# 29:194 Plasma Physics I: Lecture #1 - Introduction to Plasmas
## What? How? Why?

I. What is a plasma?

A. Definition: There is no good, single definition!

1. Fourth State of Matter: Solid ⇒ Liquid ⇒ Gas ⇒ Plasma
   Ice Water Steam
2. A gas that is significantly ionized.
3. Ionized gas that display collective behavior

B. Variety:

1. Plasma physics concepts are used to describe an enormous variety of topics — not just "traditional" plasmas

Ex: Electrons in a semi-conductor in solid state physics

C. Common:

1. Most of the matter in the universe exists in the plasma state, except "dark matter"
2. Some say 95-99% of baryonic matter is plasma

D. Applications:

1. Unfortunately, many plasma courses tend to be a "grab bag" of seemingly unrelated topics
2. GOAL: I hope to give you a sense of the overall framework of plasma physics in this course <u>and</u> how different approaches are connected.
3. I will not always be formal — formality is often a barrier to understanding
4. There will be a heavy emphasis on skills — both mathematical & numerical

Lect #1 (Continued)

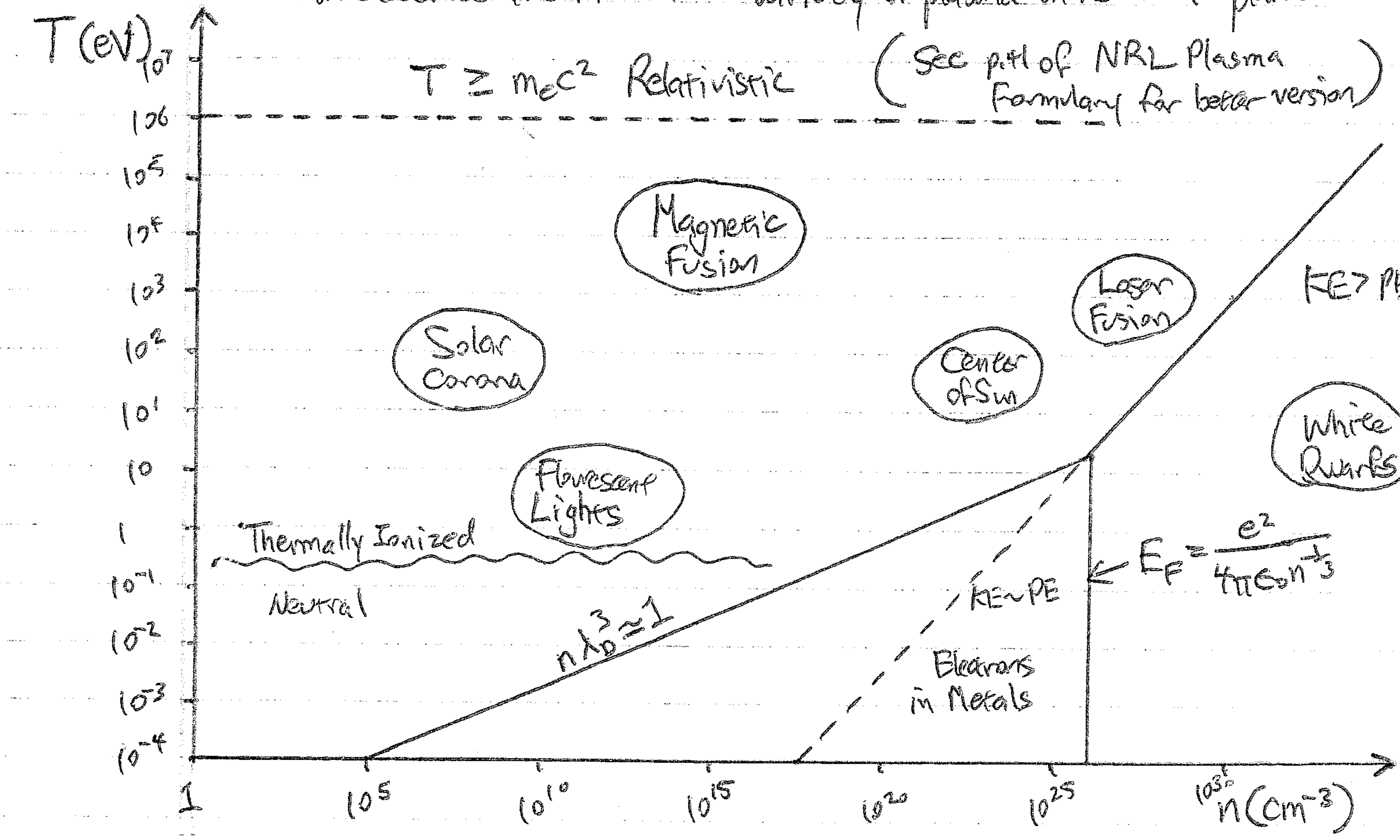
# II. How do we describe plasmas?

## A. Basic Parameters:

1. A plasma consists of one, or more, ion species and electrons.
2. Intensive variables:
   a. Density $n_s$
   b. Temperature $T_s$
   c. Magnetic Field $B_0$
3. Physical properties
   a. mass $m_s$
   b. charge $q_s$

**NOTE:** We often use $m_e/m_i \ll 1$ to simplify a problem.

4. Most plasmas have magnetic fields (although possibly no <u>mean</u> magnetic field) due to currents from charged particles moving within the plasma
   (Ampere's Law: $\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{j} + \epsilon_0 \mu_0 \frac{\partial \underset{\sim}{E}}{\partial t}$)
5. We can describe the incredible variety of plasma on the n–T plane.



Lect #1 (Continued)

II. (Continued)

B. Units:

1. Two major systems: a. SI Units (mks)
   b. Gaussian Units (cgs)
2. We'll be using SI to agree with text (Chen)

III. How do we study the physics of plasmas?

A. The Basic Picture:

1. Plasma physics is really nothing more than the study of fluctuating magnetic and electric fields and the motion of charged particles in those fields.

Particles: Ions & Electrons

position $\underset{\sim}{r}$, velocity $\underset{\sim}{v}$ $\Rightarrow$ $\rho$, $\underset{\sim}{j}$

Lorentz Force Law

$$\frac{d\underset{\sim}{p}}{dt} = q(\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B})$$

$\underset{\sim}{E}$ & $\underset{\sim}{B}$ $\Rightarrow$ $\rho$ & $\underset{\sim}{j}$

Maxwells Equations

$$\nabla \cdot \underset{\sim}{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \underset{\sim}{B} = \mu_0 \underset{\sim}{j} + \epsilon_0 \mu_0 \frac{\partial \underset{\sim}{E}}{\partial t}$$

$$\nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t}$$

$$\nabla \cdot \underset{\sim}{B} = 0$$

$\rho$ & $\underset{\sim}{j}$ $\Rightarrow$ $\underset{\sim}{E}$ & $\underset{\sim}{B}$

$\underset{\sim}{B}$ & $\underset{\sim}{E}$ fields

2. This coupling is precisely what presents the challenge of plasma physics!

B. Inconsistent Models: $\boxed{\text{Single Particle Motion}}$

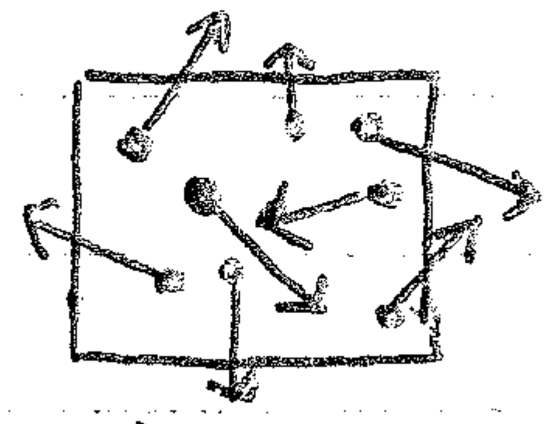
1. Assume we know $\underset{\sim}{E}$ & $\underset{\sim}{B}$ fields
2. Determine motion of a particle in these fields
3. Good approach to develop intuition
4. Does not account for the collective of other particles' motion (and the resulting $\underset{\sim}{E}$ & $\underset{\sim}{B}$ fields) on that single particle.

III. (Continued)

C. Consistent Models:

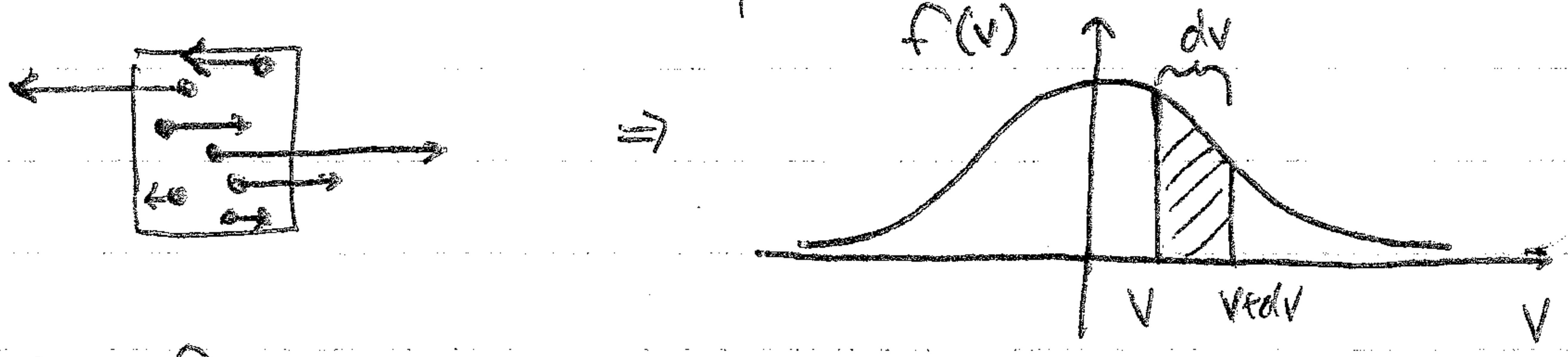
1. $\boxed{\text{Kinetic Theory!}}$

a. Statistical theory averages over the motion of many particles



$\Rightarrow$ Distribution function $f(\underset{\sim}{r}, \underset{\sim}{v}, t)$

Ex: For a 1-D system, at a single point in space, particle velocities may be distributed as follows:




Here $f(v)$ is number of particles with velocity between $v$ & $v+dv$.

We'll discuss this more later.

b. Assuming a Maxwellian distribution (thermal equilibrium) allows some closed form results to be derived.

c. Any more detailed treatment quickly becomes cumbersome, not possible in closed form.

2. $\boxed{\text{Fluid Theory}}$:

a. Assumes all particles at a given point move similarly

b. One may take moments of the distribution function:

i) Density $\quad n = \int d^3\underset{\sim}{v}\, f(\underset{\sim}{v})$

ii) Fluid velocity $\quad \underset{\sim}{U} = \dfrac{\int d^3\underset{\sim}{v}\, \underset{\sim}{v}\, f(\underset{\sim}{v})}{\int d^3\underset{\sim}{v}\, f(\underset{\sim}{v})}$

etc....

c. One must assume some closure to obtain a closed set of fluid equations.

III. C. 2.

d. $\boxed{\text{Two Fluid Theory}}$ allows for different behavior between ions and electrons

e. If you assume that ions and electrons move together, you get a single fluid theory,

$\boxed{\text{Magnetohydrodynamics}}$ or MHD.

i) This is the most simple, consistent description of a plasma.
ii) Probably the most widely used system, especially by "non-plasma physicists"

f. Even fluid theory is difficult in the complicated, nonlinear situations of most practical interest.

D. Computational Studies:
1. Numerical Solution: Solving a system of equations using the computer.
   Ex: a. Boundary value or eigenvalue problems.
   b. May use ODE integrators, shooting methods, etc.

2. Simulation: (Initial Value codes) Big business / big codes
   a. Particle-in-Cell (particle method)
   b. Fokker-Planck (Continuum method)
   c. Gyrokinetic
   d. Two Fluid / MHD

## IV. Why do we study plasma physics?

A. Plasma physics and the methods used in plasma physics contribute significantly to a wide variety of fields:

1. Stellar and Galactic Astrophysics
2. Earth's magnetosphere and the heliosphere
3. Fusion Energy research
    a. Magnetic confinement
    b. Inertial confinement (laser fusion)
4. Advanced Accelerators
5. Laser-plasma interactions
6. Industrial processing (especially computer chips)
7. Solid state, electrons in a metal

## V. The NRL Plasma Formulary

Meet your new best friend!

A. <u>Most Frequently Used Sections:</u>

1. Vector Identities p. 4-5
2. Differential Operators in Curvilinear Coordinates p. 6-9
3. Physical Constants (SI) p. 14-15
4. Fundamental Plasma Parameters p. 28-29

<u>NOTE:</u> I will refer to the Plasma Formulary in the lecture notes as NRL.

<u>Ex:</u> Use $\underset{\sim}{A}\times(\underset{\sim}{B}\times\underset{\sim}{C}) = \underset{\sim}{B}(\underset{\sim}{A}\cdot\underset{\sim}{C}) - \underset{\sim}{C}(\underset{\sim}{A}\cdot\underset{\sim}{B})$ [NRL, p. 4, (2)]

# VI. Vector Notation Review & Vector Calculus

A. Why? 1. Vector notation simplifies the mathematical notation

2. You will get lots of practice with vector algebra & calculus.

B. Notation:

1. Under-tilde denotes vector quantity $\underset{\sim}{B}$

a. In cartesian coordinates $\hat{\underset{\sim}{x}}$, $\hat{\underset{\sim}{y}}$, & $\hat{\underset{\sim}{z}}$,

$$\underset{\sim}{B} = B_x \hat{\underset{\sim}{x}} + B_y \hat{\underset{\sim}{y}} + B_z \hat{\underset{\sim}{z}}$$

2. Unit vectors: $\hat{\underset{\sim}{b}} \equiv \frac{\underset{\sim}{B}}{|\underset{\sim}{B}|}$

3. Magnitude: $|\underset{\sim}{B}| = \sqrt{\underset{\sim}{B} \cdot \underset{\sim}{B}} = \sqrt{B_x^2 + B_y^2 + B_z^2}$

4. Tensor: Denoted by double under-tilde $\underset{\approx}{\epsilon} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$

Vector Calculus:

5. a. $\frac{\partial f}{\partial \underset{\sim}{v}} \equiv \frac{\partial f}{\partial v_x} \hat{\underset{\sim}{x}} + \frac{\partial f}{\partial v_y} \hat{\underset{\sim}{y}} + \frac{\partial f}{\partial v_z} \hat{\underset{\sim}{z}}$

b. $\nabla f = \frac{\partial f}{\partial \underset{\sim}{x}} = \frac{\partial f}{\partial x} \hat{\underset{\sim}{x}} + \frac{\partial f}{\partial y} \hat{\underset{\sim}{y}} + \frac{\partial f}{\partial z} \hat{\underset{\sim}{z}}$

c. Thus $\nabla = \hat{\underset{\sim}{x}} \frac{\partial}{\partial x} + \hat{\underset{\sim}{y}} \frac{\partial}{\partial y} + \hat{\underset{\sim}{z}} \frac{\partial}{\partial z}$

## C. Vector Algebra and Calculus Review:

1. Dot Product: $\underset{\sim}{A} \cdot \underset{\sim}{B} = A_x B_x + A_y B_y + A_z B_z$

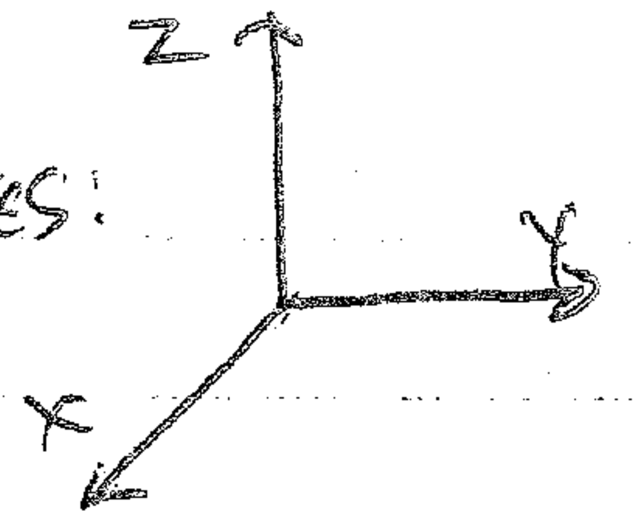
2. Cross Product: 
$$\underset{\sim}{A} \times \underset{\sim}{B} = (A_y B_z - A_z B_y) \hat{\underset{\sim}{x}} + (A_x B_z - A_z B_x) \hat{\underset{\sim}{y}} + (A_x B_y - A_y B_x) \hat{\underset{\sim}{z}}$$

$$\begin{vmatrix} \hat{\underset{\sim}{x}} & \hat{\underset{\sim}{y}} & \hat{\underset{\sim}{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Lecture #1 (Continued)

VI C. (Continued)

3. Right-handed coordinates:




$$\hat{x}\times\hat{y}=\hat{z}$$
$$\hat{y}\times\hat{z}=\hat{x}$$
$$\hat{z}\times\hat{x}=\hat{y}$$

4. Integration: $\int d^3v\, f(\mathbf{v}) \equiv \int dv_x \int dv_y \int dv_z\, f(\mathbf{v})$

5. $\mathbf{v}\cdot\nabla = (v_x\hat{x}+v_y\hat{y}+v_z\hat{z})\cdot\left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right) = v_x\frac{\partial}{\partial x}+v_y\frac{\partial}{\partial y}+v_z\frac{\partial}{\partial z}$

6. $\mathbf{v}\times\nabla = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\end{vmatrix} = \left(v_y\frac{\partial}{\partial z}-v_z\frac{\partial}{\partial y}\right)\hat{x}+\left(v_z\frac{\partial}{\partial x}-v_x\frac{\partial}{\partial z}\right)\hat{y}+\left(v_x\frac{\partial}{\partial y}-v_y\frac{\partial}{\partial x}\right)\hat{z}$

D. Examples:

1. MHD continuity: $\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0$

a. By NRL p. 4 (7), $\nabla\cdot(\rho\mathbf{v}) = \rho\nabla\cdot\mathbf{v}+(\mathbf{v}\cdot\nabla)\rho$, so

$$\boxed{\frac{\partial\rho}{\partial t}+(\mathbf{v}\cdot\nabla)\rho = -\rho\nabla\cdot\mathbf{v}}$$

2. MHD momentum: $\rho\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p+\mathbf{j}\times\mathbf{B}$

a. We also have $\mathbf{j}=\frac{1}{\mu_0}\nabla\times\mathbf{B}$ (Ampere's Law, displacement current dropped)

b. Rewrite $\mathbf{j}\times\mathbf{B}$ term in terms of only $\mathbf{B}$:

$$\mathbf{j}\times\mathbf{B} = \frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B} = -\frac{1}{\mu_0}\mathbf{B}\times(\nabla\times\mathbf{B})$$

From NRL p. 4 (12) with $\mathbf{B}=\mathbf{A} \Rightarrow \nabla(\mathbf{B}\cdot\mathbf{B}) = 2\,\mathbf{B}\times(\nabla\times\mathbf{B})+2(\mathbf{B}\cdot\nabla)\mathbf{B}$

So $\mathbf{B}\times(\nabla\times\mathbf{B}) = \frac{1}{2}\nabla(\mathbf{B}\cdot\mathbf{B})-(\mathbf{B}\cdot\nabla)\mathbf{B}$

Thus $\boxed{\mathbf{j}\times\mathbf{B} = -\frac{\nabla B^2}{2\mu_0}+\frac{(\mathbf{B}\cdot\nabla)\mathbf{B}}{\mu_0}}$