

Lecture #6: Key Results on Kinetic Turbulence: Driving Scales

Here we cover key results for understanding kinetic turbulence. A good reference is:

Howes, G.G. "Kinetic Turbulence," in Magnetic Fields in Diffuse Media, ed. de Gouveia Dal Pino, E. & Lazarian, A., Springer: Heidelberg (2015).

I. Driving or Energy Containing Scales

A. Powering the Turbulent Cascade

1. Turbulence is generally driven by violent events or instabilities at large scales that generate fluctuations in the magnetic field or plasma flows.

a. Driving Scale: $L \gg \rho_i$

b. Driving wavenumber $k_0 \sim \frac{1}{L}$

2. Isotropic Driving: In the absence of arguments to the contrary, we assume driving is isotropic relative to the magnetic field direction,

$$k_{\parallel 0} \sim k_{\perp 0} \sim k_0$$

3. Debye: Isotropic Driving Wavenumber

$$k_0 \rho_i \ll 1$$

Ref: Howes et al. JGR (2008), Howes et al. (2011) Pop.

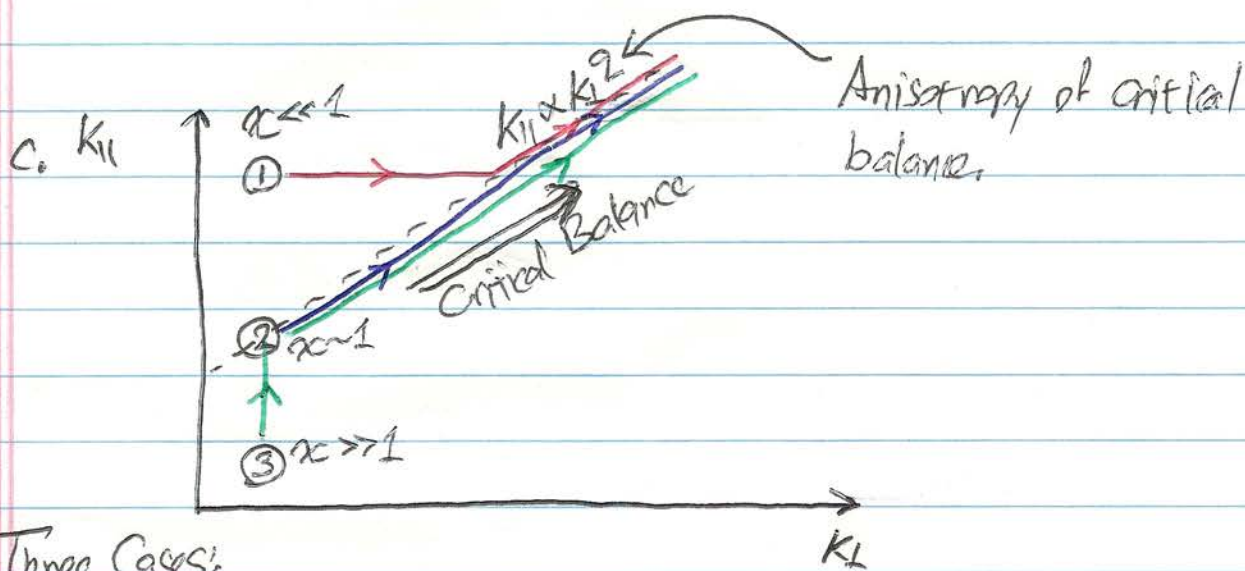
4. Amplitude of Driving:

a. If the MHD approximation is satisfied at L , then MHD turbulence theory will apply
 (Sridhar & Goldreich 1994, Goldreich & Sridhar 1995 (GS95)
 Galtier et al. 2009, Lithwick & Goldreich 2001, Boldyrev 2006)
 (B06)

b. Nonlinearity Parameter:

i. $\alpha = \frac{k_{\perp} v_k}{k_{\parallel} v_A}$ (GS95)

ii. $\alpha = \frac{k_{\perp} v_k \Theta_k}{k_{\parallel} v_A}$ (B06) At L , take $\Theta_k \sim 1$ rad.
 thus both versions are equivalent.



Three Cases:

- ① Weak Driving $\alpha \ll 1$: No parallel cascade, $k_{\parallel} \propto \text{constant}$
 $E_k \propto k_{\perp}^{-2} \Rightarrow v_k \propto k_{\perp}^{-1/2} \Rightarrow \alpha \propto k_{\perp}^{1/2}$
 $\Rightarrow \alpha$ increases to 1: $\alpha \rightarrow 1 \Rightarrow$ Strong turbulence
- ② Strong (Critically Balanced) Driving, $\alpha \sim 1$: GS95 $k_{\parallel} \propto k_{\perp}^{2/3}$ $q = +2/3$
 B06 $k_{\parallel} \propto k_{\perp}^{1/2}$ $q = +1/2$

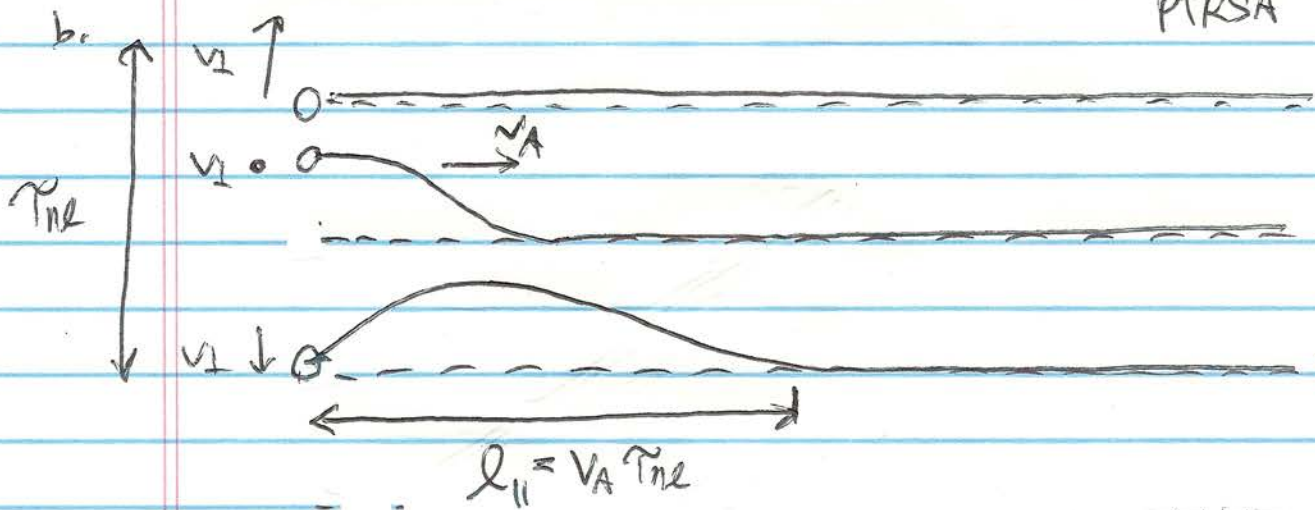
③ Over-Strong Driving $\alpha \gg 1$:

Perpendicular fluctuations ($k_{\perp} v_{\perp} \gg k_{\parallel} v_A$) will quickly become decorrelated along the magnetic field, rapidly increasing k_{\parallel} until $\alpha \rightarrow 1$.

\Rightarrow Back to strong, critically balanced turbulence.

5. Why? a. Shake a magnetic field transversely with period $\tau_{ne} \sim \frac{1}{k_{\perp} v_{\perp}} \sim \frac{\ell_{\perp}}{v_{\perp}}$

Ref: Hawes (2015)
PTRSA



c. Now, $l_{\parallel} = v_A \left(\frac{\ell_{\perp}}{v_{\perp}} \right) \Rightarrow \frac{l_{\parallel} v_{\perp}}{\ell_{\perp} v_A} = 1 \Rightarrow \frac{k_{\perp} v_{\perp}}{k_{\parallel} v_A} = \alpha = 1$

d. If you shake the magnetic field more rapidly (higher v_{\perp}) it will simply generate smaller parallel scales.

$\Rightarrow l_{\parallel}$ decreases $\Rightarrow k_{\parallel}$ increases.

e. Thus uncorrelated driving motions ($\alpha \gg 1$) will rapidly decorrelate to reach $\alpha = \frac{k_{\perp} v_{\perp}}{k_{\parallel} v_A} = 1$ by increasing k_{\parallel} .

B. The MHD Approximation

1. MHD requires the following conditions for fluctuations with scale l and frequency ω

① Non-relativistic: $\frac{v_{ts}}{c} \ll 1$

② Strongly collisional: $\frac{\lambda_{mfp}}{l} \ll 1$

③ Magnetized, large scale motions: $\frac{P_i}{l} \ll 1$

④ Low-frequency: $\frac{\omega}{\Omega_i} \ll 1$

C. Nature of Driven Fluctuations

1. If the MHD approximation is satisfied, the driving fluctuations will be a mixture of:

Alfvén waves \Rightarrow incompressible wave

Slow magnetosonic waves } \Rightarrow compressible waves
Fast magnetosonic waves }

Entropy mode \Rightarrow Non-propagating, pressure balanced structures (PBS).

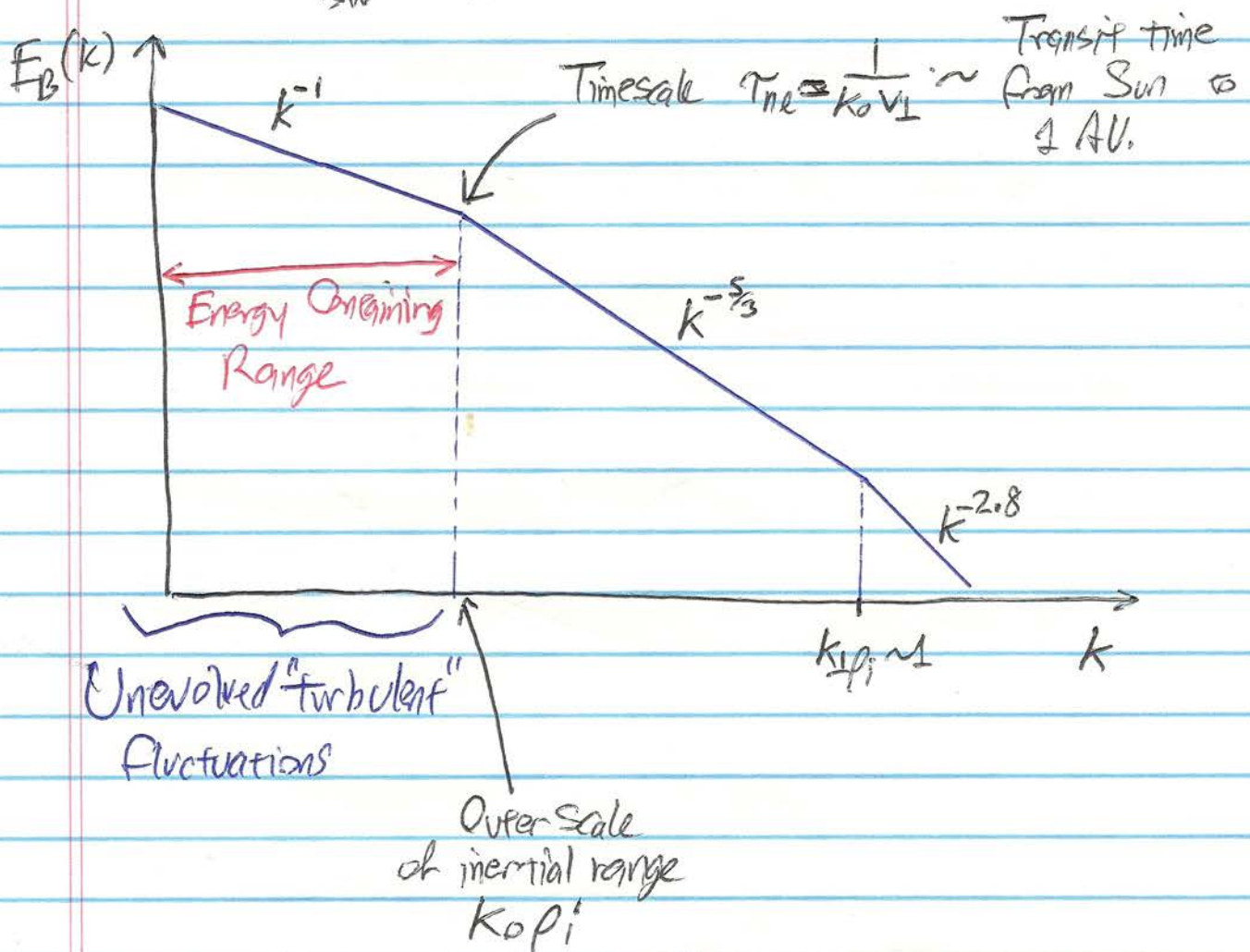
2. Resulting Turbulent Cascade will depend on mixture.

D. Energy Containing Range: The Solar Wind

Refs: Matthaeus (1994); Kiyani et al. (2015) PRSA

1. In the solar wind at 1 AU, the solar wind magnetic wavenumber spectrum (Using Taylor Hypothesis

$$k = \frac{2\pi f_{sc}}{V_{sw} \cos \theta_{kv}} \quad , \quad \text{see lecture \# 5, I.B.3}$$



a. At the largest scales, the fluctuations have not had enough time to interact nonlinearly (and thereby transfer their energy to smaller scales via a turbulent cascade) within their lifetime since leaving the sun \Rightarrow Unresolved turbulence.

b. Outer scale k_{0p1} is where fluctuations have had time to interact

2. Thus, solar wind turbulence is not being actively driven, but instead is a case of decaying turbulence.

b. However, the energy containing range is a reservoir of turbulent energy at large scales

c. As the solar ^{wind} flows from the Sun (and therefore continues to evolve), the fluctuations at the small scale (high k) end of the energy containing range have time to interact, and thus feed their energy in the inertial range turbulent cascade.

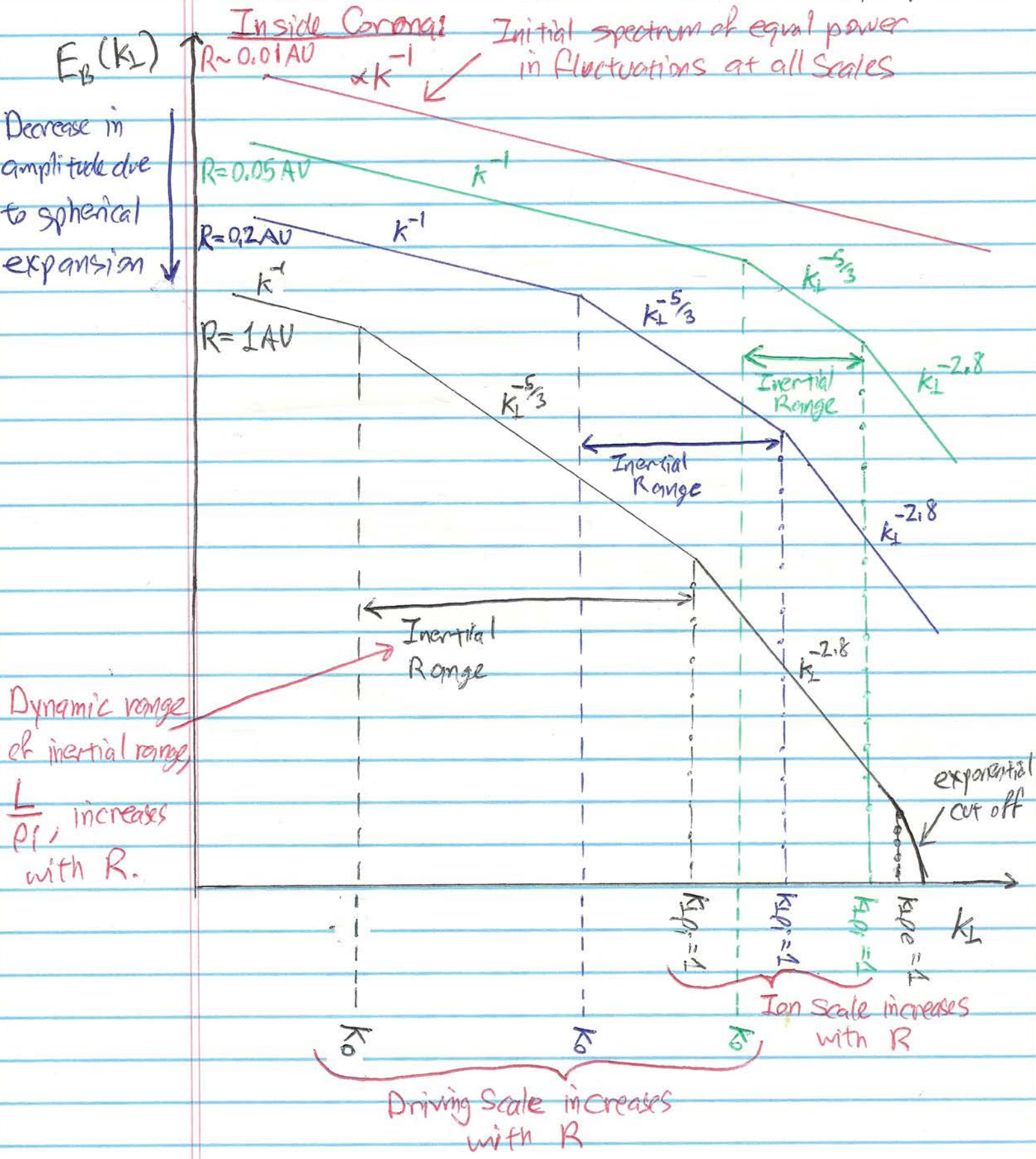
⇒ Thus k_{epi} decreases in time (or distance R from Sun).
 → (Roberts, D.A., ed. 1992)

3. Some studies claim solar wind turbulence is driven by Kelvin-Helmholtz instabilities at velocity shears, but the instability growth rates are much too slow at the large scales to drive the turbulence.

4. a. The fluctuations in the energy containing range should provide information about the large-scale conditions of the solar wind when it was launched from the Sun.

b. Fluctuations at wavenumbers above k_{epi} have been processed by strong turbulent interactions

E. Predicted Evolution of Solar Wind Magnetic Energy Spectrum



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