Electricity and Magnetism 1 [3811] Final Exam
Tuesday December 11th, 2018

Directions:

This exam is closed book. You are allowed a copy of the latest equation sheet posted on the course website. You may annotate your equation sheet.

Read all the questions carefully and answer every part of each question. Show your work on all problems – partial credit may be granted for correct logic or intermediate steps, even if your final answer is incorrect.

Unless otherwise instructed, express your answers in terms of fundamental constants like μ_0 and ε_0, rather than calculating numerical values.

If the question asks for an explanation, write at least one full sentence explaining your reasoning.

Please ask if you have any questions, including clarification about the instructions, during the exam.

<u>This test is designed to be gender and race neutral.</u>

Good luck!

Honor Pledge: I understand that sharing information with anyone during this exam by talking, looking at someone else's test, or any other form of communication, will be interpreted as evidence of cheating. I also understand that if I am caught cheating, the result will be no credit (0 points) for this test, and disciplinary action may result.

Sign Your Name___________________________________

Print Your Name___________________________________

Problem 1 (15 points): Consider a sphere of radius R, with a given charge density inside the sphere, and no charge outside the sphere.

1a (5 points). If the electric field inside the sphere is $\vec{E}(\vec{r}) = Ar^2\hat{r}$, find the charge density *ρ(r)* inside the sphere.

1b (5 points). Find the total charge contained in the sphere, and from this derive the electric field outside the sphere.

1c (5 points). Find the electric potential as a function of radius inside and outside the sphere, with respect to infinity.

Problem 2 (10 points): Consider an arrangement with one point charge *+q* at source position $\vec{r}_1$ and a second point charge *-q* at source position $\vec{r}_2$. Use Dirac delta functions to write the corresponding total volume charge density $\rho(\vec{r})$ as a function of position $\vec{r}$.

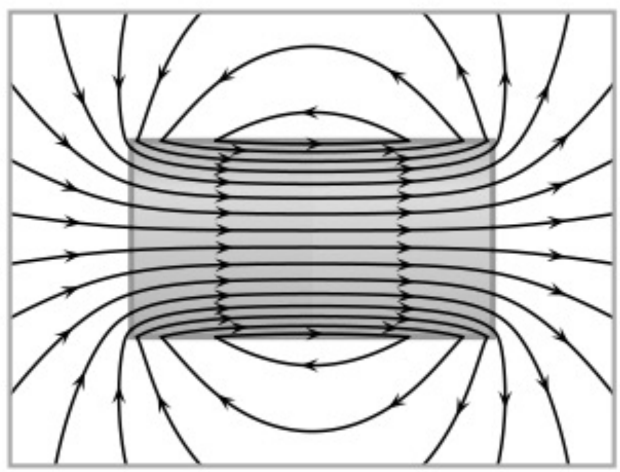
Problem 3 (15 points): Consider a spherical shell of radius R, held at a fixed electric potential $V_o(\theta) = V_a + V_b \cos(\theta)$, with vacuum inside and surrounding it.

3a (6 points). Find the full solutions $V_{in}(r,\theta)$ and $V_{out}(r,\theta)$ valid for the regions inside and outside of the sphere.

Note: The first four Legendre polynomials are $P_0(\cos\theta) = 1$, $P_1(\cos\theta) = \cos\theta$, $P_2(\cos\theta) = (3\cos^2\theta - 1)/2$, $P_3(\cos\theta) = (5\cos^3\theta - 3\cos\theta)/2$.

3b (9 points). From the solutions for *V*, find the charge density $\sigma(\theta)$ on the shell.

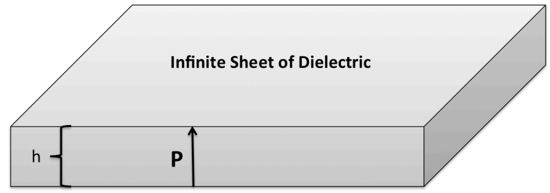
Problem 4 (10 points): Given the image at right:



4a (5 points). If the figure shows a uniformly polarized object (with polarization pointing to the right) does it show lines of *D* or lines of *E*? *Explain why in a sentence, or using equations.*

4b (5 points). If the figure instead shows a uniformly magnetized object (with magnetization pointing to the right), does it show lines of *B* or lines of *H*? *Explain why in a sentence, or using equations.*

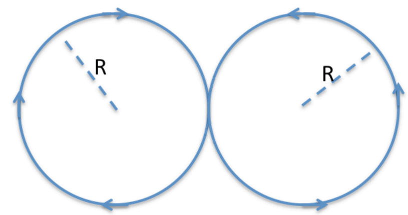
Problem 5 (15 points): Consider an infinite sheet of dielectric material with thickness *h*, with a uniform polarization *P* perpendicular to the plane of the sheet, and no free charge.



5a (10 points). Find all of the bound charge, and then use Gauss's law to find the electric field *E* (magnitude and direction) above, below, and inside the dielectric. Be explicit about the Gaussian surface(s) you use to find *E* in the three regions.

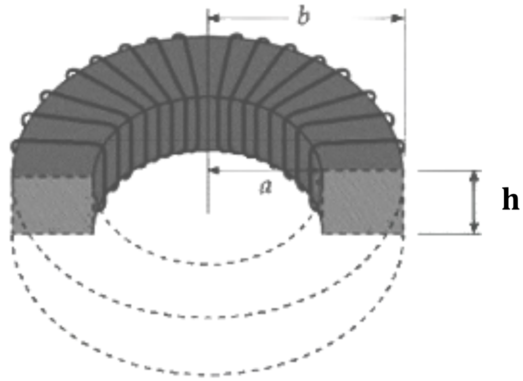
5b (5 points). Find *D* (magnitude and direction) in the three regions, and show that your answer satisfies the boundary conditions on *D* at the top and bottom of the polarized sheet.

Problem 6 (10 points): Find the magnetic dipole moment (magnitude and direction) of a current carrying wire twisted into a figure eight (as shown).



Problem 7 (10 points): Consider an infinitely long cylinder of radius R, with an axial volume current density that varies inversely with radius from the center to the surface of the cylinder (i.e. $\vec{J}(\vec{r}) = \frac{k}{s}\hat{z}$), and no current density outside the cylinder. Find the magnetic field (magnitude and direction) inside and outside of the cylinder.

Problem 8 (15 points): Consider an N-loop toroid carrying current I, with a rectangular cross section with inner and outer radii *a* and *b* and height *h.* The toroid is completely filled with a core of paramagnetic material.



8a (5 points). Use Ampere's law to find *H* (magnitude and direction) as a function of position inside the toroid. Be explicit about the Amperian loop(s) you use to find *H*.

8b (5 points). If the material inside the toroid has a linear magnetic susceptibility χ_m, find the magnetization *M* and the magnetic field *B* inside the toroid.

8c (5 points). From the magnetization you found above, compute the bound surface and volume current densities in and on the surface of the paramagnetic core. How are these related to the free current I?