1a. $\rho = \varepsilon_0 \nabla \cdot \vec{E}$

$= \varepsilon_0 \cdot \frac{1}{r^2} \partial/\partial r (r^2 E_r)$

$= \varepsilon_0 \cdot \frac{1}{r^2} \cdot \partial/\partial r (A r^4)$

$= 4A\varepsilon_0 r$

b. $Q = \int \rho \, dV = 4\pi \int_0^R 4A\varepsilon_0 r \cdot r^2 \, dr$

$= 16\pi \varepsilon_0 A \cdot \frac{r^4}{4} \Big|_0^R$

$= 4\pi\varepsilon_0 A R^4$

$\vec{E} = Q/4\pi\varepsilon_0 r^2 \hat{r} = AR^4/r^2 \hat{r}$

c. $V(r) = -\int_\infty^r E_r \, dr$

$r > R$; $V(r) = -\int_\infty^r \frac{AR^4}{r^2} dr$

$= \frac{AR^4}{r} \Big|_\infty^r = \frac{AR^4}{r}$

$r < R$; $V(r) = -\int_\infty^R \frac{AR^4}{r^2} dr - \int_R^r Ar^2 dr$

$= AR^3 - Ar^3/3 \Big|_R^r$

$= AR^3 - Ar^3/3 + AR^3/3$

$= 4/3 AR^3 - Ar^3/3$

V continuous @ $r = R$

w/ $V(R) = AR^3$

2. $\rho(\vec{r}) = q\,\delta^3(\vec{r}-\vec{r}_1) - q\,\delta^3(\vec{r}-\vec{r}_2)$

3.a. $V_{in}(r,\theta) = V_A + V_0 \frac{r}{R}\cos\theta$

$$V_{out}(r,\theta) = V_A \frac{R}{r} + V_0 \frac{R^2}{r^2}\cos\theta$$

Matches @ $r = R$

b. $\Delta\left(\partial V/\partial n\right) = \Delta\, \partial V/\partial r\big|_R = -\sigma/\varepsilon_0$

$$\partial V_{out}/\partial r\big|_R = -V_A R/r^2 - 2V_0R^2/r^3 \cos\theta\,\big|_R$$
$$= -V_A/R - 2V_0/R\,\cos\theta$$

$$\partial V_{in}/\partial r\big|_R = \frac{V_0}{R}\cos\theta$$

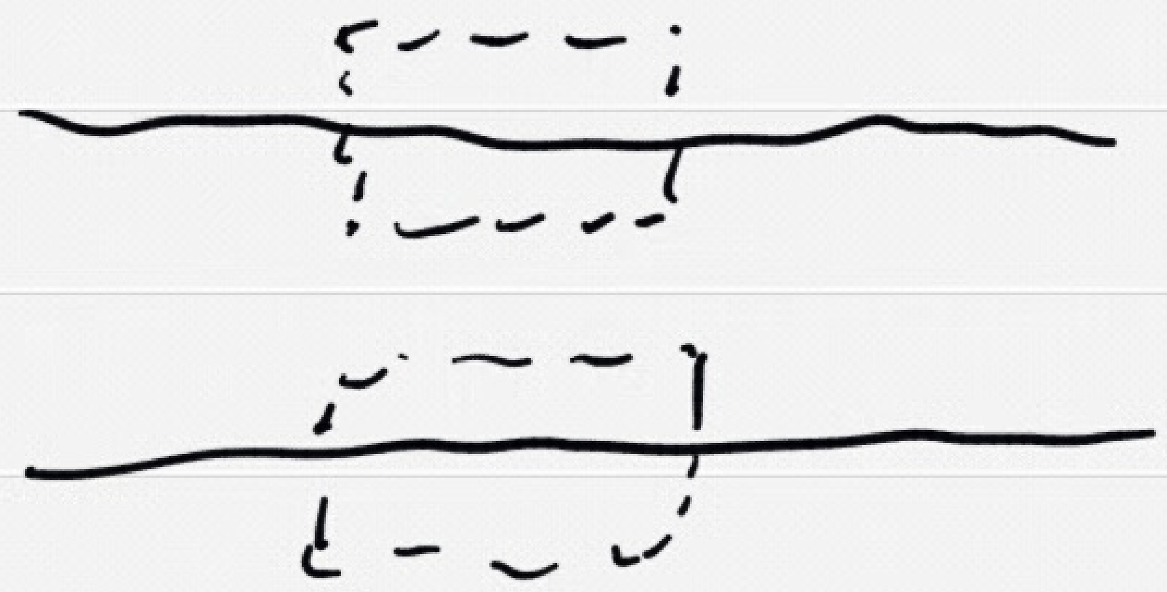
$$\sigma = -\varepsilon_0\, \Delta\, \partial V/\partial r$$
$$= \varepsilon_0\left(V_A/R + \frac{2V_0}{R}\cos\theta + V_0/R\cos\theta\right)$$
$$= \boxed{\frac{\varepsilon_0}{R}\left(V_A + 3V_0\cos\theta\right)}$$

4a. perp. continuous $\Rightarrow$ D

b. perp. continuous $\Rightarrow$ B

5. a. $\vec{P}$ uniform $\Rightarrow$ $\rho_b = 0$

$\sigma_b = \vec{P}\cdot\hat{n} =$ $+P$ on top
$-P$ on bottom



$$\oint \vec{E}_{top}\cdot d\vec{a} = 2E_{top}\cdot A = PA/\varepsilon_0$$

$$\Rightarrow E_{top} = \frac{P}{2\varepsilon_0}\hat{n}$$

similarly $E_{bottom} = -\frac{P}{2\varepsilon_0}\hat{n}$

$E_{above} = 0$
$E_{in\ between} = P/\varepsilon_0$ downward
$E_{below} = 0$

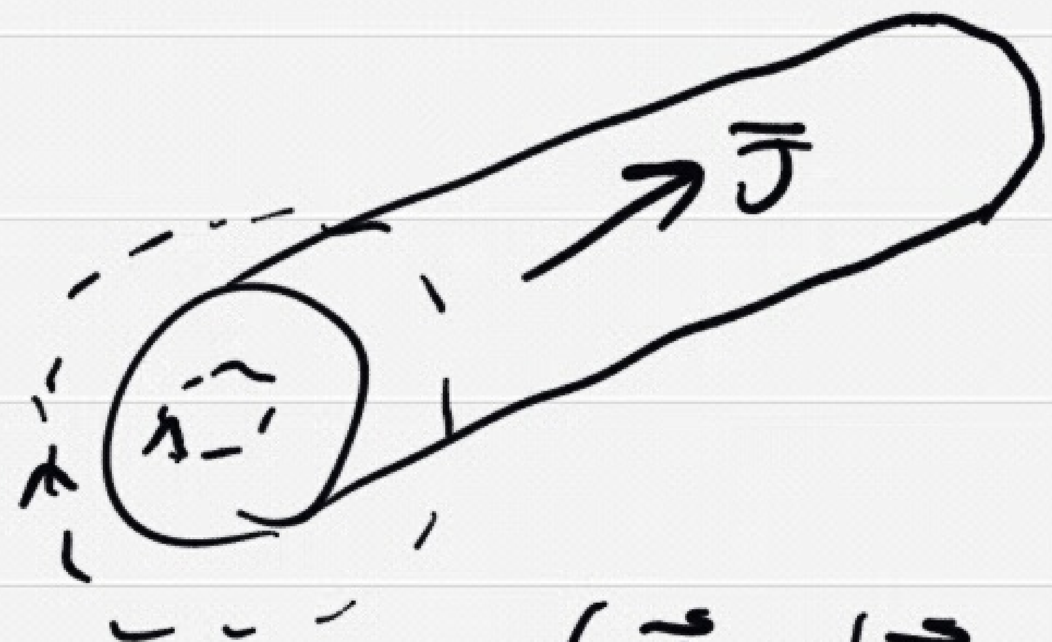
b. $D = \varepsilon_0\vec{E} + \vec{P}$

$D_{above} = 0$
$D_{in\ between} = 0$
$D_{below} = 0$

No free charge, so D continuous

6. $\mu = i \cdot \pi R^2 - i \pi R^2$
$= 0$

7. $\vec{J} = \frac{K}{s}\hat{z}$



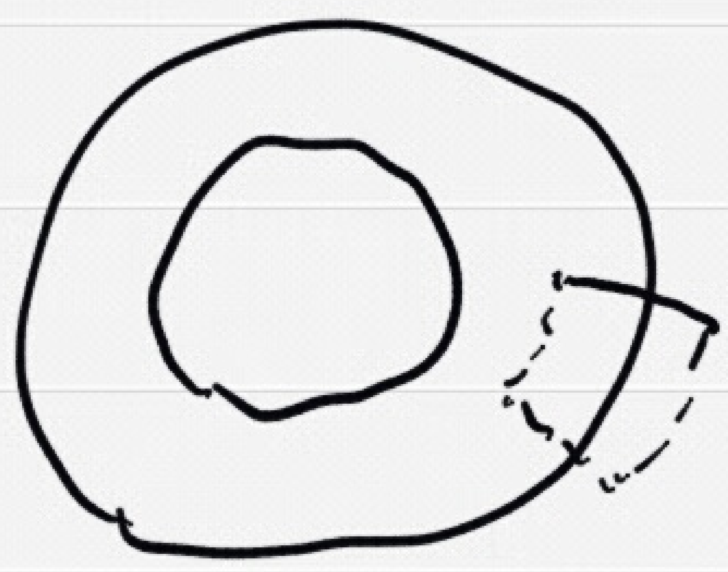
$$\oint \vec{B} \cdot d\vec{l} = B_\varphi \cdot 2\pi s$$
$$= \mu_0 I_{enc}$$
$$= \mu_0 \int \vec{J} \cdot d\vec{a}$$
$$= \mu_0 \int \frac{K}{s} \cdot 2\pi s \, ds$$
$$= \mu_0 \int 2\pi K \, ds$$
$$= \mu_0 \cdot 2\pi K s$$

$$\Rightarrow \vec{B}(s<R) = \mu_0 K \hat{\varphi}$$

$$\vec{B}(s>R) = \frac{\mu_0 K R}{s} \hat{\varphi}$$

check $\nabla \times \vec{B} = \frac{\mu_0 K}{s} \hat{z}$ $\quad s<R$
$= 0$ $\quad s>R$ //

8. a.



$$\oint \vec{H} \cdot d\vec{l} = H \cdot L$$
$$= I_{f\_enc}$$
$$= NI \cdot \frac{L}{2\pi s}$$

$$\Rightarrow \vec{H} = \frac{NI}{2\pi s} \hat{\varphi}$$

b. $\vec{M} = \chi_m \vec{H} = \chi_m \frac{NI}{2\pi s} \hat{\varphi}$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$
$$= (1+\chi_m) \frac{\mu_0 N I}{2\pi s} \hat{\varphi}$$

c. $\vec{J}_b = \nabla \times \vec{M}$

$$= \frac{1}{s} \frac{\partial}{\partial s} (s M_\varphi) \hat{z}$$
$$= 0$$

$$\vec{K}_b = \vec{M} \times \hat{n}$$
$$= -\chi_m \cdot \frac{NI}{2\pi b} \hat{z} \quad \text{outer}$$
$$= \chi_m \frac{NI}{2\pi s} \hat{s} \quad \text{top}$$
$$= \chi_m \frac{NI}{2\pi a} \hat{z} \quad \text{inner}$$
$$= -\chi_m \frac{NI}{2\pi s} \hat{s} \quad \text{bottom}$$

$$= \chi_m \vec{K}_{free}$$