

Introduction to Shocks in Fluids:

I. The Basic Equations of Fluid Dynamics

A. The Chapman-Enskog Procedure

1. Beginning with the kinetic description of a neutral fluid given by the Boltzmann Equation, we may derive a hierarchy of moment equations.

2. To obtain a closed set of equations, the Chapman-Enskog procedure orders equations by the small parameter

$$\epsilon \equiv \frac{\lambda}{L} \ll 1$$

where λ = mean free path of particles

L = characteristic length scale (for gradients)

B. The Euler Equations: $\mathcal{O}(\epsilon)$

1. To lowest order, $\mathcal{O}(1)$, we obtain the Euler Equations

① a. Continuity Eq: $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0$

② b. Force Eq: $\rho \left[\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right] = -\nabla p$

③ c. Entropy Eq: $\rho T \frac{ds}{dt} = 0$ \leftarrow No heat flow in Lagrangian frame \rightarrow isentropic

where $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \underline{u} \cdot \nabla$ is the substantial (or Lagrangian) derivative

and $S \equiv c_v \ln \left(\frac{p}{\rho^\gamma} \right)$ is the specific entropy,

Here $c_v = \frac{3k}{2m}$ is the specific heat at constant volume

and $\gamma = \frac{5}{3}$ for a monatomic gas,

I.B. (Continued)

Haves ②

2. Ideal gas law: $p = nkT$

3. NTE: The entropy equation may also be written

$$\textcircled{4} \quad \frac{d}{dt} \left(\frac{p}{\rho \theta} \right) = 0$$

a. This form clearly shows that we have a closed set of equations for variables ρ , \underline{u} , and p .

C. Navier-Stokes Equations: $\mathcal{O}(\epsilon)$

$$\textcircled{5} \text{ a. Continuity Eq: } \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0$$

$$\textcircled{6} \text{ b. Force Equation: } \rho \left[\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right] = -\nabla p - \nabla \cdot \underline{\underline{\pi}}$$

$$\textcircled{7} \text{ c. Energy Equation: } \rho T \frac{ds}{dt} = -\nabla \cdot \underline{\underline{F}}_h + \underline{\underline{\pi}} : (\nabla \underline{u})$$

where: $\underline{\underline{F}}_h = -\kappa \nabla T$ is conductive heat flux

κ is coefficient of thermal conductivity

$\underline{\underline{\pi}}$ is viscous stress tensor

2. Viscosity

a. To $\mathcal{O}(\epsilon)$ [linear in $\frac{\lambda}{L}$], viscosity is expressed by

$$\text{Hooke's Law } \underline{\underline{\pi}} = \mu \underline{\underline{D}} \quad (\text{stress} \propto \text{strain})$$

$$\text{b. Here: } \underline{\underline{D}}_{ik} \equiv \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3} (\nabla \cdot \underline{u}) \delta_{ik}$$

This is a traceless, symmetric tensor for the rate of strain.

a. μ is the coefficient of shear viscosity.

I.C. (Continued)

Hawes ③

3. Ordering of viscous to pressure term in Force eq.

a. Take $\pi \sim \mu D$ where $D \sim \frac{\partial v}{\partial x} \sim \frac{v_f}{L}$
and $\mu \sim n m v_f \lambda$

b. Use $p \sim n k T$ and note that $v_f^2 \equiv \frac{2kT}{m}$

c. Thus

$$\frac{\mathcal{O}(-\nabla \cdot \pi)}{\mathcal{O}(-\nabla p)} \sim \frac{\left(\frac{\pi}{L}\right)}{\left(\frac{p}{L}\right)} \sim \frac{\mu D}{p} \sim \frac{(n m v_f \lambda) \frac{v_f}{L}}{n k T} \sim \frac{(m) v_f^2 \lambda}{k T} \sim \frac{\lambda}{L} \sim \frac{\lambda}{L}$$

$= v_f^2$

d. Thus, viscosity becomes important when gradient scale $L \sim \lambda$!
We'll see that viscosity plays an important role in fluid shocks!

II. Acoustic Waves in Fluids

A. Linear Acoustic Waves

1. Small amplitude acoustic waves are isentropic, $\frac{ds}{dt} = 0$

a. $\Rightarrow \frac{d}{dt} \left(\frac{p}{\rho^\gamma} \right) = 0 \Rightarrow p \propto \rho^\gamma$

b. Thus, we can write the pressure gradient in the Euler force eq. ②
in terms of density:

$$\nabla p = \left(\frac{\partial p}{\partial \rho} \right)_s \nabla \rho$$

c. Define: Adiabatic Sound Speed: $c^2 \equiv \left(\frac{\partial p}{\partial \rho} \right)_s = \frac{\partial p}{\partial \rho}$

③ $\Rightarrow \nabla p = c^2 \nabla \rho$

2. For simplicity, we'll take systems that vary only in one dimension throughout this lecture. $\nabla \Rightarrow \frac{\partial}{\partial x}$.

b. For linear waves, let's look at the Euler Equations!

II. A. (Continued)

Hawes (4)

3. We may linearize ~~the system of~~ ^{the Euler} equations as usual, taking $\rho = \rho_0 + \rho_1$, $u = u_1$, $p = p_0 + p_1$ and using $c_0^2 = \frac{\gamma p_0}{\rho_0}$ to find:

$$a. \quad \frac{\partial^2 \rho_1}{\partial t^2} - c_0^2 \frac{\partial^2 \rho_1}{\partial x^2} = 0$$

b. This homogeneous wave equation has the general solution

$$\rho_1(x, t) = f(x - c_0 t) + g(x + c_0 t)$$

4. Wave forms of linearized system do not evolve!

a. Disturbance maintains its original waveforms

$f(x)$ traveling in $+\hat{x}$ direction

$g(x)$ traveling in $-\hat{x}$ direction.

For all time, \Rightarrow This is a consequence of the linearity.

b. But, the fluid dynamic equations are fundamentally nonlinear.

\Rightarrow Finite amplitude acoustic waves steepen (as we shall see)

c. In contrast, Maxwell's equations in vacuum are linear.

~~4.~~

5. Nonlinear Waves:

a. If viscosity is neglected, waves of any amplitude will eventually steepen.

b. One exception are dispersive waves. If wave steepening is balanced by the dispersive tendency to spread out, one can find finite amplitude, nonlinear solutions that maintain their waveforms while propagating \Rightarrow Solitons.

II. (Continued)

Hawes (5)

B. Nonlinear Waves and Riemann Invariants

1. The Euler Equations can be expressed (Using (5) to replace ∇p)

$$\left. \begin{aligned} \frac{\partial p}{\partial t} + U \frac{\partial p}{\partial x} + \rho \frac{\partial u}{\partial x} &= 0 \\ \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + \frac{c^2}{\rho} \frac{\partial \rho}{\partial x} &= 0 \end{aligned} \right\} \text{1-D, Nonlinear}$$

2. These equations can be put into a nicely symmetric form by eliminating ρ in favor of c as ~~the~~ a variable:

a. The entropy equation gives $p \propto \rho^\gamma$. Using $c^2 = \frac{\partial p}{\partial \rho}$, we can obtain

$$c = c_0 \left(\frac{\rho}{\rho_0} \right)^{\frac{\gamma-1}{2}}$$

b. Differentiating, we find $\frac{dp}{\rho} = \frac{2}{\gamma-1} \frac{dc}{c}$

c. Substituting for dp everywhere yields

$$(9) \quad \frac{\partial}{\partial t} \left(\frac{2}{\gamma-1} c \right) + U \frac{\partial}{\partial x} \left(\frac{2}{\gamma-1} c \right) + c \frac{\partial u}{\partial x} = 0$$

$$(10) \quad \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + c \frac{\partial}{\partial x} \left(\frac{2}{\gamma-1} c \right) = 0$$

3. Adding & Subtracting (9) & (10) yields:

Lagrangian derivative moving at $u+c$

$$\left[\frac{\partial}{\partial t} + (U+c) \frac{\partial}{\partial x} \right] \left(U + \frac{2}{\gamma-1} c \right) = 0$$

$$\left[\frac{\partial}{\partial t} + (U-c) \frac{\partial}{\partial x} \right] \left(U - \frac{2}{\gamma-1} c \right) = 0$$

Lagrangian derivative moving at $U-c$

4. Riemann Invariants: $J_+ \equiv U + \frac{2}{\gamma-1} c$

$$J_- \equiv U - \frac{2}{\gamma-1} c$$

II. B. (Continued)

Hawes 6

5. Method of Characteristics:

a. Solutions to Nonlinear Euler Equations can be found:

$$J_+ = \text{constant} \quad \text{on plus characteristic} \quad \frac{dx}{dt} = U + C$$

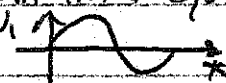
$$J_- = \text{constant} \quad \text{on minus characteristic} \quad \frac{dx}{dt} = U - C$$

C. Wave Steepening

1. Define: Simple Wave:

Solution for which one Riemann invariant, say J_+ , is strictly constant for all (x, t) , while J_- is a different constant on different characteristics.

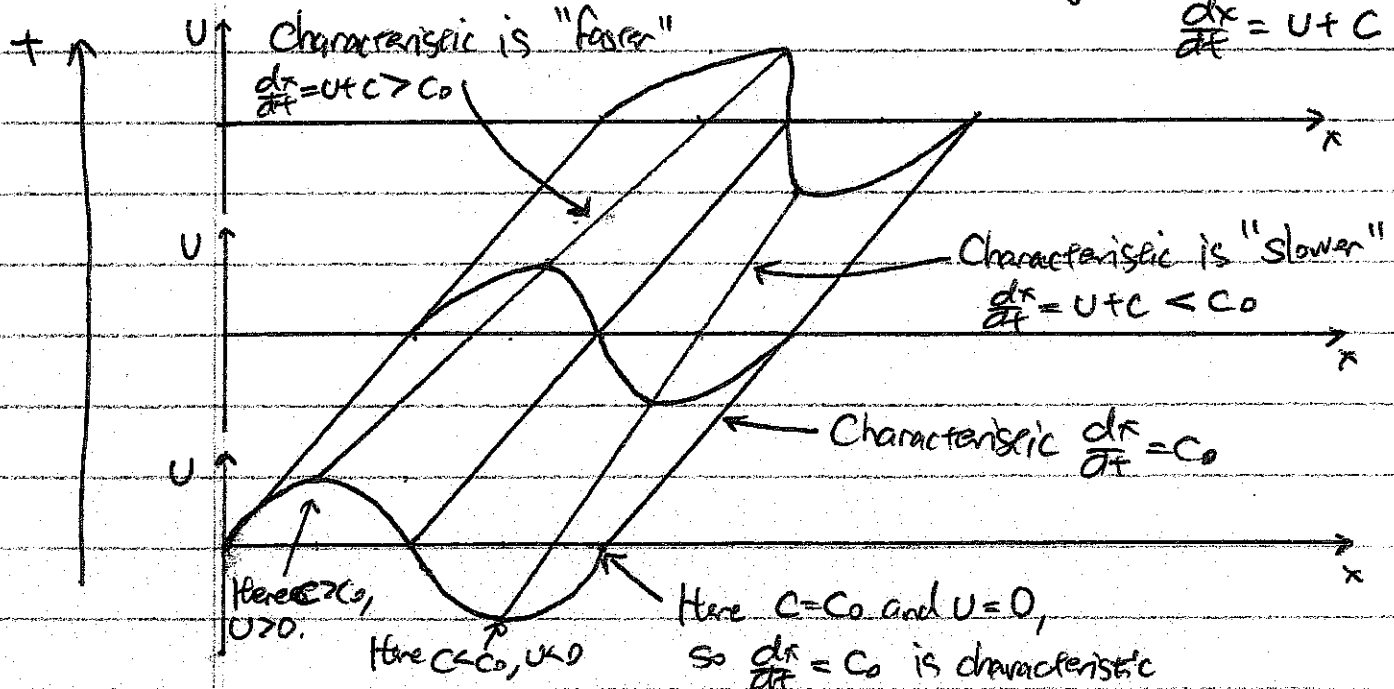
2. Consider initial conditions from linear equations for the "plus" wave:

$$\frac{p_1}{p_0} = \frac{u_1}{c_0} \Rightarrow u_1 \uparrow$$


b. Construct Riemann Invariant at $t=0$: $J_+(x(t), t) = U(x(t), t) + \frac{2}{\gamma-1} C(x(t), t)$

c. For all time $t > 0$, $J_+(x(t), t) = J_+(x(0), 0)$ along $x(t)$ given by characteristic

$$\frac{dx}{dt} = U + C$$



II, C. (Continued)

Hawes 7

3. Solution using Riemann invariant shows wave steepening

a. Crest of wave moves on "faster" characteristic

⇒ Catches up with trough on "slower" characteristic.

b. Eventually, the characteristics will cross, leading to a Riemann invariant solution:



c. Fluid attempts to take multiple values at a single point in space

⇒ UNPHYSICAL!

d. So what went wrong?

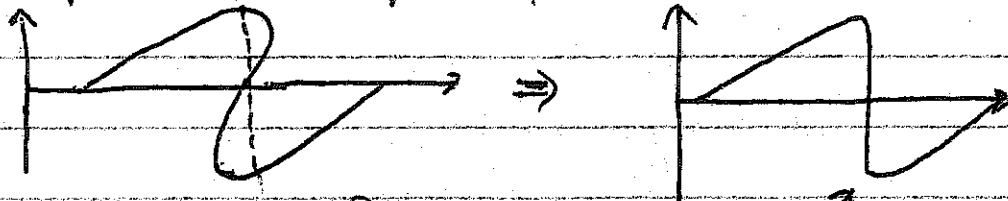
4. Failure of Inviscid Treatment by Euler Equations

a. As the wave steepens, the approximation $\lambda \ll L$ fails.

⇒ Viscosity can no longer be neglected!

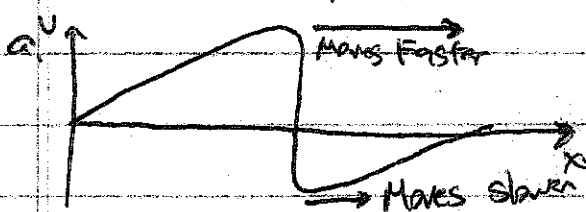
b. Viscosity halts the steepening predicted by the characteristics.

c. The profile cannot steepen beyond a discontinuous jump.

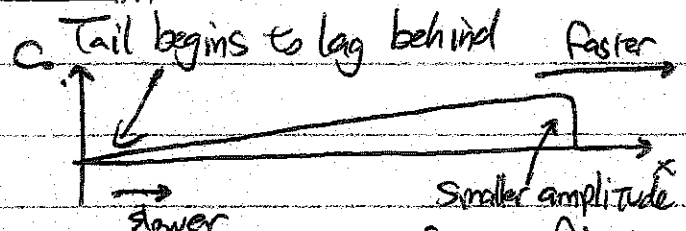
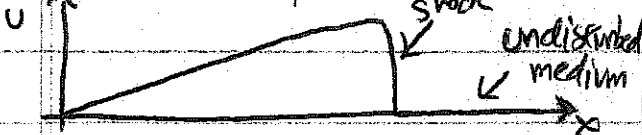


We have formed a shock!

5. Qualitative picture of shock formation and evolution



b. Crest eventually overruns trough



Shock spreads over more & more fluid, weakening the shock

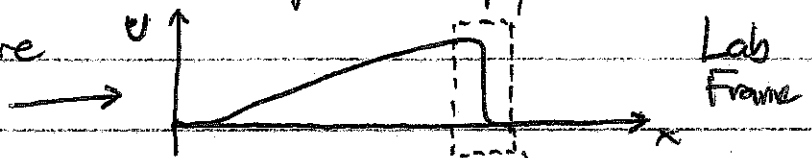
d. Shock dissipates, becoming a small amplitude acoustic wave.

III. Shocks

A. The Structure of Viscous Shocks

1. To study the fluid dynamics as the nonlinear wave steepens to form a shock, we must abandon the inviscid Euler description and employ Navier-Stokes eq's.

2. Let's consider the structure near the shock front

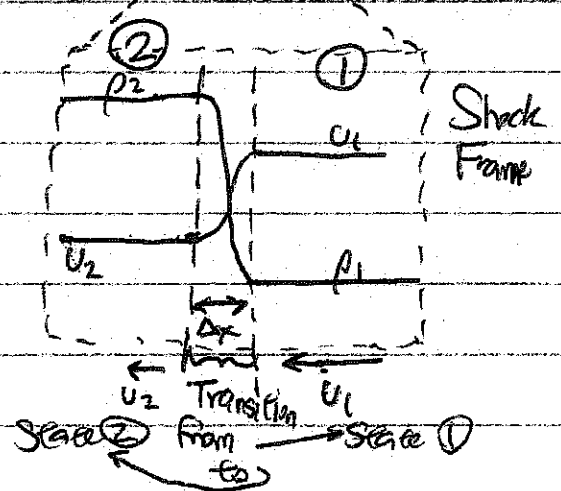


a. Consider upstream & downstream regions constant

b. Transform to frame of reference moving with the shock, the "Shock" frame.

c. The shock represents a transition layer from State ① to State ②

d. Let's estimate the thickness of the shock transition, Δx .



3. Conservation Form of 1-D Navier Stokes Equations

a. Mass: $\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$

b. Momentum: $\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p - \frac{4}{3} \mu \frac{\partial u}{\partial x})}{\partial x} = 0$

c. Energy: $\frac{\partial (\underbrace{\frac{\rho u^2}{2}}_{\text{Kinetic Energy Density}} + \underbrace{\frac{p}{\gamma-1}}_{\text{Thermal Energy Density}})}{\partial t} + \frac{\partial (\underbrace{\frac{\rho u^3}{2}}_{\text{K.E. Flux}} + \underbrace{\frac{\delta p}{\gamma-1} u}_{\text{Enthalpy Flux}} - \frac{4}{3} \mu \frac{\partial u}{\partial x} u - \kappa \frac{\partial T}{\partial x})}{\partial x} = 0$

where we have taken $\pi_{xx} = \frac{4}{3} \mu \frac{\partial u}{\partial x}$

and Define: Specific enthalpy $h \equiv \frac{\delta p}{(\gamma-1)\rho}$

III.A. (Continued)

Howes 9

4. The Steady State solution in the Shock frame has $\frac{\partial}{\partial t} = 0$.

a. Thus, as we pass from State ① into transition layer,

$$\rho U^2 + p + \underbrace{\frac{4}{3} \mu \frac{\partial U}{\partial x}}_{\text{viscosity is negligible outside of transition layer}} = \text{constant}$$

viscosity is negligible outside of transition layer

b. Inside the shock transition, viscous term must balance others:

$$\frac{4}{3} \mu \frac{\partial U}{\partial x} \sim \rho U^2$$

We can use this to estimate the shock thickness, Δx

c. DEFIN: Kinematic viscosity $\nu \equiv \frac{\mu}{\rho}$

where, for a neutral gas, $\nu \sim \lambda v_T$

d. Strong shock has $\Delta U \sim U$, we estimate $U \sim v_T$.

e. Thus, $\frac{4}{3} \mu \frac{\partial U}{\partial x} \sim \rho U^2 \Rightarrow \frac{4}{3} (\rho \nu) \frac{\Delta U}{\Delta x} \sim \rho U^2$

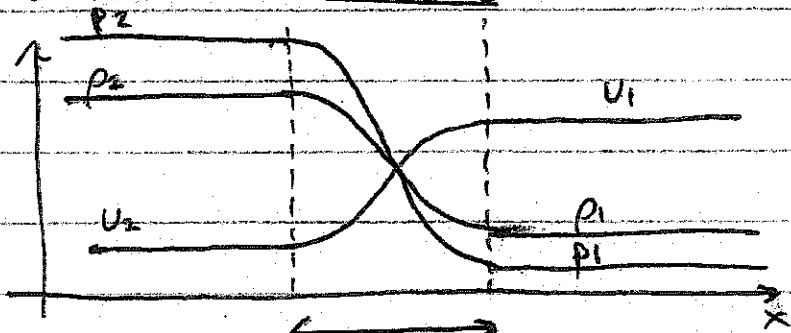
$$\Rightarrow \rho (\lambda v_T) \frac{v_T}{\Delta x} \sim \rho v_T^2 \Rightarrow \boxed{\Delta x \sim \lambda}$$

The shock thickness is of order of the mean free path λ

\Rightarrow We must include terms $O(\frac{\lambda}{L}) \Rightarrow$ Navier Stokes.

5. Viscous Shock Structure

Shock Frame



State ②
viscosity negligible
(Euler)

$\Delta x \sim \lambda$
viscosity
Required
(Navier-
Stokes)

State ①
viscosity negligible
(Euler)

II. A. (Continued)

Haves 10

6. Validity of Fluid Treatment

a. Chapman-Enskog procedure assumes Local Thermodynamic Equilibrium (LTE) and that $\epsilon = \frac{\lambda}{L}$ is a small parameter.

b. Thus, internal shock structure with $\frac{\lambda}{L} \sim 1$ requires kinetic, Boltzmann treatment

c. It turns out that the Navier-Stokes fluid treatment yields a reasonable approximation to the kinetic physics.

⇒ For a strong shock, transition layer is a few mean free paths

⇒ This is all the kinetics needs to transform from one state of LTE to another.

B. Rankine-Hugoniot Shock Jump Conditions

1. On macroscopic scales $L \gg \lambda$, one may regard the shock transition as a single, discontinuous jump.

2. The Euler Equation description, valid outside the transition layer, must still satisfy mass, momentum, & energy conservation from state ① to state ②, independent of the viscous physics in the layer.

3. Rankine-Hugoniot Jump Conditions

a. Mass: $\rho_1 U_1 = \rho_2 U_2$

b. Momentum: $\rho_1 U_1^2 + P_1 = \rho_2 U_2^2 + P_2$

c. Energy: $\rho_1 U_1 \left[\frac{U_1^2}{2} + \frac{\gamma P_1}{(\gamma-1)\rho_1} \right] = \rho_2 U_2 \left[\frac{U_2^2}{2} + \frac{\gamma P_2}{(\gamma-1)\rho_2} \right] \Rightarrow \frac{U_1^2}{2} + h_1 = \frac{U_2^2}{2} + h_2$

Using ①

Specific enthalpy $h = \frac{\gamma P}{(\gamma-1)\rho}$

4. DEFINE: Upstream Mach Number,

Key Shock Parameter $\rightarrow M_1 = \frac{U_1}{c_1}$ where $c_1^2 = \frac{\gamma P_1}{\rho_1}$

III. B. (Continued)

Hawes (11)

5. Given the values of the upstream state p_1, ρ_1 , & T_1 along with the upstream Mach number M_1 , we can solve for ρ_2, p_2 , & T_2

$$a. \frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2}{(\gamma+1) + (\gamma-1)(M_1^2-1)} = \frac{U_1}{U_2}$$

$$b. \frac{p_2}{p_1} = \frac{(\gamma+1) + 2\gamma(M_1^2-1)}{(\gamma+1)}$$

$$c. \frac{T_2}{T_1} = \frac{[(\gamma+1) + 2\gamma(M_1^2-1)][(\gamma+1) + (\gamma-1)(M_1^2-1)]}{(\gamma+1)^2 M_1^2} \quad (\text{using } p = nkT)$$

C. Properties of Fluid Shocks

1. For $M_1 > 1$, $p_2 > p_1$
 $\rho_2 > \rho_1$ but $U_2 < U_1$
 $T_2 > T_1$

2. Supersonic to Subsonic Transition:

a. It may be proven (see HW) that $M_2 < 1$ when $M_1 > 1$.
(subsonic) (supersonic)

3. Strong Shocks when $M_1 \gg 1$

$$a. \frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1} = 4 \quad (\text{for } \gamma = \frac{5}{3}) \Rightarrow \text{Density jump is capped.}$$

$$b. \frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} M_1^2 = \frac{5}{4} M_1^2 \Rightarrow \text{Pressure \& Temperature jumps are unbounded.}$$

$$c. \frac{T_2}{T_1} = \frac{2\gamma(\gamma+1)}{(\gamma+1)^2} M_1^2 = \frac{5}{16} M_1^2$$

4. Entropy: a Rankine-Hugoniot Conditions allow solutions with $M_1 < 1$, $M_2 > 1$.

\Rightarrow This would be a "rarefaction shock"

b. But, the Second Law of Thermodynamics allows only entropy $\Delta S \geq 0$.

where DEFINE: specific entropy $S \equiv c_v \ln\left(\frac{p}{\rho^\gamma}\right)$

~~where~~

c. We can estimate the change in a related quantity $S' \equiv \frac{p}{\rho^\gamma}$.

III C.4 (Continued)

Hawes (12)

$$\frac{d\Delta S'}{S_1} = \frac{S_2 - S_1}{S_1} = \frac{\left(\frac{p_2}{\rho_2^\gamma} - \frac{p_1}{\rho_1^\gamma}\right)}{S_1} = \frac{p_2}{p_1} \left(\frac{\rho_1}{\rho_2}\right)^\gamma - 1$$

It can be shown that $\frac{\Delta S'}{S_1} > 0$ for $M_1 > 1$ (Entropy increase)
 but $\frac{\Delta S'}{S_1} < 0$ for $M_1 < 1$ (Entropy decrease)

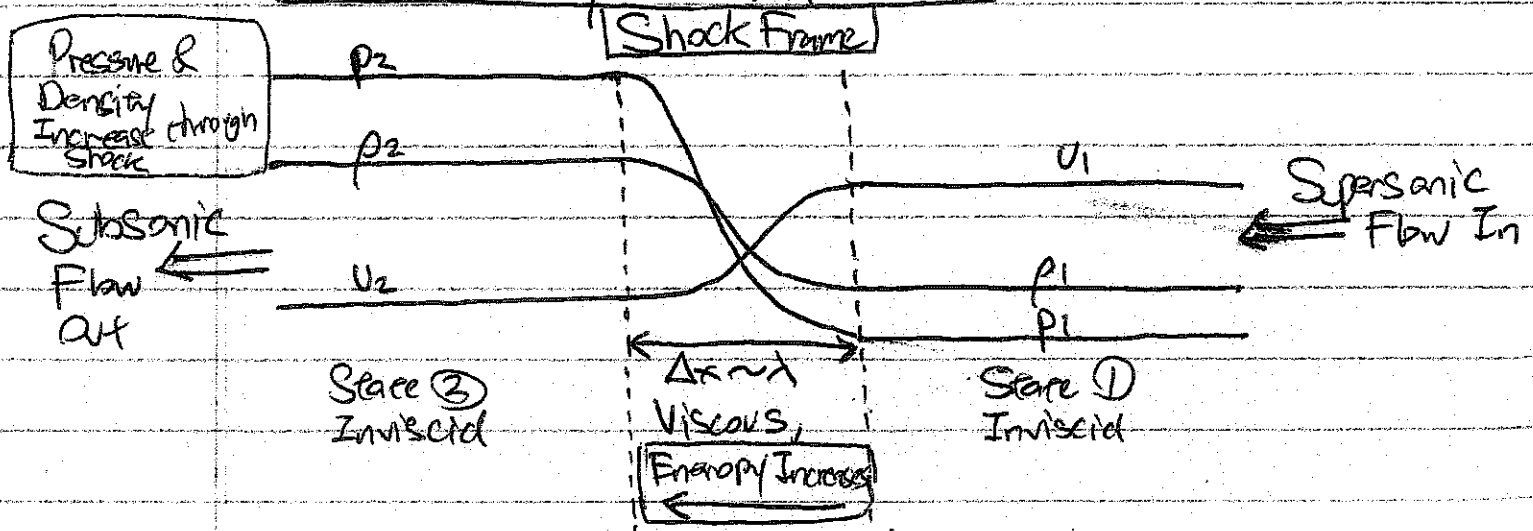
⇒ Only compressive shocks can arise in nature!

e. Viscosity can turn bulk motion into heat, but not the reverse

5. Final Note about Dissipative Mechanisms in Shocks

- Although viscosity plays a fundamental role in fluid shocks, the end state (2) in no way depends on the detailed dynamics in the shock transition.
- Physically, viscosity balances the tendency for the wave to steepen.
- Dynamically, the thickness of the shock Δx adjusts to enable the viscous force $-\frac{4}{3} \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial x} \right)$ to balance the steepening tendency.
- The end state is completely determined by mass, momentum, & energy conservation.
- ARTIFICIAL VISCOSITY: This is why any conservative scheme will suffice to capture shocks numerically. ⇒ typically spread shock out over a few grid points.

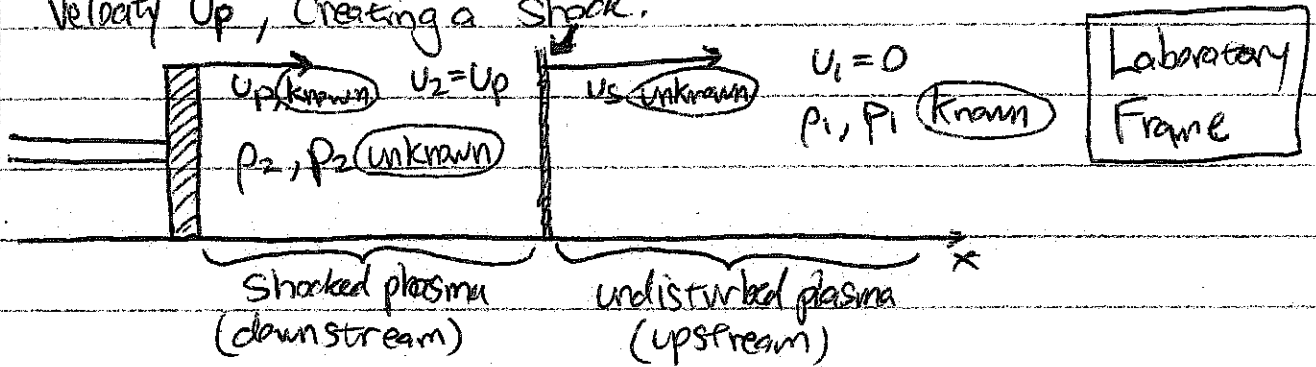
D. General Summary of Fluid Shocks



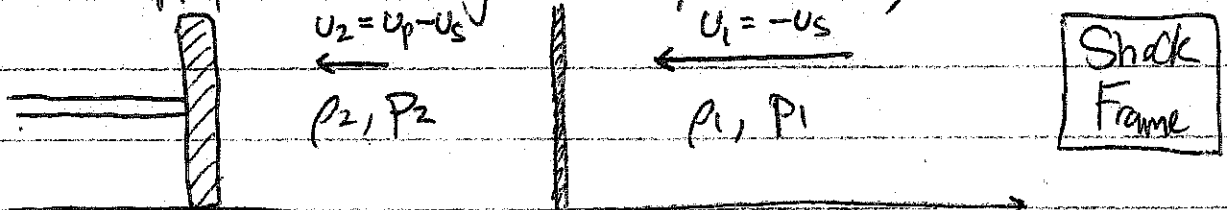
IV. Applications of Fluid Shocks

A. 1-D Piston Problem

1. Into a uniform plasma at rest, a piston is driven with velocity U_p , creating a shock.



2. To apply Rankine-Hugoniot Jump Conditions, transform to Shock Frame



$$\begin{aligned}
 \textcircled{a} \quad & \rho_1 U_1 = \rho_2 U_2 \\
 \textcircled{b} \quad & \rho_1 U_1^2 + P_1 = \rho_2 U_2^2 + P_2 \\
 \textcircled{c} \quad & \frac{U_2^2}{2} + \frac{\delta P_1}{\delta + 1 \rho_1} = \frac{U_2^2}{2} + \frac{\delta P_2}{\delta + 1 \rho_2}
 \end{aligned}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{3 equations for 3 unknowns } U_s, \rho_2, P_2 \\ U_2 = U_p - U_s \quad U_1 = -U_s \\ U_p, \rho_1, P_1 \text{ are given} \end{array}$$

3. Solve for U_2 in terms of P_1, P_2 , & U_1 above:

a. Eliminate ρ_2 from (b) using (a) $\Rightarrow P_2 = \rho_1 U_1 (U_1 - U_2) + P_1$ (d)

b. Eliminate ρ_2 & P_2 from (c) using (a) & (d)

$$\left[\frac{\delta-1}{2} \rho_1 U_1 (U_1 + U_2) + \delta P_1 - \delta \rho_1 U_1 U_2 \right] (U_1 - U_2) = 0$$

4. Solutions: a. Solution (1): $U_1 = U_2 \Rightarrow$ Trivial case of continuity, no shock

b. Solution (2): Using $C_1^2 = \frac{\delta P_1}{\rho_1}$, we solve for U_2

$$U_2 = \left(\frac{\delta-1}{\delta+1} \right) U_1 + \frac{2}{\delta+1} \frac{C_1^2}{U_1}$$

IV. A. (Continued)

Hawes (14)

5. Transforming solution (2) back to laboratory frame (U_s, u_p).

a.
$$u_p = \left(\frac{2}{\gamma+1}\right) \left(u_s - \frac{c_1^2}{u_s}\right)$$

b. In terms of Mach number in undisturbed gas, $M_p = \frac{2}{\gamma+1} \left(M_s - \frac{1}{M_s}\right)$

Thus, $M_p > 0$ for $M_s > 1$

c. Solving for u_s in terms of u_p .

$$u_s = \left(\frac{\gamma+1}{4}\right) u_p \pm \sqrt{\frac{(\gamma+1)^2}{16} u_p^2 + c_1^2} \xrightarrow{\gamma = \frac{5}{3}} \boxed{u_s = \frac{2}{3} u_p \pm \sqrt{\frac{4}{9} u_p^2 + c_1^2}}$$

6. Limits of Solution:

a. Weak Shock: For $u_p^2 \ll c_1^2$, $\boxed{u_s \approx c_1 + \frac{2}{3} u_p \approx c_1}$

This is essentially the limit of linear sound waves. Example: Stereo Speakers.

b. Strong Shock: For $u_p^2 \gg c_1^2$, $\boxed{u_s \approx \frac{4}{3} u_p}$

Shock travels into fluid 33% faster than piston.

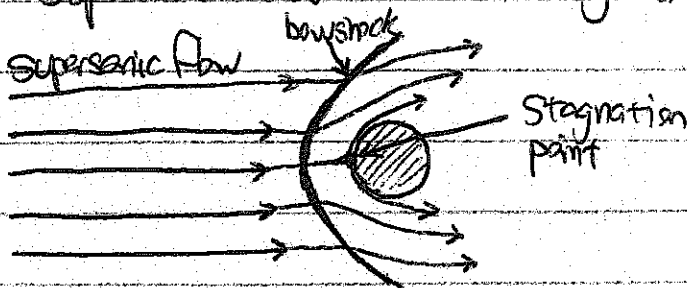
7. Entropy Considerations:

a. For $M_p > 0$, entropy increases from state (1) to state (2).

b. In shock frame, supersonic flow ($M_1 = \frac{u_1}{c_1} > 1$) is converted to subsonic flow ($M_2 = \frac{u_2}{c_2} < 1$).

B. Planetary Bow Shock

1. ~~Remember~~ This property of shocks, to convert a supersonic flow to a subsonic flow, is important in the interaction of supersonic flow with an object.



a. Bow shock reduces supersonic to subsonic flow.

b. Pressure at stagnation point is raised sufficiently to deflect flow around object.

2a. For planets such as the Earth, the interaction with the supersonic solar wind roughly obeys this principle.

b. However, the solar wind is magnetized and the earth is protected by its magnetosphere. Thus, the situation is more complicated and requires the study of MHD Shocks, our next topic.

V. References:

1. Good Introductory Reference of Astrophysical Fluid Dynamics:
Frank H. Shu, 1992, The Physics of Astrophysics Vol II: Gas Dynamics, University Science Books, Sausalito, CA.
2. Good Introductory Reference on Fluid & MHD Shocks:
Russell M. Kulsrud, 2005, Plasma Physics for Astrophysics, Princeton Univ Press, Princeton.
3. Classic Fluid Shocks Text:
Courant and Friedrichs, 1949, Supersonic Flow and Shock Waves, Interscience, NY.
4. Excellent Introduction to Shocks in Space Physics:
W. Baumjohann & R.A. Treumann, ¹⁹⁹⁷ Basic Space Plasma Physics, Imperial College Press, London.
5. Other Classic References:
Fluid → a. Landau & Lifshitz 1959, Fluid Mechanics, Pergamon Press, Oxford.
MHD → b. Kontrovich & Petschek, 1966, in Plasma Physics in Theory & Application, ed. W.B. Kunkel, McGraw Hill, NY.