7. Is it possible to have particles that travel at the speed of light? What does Eq. 2.36 require of such particles?
8. How does relativity combine space and time coordinates into spacetime?
9. Einstein developed the relativity theory after trying unsuccessfully to imagine how a light beam would look to an observer traveling with the beam at speed c. Why is this so difficult to imagine?
10. Explain in your own words the terms *time dilation* and *length contraction*.
11. Does the Moon's disk appear to be a different size to a space traveler approaching it at $v = 0.99c$, compared with the view of a person at rest at the same location?
12. According to the time dilation effect, would the life expectancy of someone who lives at the equator be longer or shorter than someone who lives at the North Pole? By how much?
13. Criticize the following argument. "Here is a way to travel faster than light. Suppose a star is 10 light-years away. A radio signal sent from Earth would need 20 years to make the round trip to the star. If I were to travel to the star in my rocket at $v = 0.8c$, to me the distance to the star is contracted by $\sqrt{1-(0.8)^2}$ to 6 light-years, and at that speed it would take me 6 light-years/0.8c = 7.5 years to travel there. The round trip takes me only 15 years, and therefore I travel faster than light, which takes 20 years."
14. Is it possible to synchronize clocks that are in motion relative to each other? Try to design a method to do so. Which observers will believe the clocks to be synchronized?
15. Suppose event A causes event B. To one observer, event A comes before event B. Is it possible that in another frame of reference event B could come before event A? Discuss.
16. Is mass a conserved quantity in classical physics? In special relativity?
17. "In special relativity, mass and energy are equivalent." Discuss this statement and give examples.
18. Which is more massive, an object at low temperature or the same object at high temperature? A spring at its natural length or the same spring under compression? A container of gas at low pressure or at high pressure? A charged capacitor or an uncharged one?
19. Could a collision be elastic in one frame of reference and inelastic in another?
20. (*a*) What properties of nature would be different if there were a relativistic transformation law for electric charge? (*b*) What experiments could be done to prove that electric charge does *not* change with velocity?

Problems

2.1 Classical Relativity

1. You are piloting a small airplane in which you want to reach a destination that is 750 km due north of your starting location. Once you are airborne, you find that (due to a strong but steady wind) to maintain a northerly course you must point the nose of the plane at an angle that is 22° west of true north. From previous flights on this route in the absence of wind, you know that it takes you 3.14 h to make the journey. With the wind blowing, you find that it takes 4.32 h. A fellow pilot calls you to ask about the wind velocity (magnitude and direction). What is your report?
2. A moving sidewalk 95 m in length carries passengers at a speed of 0.53 m/s. One passenger has a normal walking speed of 1.24 m/s. (*a*) If the passenger stands on the sidewalk without walking, how long does it take her to travel the length of the sidewalk? (*b*) If she walks at her normal walking speed on the sidewalk, how long does it take to travel the full length? (*c*) When she reaches the end of the sidewalk, she suddenly realizes that she left a package at the opposite end. She walks rapidly back along the sidewalk at double her normal walking speed to retrieve the package. How long does it take her to reach the package?

2.2 The Michelson-Morley Experiment

3. A shift of one fringe in the Michelson-Morley experiment corresponds to a change in the round-trip travel time along one arm of the interferometer by one period of vibration of light (about 2×10^{-15} s) when the apparatus is rotated by 90°. Based on the results of Example 2.3, what velocity through the ether would be deduced from a shift of one fringe? (Take the length of the interferometer arm to be 11 m.)

2.4 Consequences of Einstein's Postulates

4. The distance from New York to Los Angeles is about 5000 km and should take about 50 h in a car driving at 100 km/h. (*a*) How much shorter than 5000 km is the distance according to the car travelers? (*b*) How much less than 50 h do they age during the trip?
5. How fast must an object move before its length appears to be contracted to one-half its proper length?
6. An astronaut must journey to a distant planet, which is 200 light-years from Earth. What speed will be necessary if the astronaut wishes to age only 10 years during the round trip?

7. The proper lifetime of a certain particle is 100.0 ns. (*a*) How long does it live in the laboratory if it moves at $v = 0.960c$? (*b*) How far does it travel in the laboratory during that time? (*c*) What is the distance traveled in the laboratory according to an observer moving with the particle?
8. High-energy particles are observed in laboratories by photographing the tracks they leave in certain detectors; the length of the track depends on the speed of the particle and its lifetime. A particle moving at $0.995c$ leaves a track 1.25 mm long. What is the proper lifetime of the particle?
9. Carry out the missing steps in the derivation of Eq. 2.17.
10. Two spaceships approach the Earth from opposite directions. According to an observer on the Earth, ship *A* is moving at a speed of $0.753c$ and ship *B* at a speed of $0.851c$. What is the velocity of ship *A* as observed from ship *B*? Of ship *B* as observed from ship *A*?
11. Rocket *A* leaves a space station with a speed of $0.826c$. Later, rocket *B* leaves in the same direction with a speed of $0.635c$. What is the velocity of rocket *A* as observed from rocket *B*?
12. One of the strongest emission lines observed from distant galaxies comes from hydrogen and has a wavelength of 122 nm (in the ultraviolet region). (*a*) How fast must a galaxy be moving away from us in order for that line to be observed in the visible region at 366 nm? (*b*) What would be the wavelength of the line if that galaxy were moving toward us at the same speed?
13. A physics professor claims in court that the reason he went through the red light ($\lambda = 650$ nm) was that, due to his motion, the red color was Doppler shifted to green ($\lambda = 550$ nm). How fast was he going?

2.5 The Lorentz Transformation

14. Derive the Lorentz velocity transformations for v'_x and v'_z.
15. Observer O fires a light beam in the y direction ($v_y = c$). Use the Lorentz velocity transformation to find v'_x and v'_y and show that O' also measures the value c for the speed of light. Assume that O' moves relative to O with velocity u in the x direction.
16. A light bulb at point x in the frame of reference of O blinks on and off at intervals $\Delta t = t_2 - t_1$. Observer O', moving relative to O at speed u, measures the interval to be $\Delta t' = t'_2 - t'_1$. Use the Lorentz transformation expressions to derive the time dilation expression relating Δt and $\Delta t'$.
17. A neutral K meson at rest decays into two π mesons, which travel in opposite directions along the x axis with speeds of $0.828c$. If instead the K meson were moving in the positive x direction with a velocity of $0.486c$, what would be the velocities of the two π mesons?
18. A rod in the reference frame of observer O makes an angle of 31° with the x axis. According to observer O', who is in motion in the x direction with velocity u, the rod makes an angle of 46° with the x axis. Find the velocity u.
19. According to observer O, two events occur separated by a time interval $\Delta t = +0.465\ \mu$s and at locations separated by $\Delta x = +53.4$ m. (*a*) According to observer O', who is in motion relative to O at a speed of $0.762c$ in the positive x direction, what is the time interval between the two events? (*b*) What is the spatial separation between the two events, according to O'?
20. According to observer O, a blue flash occurs at $x_b = 10.4$ m when $t_b = 0.124\ \mu$s, and a red flash occurs at $x_r = 23.6$ m when $t_r = 0.138\ \mu$s. According to observer O', who is in motion relative to O at velocity u, the two flashes appear to be simultaneous. Find the velocity u.

2.6 The Twin Paradox

21. Suppose the speed of light were 1000 mi/h. You are traveling on a flight from Los Angeles to Boston, a distance of 3000 mi. The plane's speed is a constant 600 mi/h. You leave Los Angeles at 10:00 A.M., as indicated by your wristwatch and by a clock in the airport. (*a*) According to your watch, what time is it when you land in Boston? (*b*) In the Boston airport is a clock that is synchronized to read exactly the same time as the clock in the Los Angeles airport. What time does that clock read when you land in Boston? (*c*) The following day when the Boston clock that records Los Angeles time reads 10:00 A.M., you leave Boston to return to Los Angeles on the same airplane. When you land in Los Angeles, what are the times read on your watch and on the airport clock?
22. Suppose rocket traveler Amelia has a clock made on Earth. Every year on her birthday she sends a light signal to brother Casper on Earth. (*a*) At what rate does Casper receive the signals during Amelia's outward journey? (*b*) At what rate does he receive the signals during her return journey? (*c*) How many of Amelia's birthday signals does Casper receive during the journey that he measures to last 20 years?
23. Suppose Amelia traveled at a speed of $0.80c$ to a star that (according to Casper on Earth) is 8.0 light-years away. Casper ages 20 years during Amelia's round trip. How much younger than Casper is Amelia when she returns to Earth?
24. Make a drawing similar to Figure 2.20 showing the world-lines of Casper and Amelia from Casper's frame of reference. Divide the world line for Amelia's outward journey into 8 equal segments (for the 8 birthdays that Amelia celebrates). For each birthday, draw a line that represents a light signal that Amelia sends to Casper on her birthday. Do the same for Amelia's return journey. (*a*) According to Casper's time, when does he receive the signal showing Amelia celebrating her 8th birthday after leaving Earth? (*b*) How long does it take for Casper to receive the signals showing Amelia celebrating birthdays 9 through 16?

2.7 Relativistic Dynamics

25. (*a*) Using the relativistically correct final velocities for the collision shown in Figure 2.21*a* ($v'_{1f} = -0.585c$, $v'_{2f} = +0.294c$), show that relativistic kinetic energy is conserved

according to observer O'. (b) Using the relativistically correct final velocities for the collision shown in Figure 2.21b ($v_{1f} = -0.051c, v_{2f} = +0.727c$), show that relativistic kinetic energy is conserved according to observer O.

Find the momentum, kinetic energy, and total energy of a proton moving at a speed of $0.756c$.

An electron is moving with a kinetic energy of 1.264 MeV. What is its speed?

The work-energy theorem relates the change in kinetic energy of a particle to the work done on it by an external force: $\Delta K = W = \int F\,dx$. Writing Newton's second law as $F = dp/dt$, show that $W = \int v\,dp$ and integrate by parts using the relativistic momentum to obtain Eq. 2.34.

For what range of velocities of a particle of mass m can we use the classical expression for kinetic energy $\frac{1}{2}mv^2$ to within an accuracy of 1%?

For what range of velocities of a particle of mass m can we use the extreme relativistic approximation $E = pc$ to within an accuracy of 1%?

Use Eqs. 2.32 and 2.36 to derive Eq. 2.39.

Use the binomial expansion $(1+x)^n = 1 + nx + [n(n-1)/2!]x^2 + \cdots$ to show that Eq. 2.34 for the relativistic kinetic energy reduces to the classical expression $\frac{1}{2}mv^2$ when $v \ll c$. This important result shows that our familiar expressions are correct at low speeds. By evaluating the first term in the expansion beyond $\frac{1}{2}mv^2$, find the speed necessary before the classical expression is off by 0.01%.

(a) According to observer O, a certain particle has a momentum of 817 MeV/c and a total relativistic energy of 1125 MeV. What is the rest energy of this particle? (b) An observer O' in a different frame of reference measures the momentum of this particle to be 953 MeV/c. What does O' measure for the total relativistic energy of the particle?

An electron is moving at a speed of $0.81c$. By how much must its kinetic energy increase to raise its speed to $0.91c$?

What is the change in mass when 1 g of copper is heated from 0 to 100°C? The specific heat capacity of copper is 0.40 J/g·K.

Find the kinetic energy of an electron moving at a speed of (a) $v = 1.00 \times 10^{-4}c$; (b) $v = 1.00 \times 10^{-2}c$; (c) $v = 0.300c$; (d) $v = 0.999c$.

An electron and a proton are each accelerated starting from rest through a potential difference of 10.0 million volts. Find the momentum (in MeV/c) and the kinetic energy (in MeV) of each, and compare with the results of using the classical formulas.

n a nuclear reactor, each atom of uranium (of atomic mass 35 u) releases about 200 MeV when it fissions. What is the hange in mass when 1.00 kg of uranium-235 is fissioned?

2.8 Conservation Laws in Relativistic Decays and Collisions

39. A π meson of rest energy 139.6 MeV moving at a speed of $0.906c$ collides with and sticks to a proton of rest energy 938.3 MeV that is at rest. (a) Find the total relativistic energy of the resulting composite particle. (b) Find the total linear momentum of the composite particle. (c) Using the results of (a) and (b), find the rest energy of the composite particle.
40. An electron and a positron (an antielectron) make a head-on collision, each moving at $v = 0.99999c$. In the collision the electrons disappear and are replaced by two muons ($mc^2 = 105.7$ MeV), which move off in opposite directions. What is the kinetic energy of each of the muons?
41. It is desired to create a particle of mass 9700 MeV/c^2 in a head-on collision between a proton and an antiproton (each having a mass of 938.3 MeV/c^2) traveling at the same speed. What speed is necessary for this to occur?
42. A particle of rest energy mc^2 is moving with speed v in the positive x direction. The particle decays into two particles, each of rest energy 140 MeV. One particle, with kinetic energy 282 MeV, moves in the positive x direction, and the other particle, with kinetic energy 25 MeV, moves in the negative x direction. Find the rest energy of the original particle and its speed.

2.9 Experimental Tests of Special Relativity

43. In the muon decay experiment discussed in Section 2.9 as a verification of time dilation, the muons move in the lab with a momentum of 3094 MeV/c. Find the dilated lifetime in the laboratory frame. (The proper lifetime is 2.198 μs.)
44. Derive the relativistic expression $p^2/2K = m + K/2c^2$, which is plotted in Figure 2.28a.

General Problems

45. Suppose we want to send an astronaut on a round trip to visit a star that is 200 light-years distant and at rest with respect to Earth. The life support systems on the spacecraft enable the astronaut to survive at most 20 years. (a) At what speed must the astronaut travel to make the round trip in 20 years of spacecraft time? (b) How much time passes on Earth during the round trip?
46. A "cause" occurs at point 1 (x_1, t_1) and its "effect" occurs at point 2 (x_2, t_2). Use the Lorentz transformation to find $t'_2 - t'_1$, and show that $t'_2 - t'_1 > 0$; that is, O' can never see the "effect" coming before its "cause."
47. Observer O sees a red flash of light at the origin at $t = 0$ and a blue flash of light at $x = 3.26$ km at a time $t = 7.63$ μs. What are the distance and the time interval between the flashes according to observer O', who moves relative to O in the direction of increasing x with a speed of $0.625c$?