
Comments on The Use of the Epstein Formula for the Neutral Drag Force on Dust Particles

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The neutral drag force, $\vec{F}_{dn}$, is one of the most important forces acting on micron-size dust grains in typical laboratory dusty plasmas. This force can be expressed generally as

$$\vec{F}_{dn} = -m_d \nu_{dn} \vec{u}_{dn} , \tag{1}$$

where m_d is the mass of the dust particles, ν_{nd} is the neutral-dust collision frequency, and $\vec{u}_{dn}$ is the velocity of the dust particle relative to that of the neutral gas atoms (or molecules). When the mean free path, λ_n for atom-atom collisions is much larger than the radius (a) of the dust particle, i.e., $\lambda_n >> a$, and the dust particles move slowly relative to the average thermal speed of the neutral atoms, $u_{dn} << \overline{c}_n$, $\vec{F}_{dn}$ is given by the Epstein formula [1]

$$F_{dn} = \delta \frac{4\pi}{3} a^2 N m_n \overline{c}_n u_{dn} , \tag{2}$$

where, N and m_n are the number density and mass of the atoms, and $\overline{c}_n$ is the average thermal speed of the gas atoms at a temperature T_n,

$$\overline{c}_n = \sqrt{\frac{8kT_n}{\pi m_n}} , \tag{3}$$

where k is the Boltzmann constant. The coefficient δ (~ 1) in (2) accounts for the microscopic mechanism of the collision between the gas atom and the surface of the dust particle, and was discussed by Epstein. For specular reflection of the atoms from the dust surface, δ = 1. In general, δ should be determined experimentally for the particular combination of atoms and dust particles. For melamine-formaldehyde microspheres and argon gas, δ was measured by Liu et al., who found $\delta = 1.26 \pm 0.13$ using a single particle laser acceleration method. [2]

If (1) and (2) are combined, we obtain the expression for the Epstein neutral-dust collision frequency

$$\nu_{dn} = \delta \frac{4\pi}{3} a^2 N \frac{m_n}{m_d} \bar{c}_n . \tag{4}$$

Potential confusion (and error) may occur when (4) is used. The confusion centers around the term $\bar{c}_n$. It is common in plasma physics to use the term "thermal speed" V_{T_α} by which is meant

$$V_{T_\alpha} = \sqrt{\frac{kT_\alpha}{m_\alpha}} . \tag{5}$$

If (5) is used in (4) then (4) becomes

$$\nu_{dn} = \delta \frac{8\sqrt{2\pi}}{3} a^2 N \frac{m_n}{m_d} V_{T_\alpha} . \tag{6}$$

An error may be incurred if (4) is applied, but the thermal speed (5) is used (in place of $\bar{c}_n$) while retaining the parameter (4π/3). Note that

$\left[\left(8\sqrt{2\pi}/3\right)\big/\left(4\pi/3\right)\right] = \left(6.68/4.19\right) = 1.6$.

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[1] P. Epstein, Phys. Rev. **23**, 710 (1924).

[2] Bin Liu, J. Goree, V. Nosenko, and L. Boufendi, Phys. Plasmas **10**, 9 (2003).

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