{8kT_n / \pi m_n}$
- neutral pressure = P
- neutral atom density = N
- magnetic field strength = B
- dust gyro-frequency = $\omega_{cd} = Q_d B / m_d = 2\pi f_{cd} = 2\pi / T_{cd}$
- dust gyro-radius = $r_{cd} = v_{Td} / \omega_{cd}$
- dust-neutral collision frequency = ν_{dn}
- dust weight = $F_g = m_d\, g$
- magnetic force on dust = F_m

A. Calculation of the dust charge

The dust charge is calculated using OML theory, assuming the dust grains are isolated, i.e., not taking into account the 'close-packing' effect. The following transcendental equation is solved numerically for the dust surface potential, V_s which is related to the dust charge using the capacitance model, $Q_d = 4\pi\varepsilon_o a V_s$:

$$-\left(\frac{kT_e}{m_e}\right)^{\frac{1}{2}} \exp\left(\frac{eV_s}{kT_e}\right) + \left(\frac{kT_i}{m_i}\right)^{\frac{1}{2}} \left(1 - \frac{eV_s}{kT_i}\right) = 0\,. \tag{5}$$

The isolated grain assumption is valid for small values of the Havnes' parameter $P_H = n_d Z_d / n_i$, where n_d and n_i are the dust and ion densities, respectively.

Of course, in a magnetized dusty plasma, the electrons and ions will be magnetized. This brings up the question of how to calculate the dust charge when the electrons and ions are magnetized. This is an open issue in dusty plasma physics that has only recently begun to be addressed theoretically. When the electrons and ions are magnetized, they can only be collected along magnetic field lines that intersect the particle. For a 1 T magnetic field with $T_e = 2.5$ eV and singly charged argon ions with $T_i = 0.025$ eV, the electron and ion gyro-radii are respectively, $r_{ce} = 3.7$ µm and $r_{ci} = 37$ µm.

B. Dust-neutral collision frequency

The dust-neutral collision frequency is calculated using the Epstein formula

$$\nu_{dn} = \delta \frac{4\pi}{3} \frac{m_n N v_{Tn} a^2}{m_d}\,, \tag{6}$$

where δ is a numerical factor which John Goree has determined to be 1.26 for argon. The neutral atom density, N is related to the gas pressure by

$$N\,[m^{-3}] = 3.22\times10^{19} P[mtorr]\,. \tag{7}$$

(c) scaling of the fundamental parameters with a, B, T_d, and P

- $r_{cd} \sim \dfrac{a^{1/2} T_d^{1/2}}{B}$

- $\omega_{cd} \sim \dfrac{B}{a^2}$
- $\nu_{dn} \sim \dfrac{P}{a}$
- $\dfrac{\omega_{cd}}{\nu_{dn}} \sim \dfrac{B}{aP}$
- magnetic force to weight ratio = $\dfrac{F_m}{F_g} = \dfrac{Q_d v_{Td} B}{m_d g} \sim \dfrac{T_d^{1/2} B}{a^{7/2}}$

IV. RESULTS

We present the results of calculations of the fundamental parameters as series of graphs. The choice of the range of the relevant parameters reflects the arguments made in Sec. 2.

A. Dust gyro-radius

Plots of r_{cd} vs. magnetic field strength for dust temperatures ranging from 0.025 eV (cold dust) to 10 eV (hot dust) and for dust radii of (a) 0.1 μm and (b) 0.5 μm are shown in Fig. 1.

B. Ratio of dust-gyro-frequency to dust-neutral collision frequency

Plots of the ratio ω_{cd}/ν_{dn} vs. magnetic field strength for neutral gas pressures ranging from0.001 mtorr to 100 mtorr and for (a) 0.1 μm and (b) 0.5 μm are shown in Fig. 2.

C. Dust gyro-frequency and gyro-period

To indicate the various timescales pertinent to a dust grain in a magnetic field, we show in Fig. 3 a plot of both the gyro-radius, $\omega_{cd}/2\pi$ (solid lines), and gyro-period, $2\pi/\omega_{cd}$, (dashed lines) for a = 0.1 μm and 0.5 μm.

D. Ratio of magnetic force, F_m, on a dust grain to its weight F_g

Plots of F_m / F_g vs. magnetic field strength for dust radii ranging from 0.01 μm to 0.5 μm are shown in Fig. 4 for (a) T_d = 0.025 eV and (b) T_d = 10 eV.

E. Q machine plasma

Since the Q machine source produces a plasma with rather different properties than discharge generated plasmas, we include here two plots of the relevant dust parameters relevant to the Q machine. Fig. 5 (a) gives the dust gyro-radius vs. dust radius for B = 0.5 T and B = 2 T, and for dust temperatures of 0.025 and 10.0 eV. Fig. 5 (b) shows the variation of the dust gyro-period with dust radius and the ratio of dust gyro-frequency to dust-neutral collision frequency for a neutral pressure $P = 10^{-3}$ torr.

V. DISCUSSION

Even a cursory glance at the plots should convince you of the difficulties involved in producing a magnetized dusty plasma. From Fig. 1(b) we see that for $a = 0.5$ μm, even with room temperature dust, a magnetic field of at least 0.6 T is needed to keep the dust gyro-radius below 1 cm. The effect of the dust temperature is also very critical and T_d is not a parameter that can easily be controlled by the experimenter. This, of course is only true if the dust-gyroradius is determined by the dust thermal velocity. If the gyro-radius is determined by some other velocity, e.g. the velocity acquired by the dust as a result of how the dust is dispersed into the plasma, then this would have to be taken into account in computing $r_{cd.}$ It seems inescapable, however, that if the dust is relatively hot, it will be nearly impossible to magnetize it.

The plots in Fig. 2 illustrate the difficulties connected with the neutral pressure in producing magnetized dust. At a minimum, ω_{cd}/ν_{dn} should be at least ~ 1 to maintain the dust magnetized. This condition for the dust sizes investigated suggest that the neutral pressure must be less than 0.1 mtorr. This has important implications for the type of plasma source that could be used.

Fig. 4 gives some indication of the possibility that the magnetic force on the dust could be used to levitate it against gravity. This assumes, of course that the device is oriented so that $\vec{g} \perp \vec{B}$. In this case, Fig. 4 shows that the magnetic field force can only balance gravity for very small grains < 0.1 μm. If $\vec{g} \perp \vec{B}$, this opens up the possibility of a $\vec{g} \times \vec{B}$ drift that would need to be taken into account.

Another possibility, however is to have $\vec{g} \parallel \vec{B}$ in which case the gyro-radius of the dust would probably be determined by its thermal velocity. This geometry may heav some advantage in allowing the dust to be introduced into the plasma by dropping it along the magnetic field lines. If the dust becomes magnetized, it is not clear how to get it into the plasma across the magnetic field lines. A start-up scenario might need to be considered in which the plasma is first produced at low magnetic field, then the dust is introduced, and finally the magnetic field is ramped up to a value necessary to magnetize the dust.

The results shown in Fig. 5 allow us to access more readily the possibility of using a Q machine source. The main advantage is that the Q machine operates at a low pressure, so that even for the lowest B field and largest dust considered $\omega_{cd} / \nu_{dn} > 10$. Another advantage of the Q machine is that the plasma remains relatively uniform even at high values of B, although no one has attempted to operate a Q machine at 2 T. The low value of the electron temperature in the Q machine results in lower Z_d's (5, 28, 55, 277 for a = 0.01, 0.05, 0.1, and 0.5 μm respectively).

VI. CONCLUSIONS AND OUTLOOK

There are some other effects that have not been considered. For example, forces on the dust due to electric fields in the plasma. In magnetized plasmas, the electric potential in the direction ⊥ to B is usually not uniform, so that radial E fields are present. This effect would also have to be taken into account. The ion drag force on the dust has also not been taken into account. If there are radial electric fields in the plasma, the ions will acquire an $\vec{E} \times \vec{B}$ drift which may produce a significant ion drag force on the dust.

There does appear to be some parameter space which is technologically accessible for the production of a magnetized dusty plasma. The parameter space, however, is relatively restricted will place severe restraints on the design of such a device.

There are a number of outstanding issues which should be addressed before proceeding further:

- Should the dust particles be large enough to image (If particle imaging is not necessary, then smaller particles could be used which lowers the strength of the magnetic field required to magnetize them.)

- What techniques could be used to diagnose the properties of this dusty plasma
- What type of plasma source could be used
- What dusty plasma physics phenomena should this device be designed to investigate (This discussion should perhaps precede all others.)

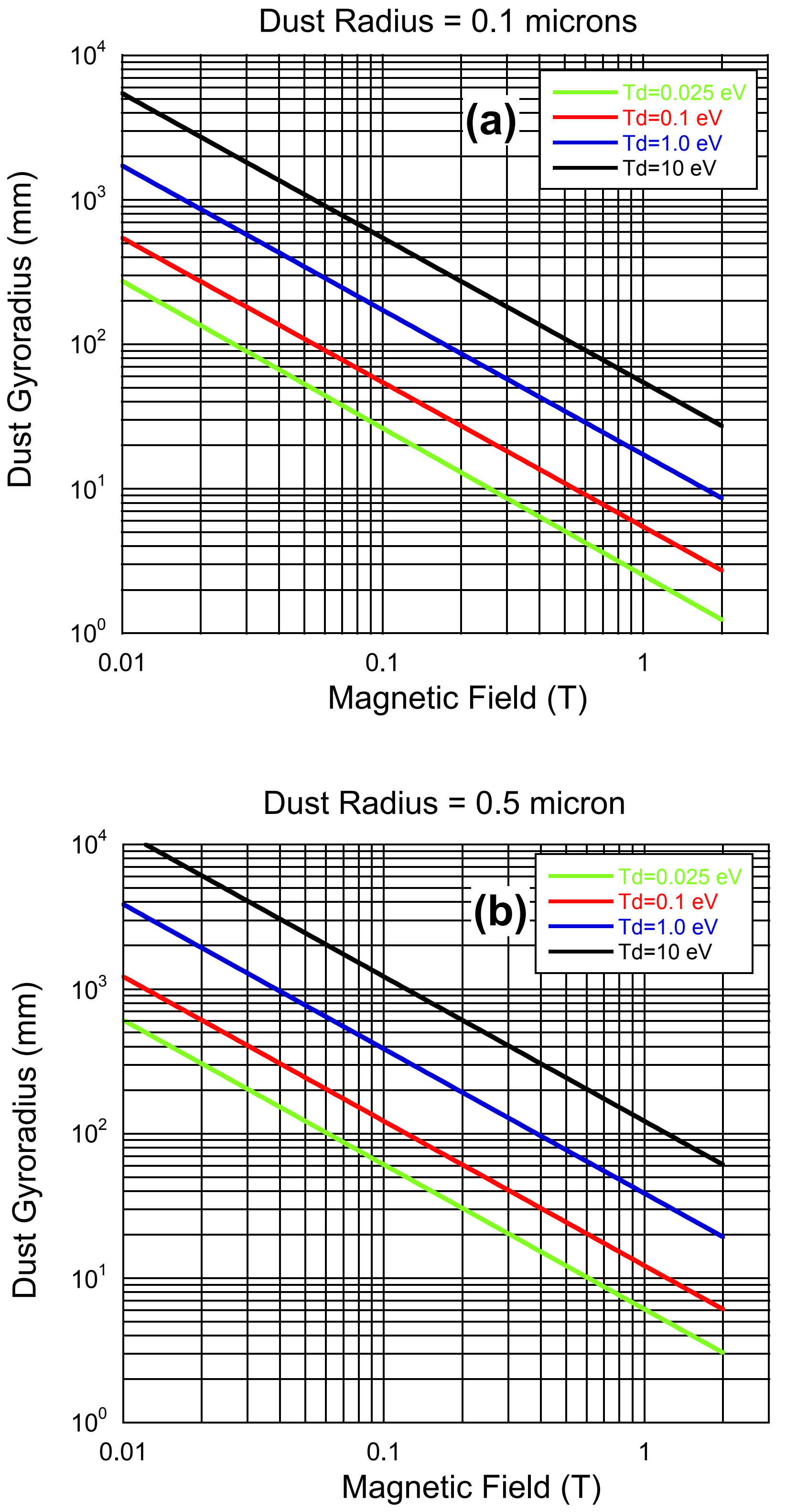

Fig. 1 Dust gyro-radius vs. magentic field strength for various values of the dust temperature. (a) a = 0.1 μm, (b) a = 0.5 μm.

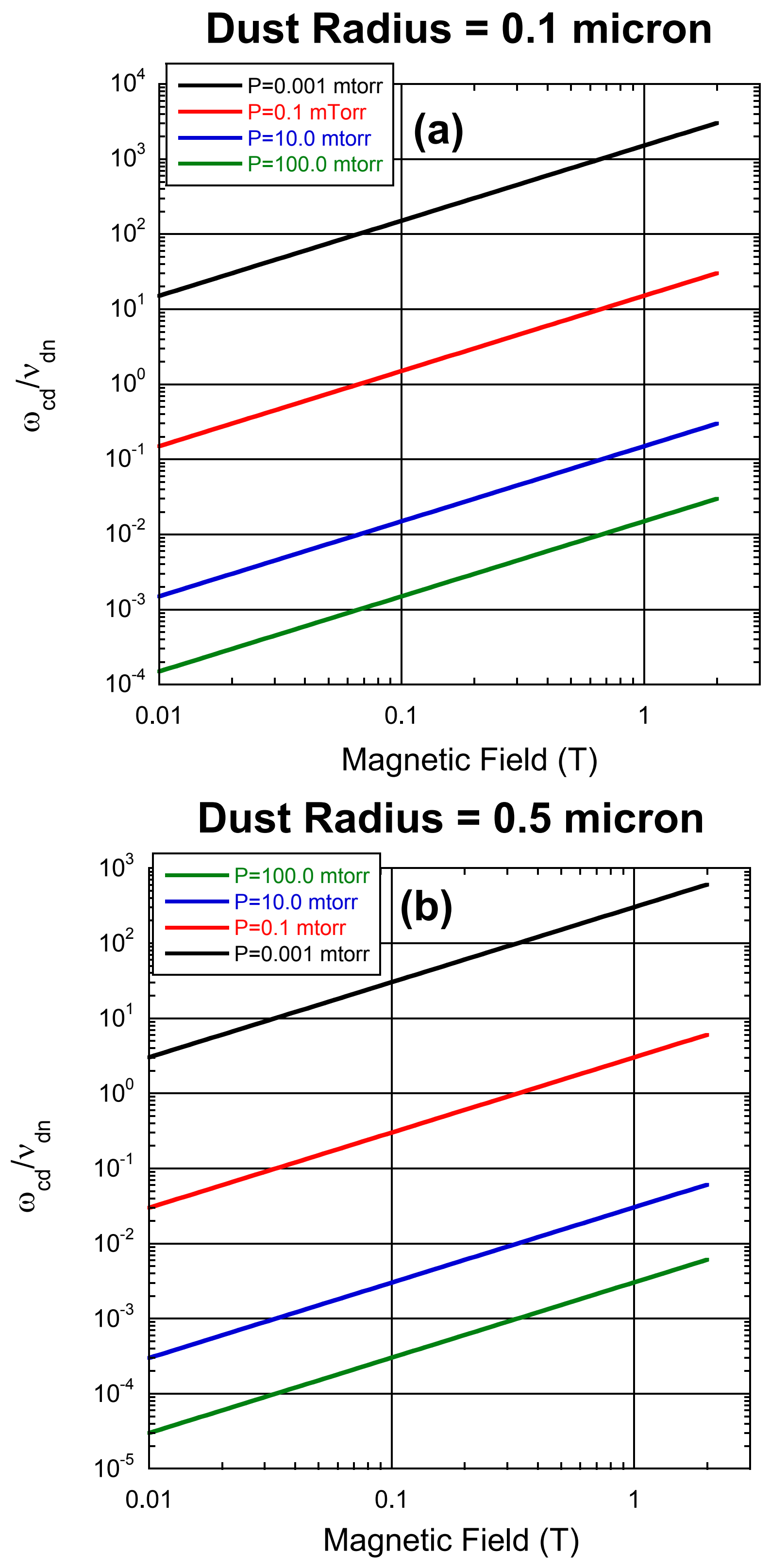

Fig. 2 $\omega_{cd/}\nu_{dn}$ vs. magnetic field for various neutral pressures and (a) a = 0.1 μm and (b) a = 0.5 μm.

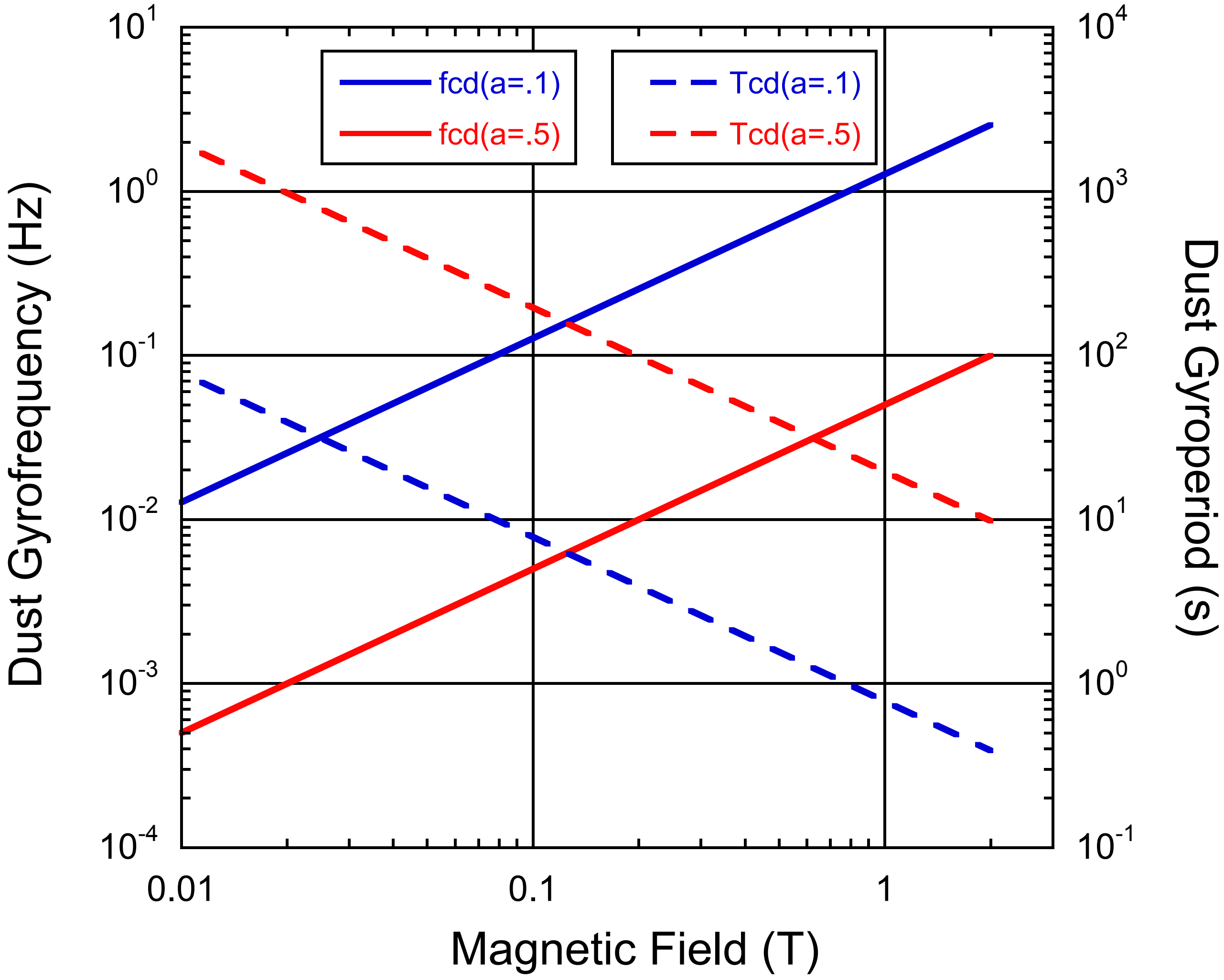

Fig. 3 Dust gyro-frequency and gyro-period vs. magnetic field for a = 0.1 μ mand a=0.5 μm.

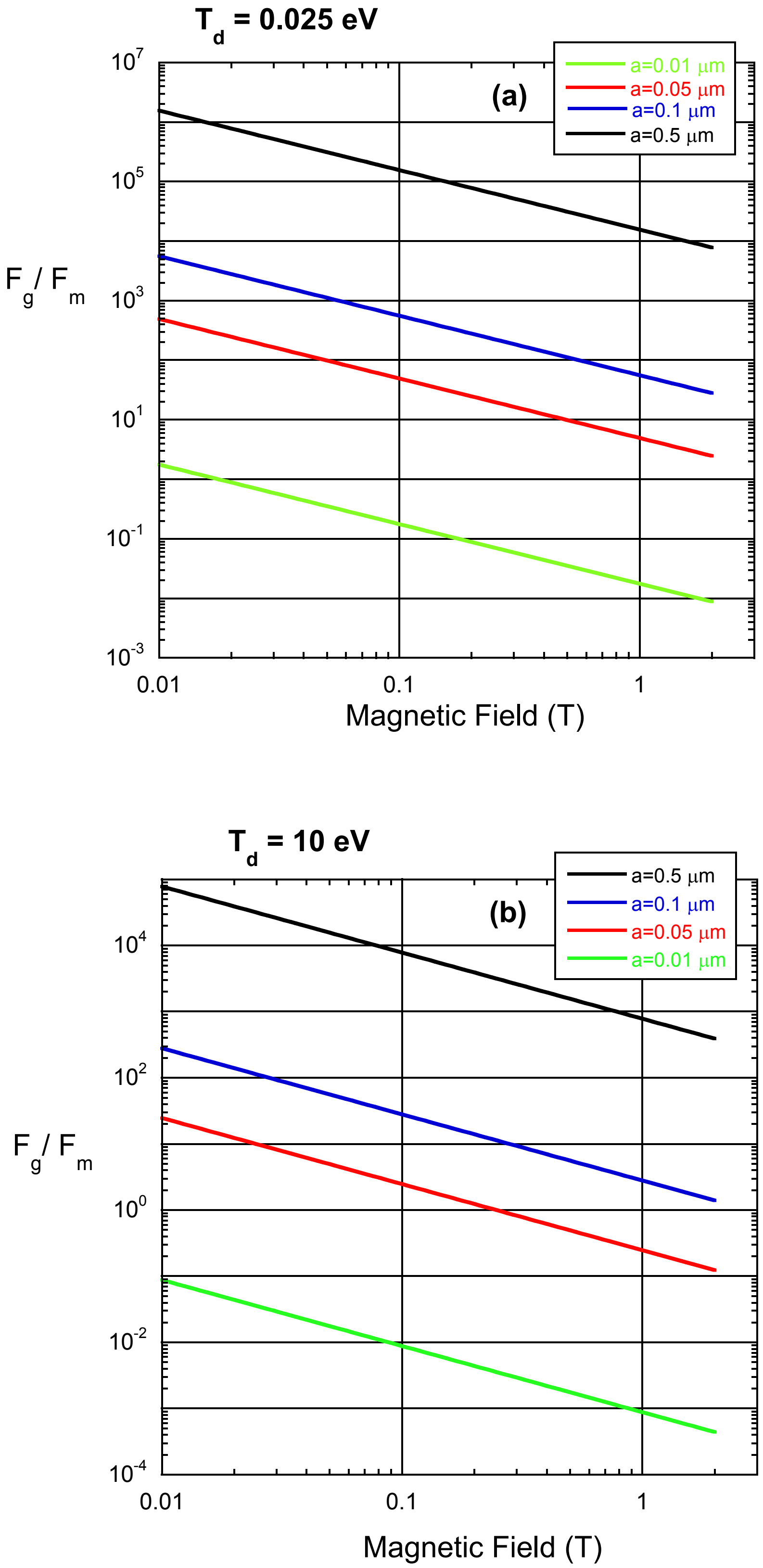

Fig. 4 Ratio of gravitational to magentic force on dust vs. magnetic field for various dust radii for (a) T_d = 0.025 eV, and (b) T_d = 10 eV.

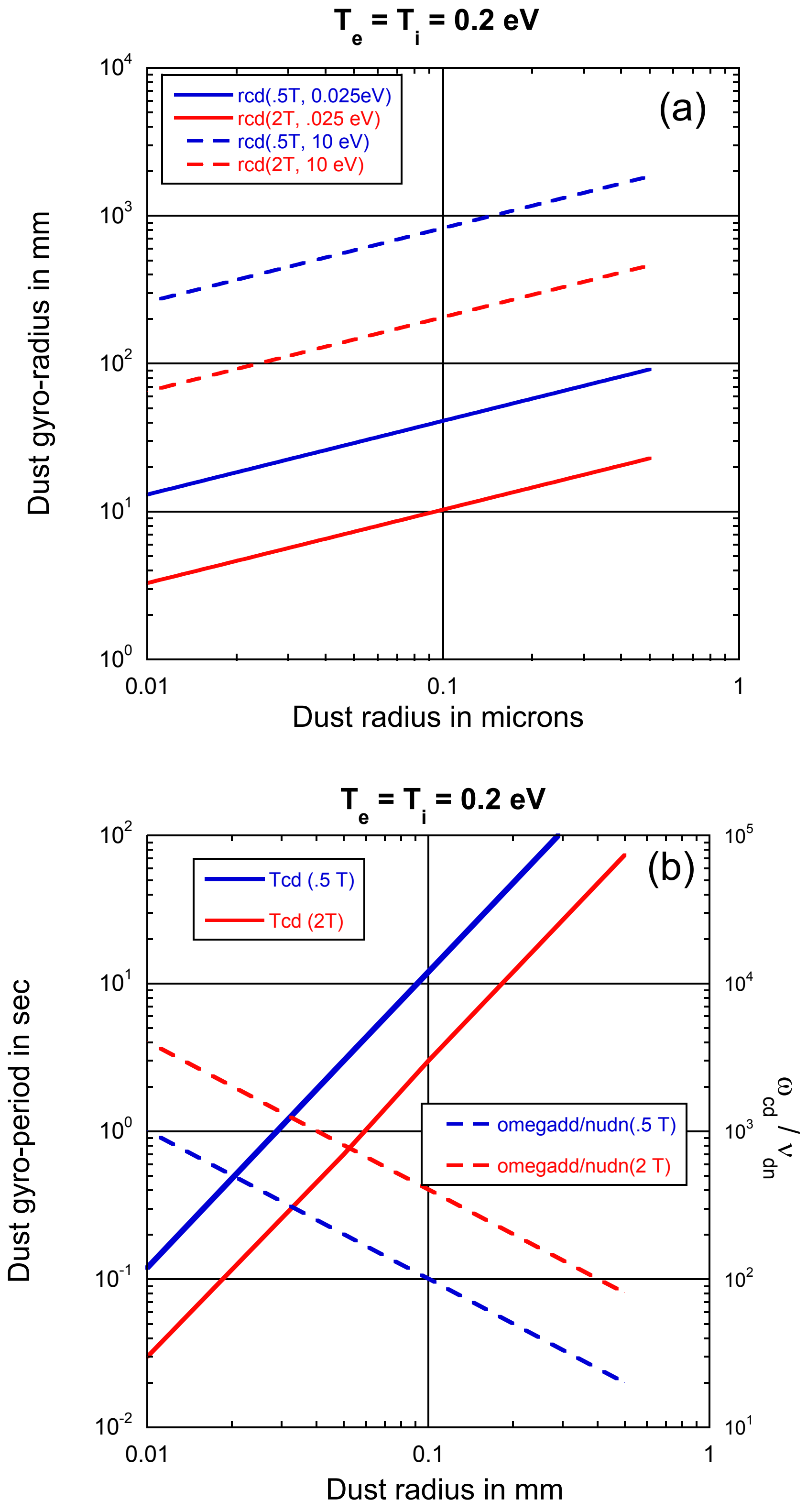

Fig. 5 For Q machine parameters. (a) dust gyro-radius vs. a (b) dust gyroperiod and ω_{cd} / ν_{dn} vs a.