

29:194 Homework #8

Reading: Gurnett & Bhattacharjee, Sec 6.5 (p.195–204)

Due at the beginning of class, Thursday, November 4, 2010.

1. Fluid Electron Waves

Assuming that the ions are stationary in a homogeneous, unmagnetized plasma and that the electrons respond to an applied electrostatic wave of the form

$$\phi(x, t) = \phi_1 e^{i(kx - \omega t)},$$

use the electron equations from the Two Fluid Equations (neglecting the drag term)

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{U}_e) = 0$$

$$m_e n_e \left(\frac{\partial \mathbf{U}_e}{\partial t} + \mathbf{U}_e \cdot \nabla \mathbf{U}_e \right) = -\nabla p_e - e n_e (\mathbf{E} + \mathbf{U}_e \times \mathbf{B})$$

$$\frac{d}{dt} \left(\frac{p_e}{n_e^\gamma} \right) = 0$$

along with Poisson's Equation

$$\nabla \cdot \mathbf{E} = \frac{\sum_s n_s q_s}{\epsilon_0}$$

to derive the dispersion relation $\omega = \omega(k)$. Hint: Use $\mathbf{E} = -\nabla\phi$ and you may use the expression $p_e = n_e k T_e$.

- (a) Treating the perturbation as small, write down the linearized set of equations. Hint: Don't forget to use the property of quasineutrality.
- (b) Solve for the dispersion relation $\omega = \omega(k)$ in terms of the electron plasma frequency ω_{pe} and the electron thermal velocity v_{te} .
- (c) What is the appropriate adiabatic index γ for these waves if they are very fast (and thus adiabatic) and strictly one-dimensional?
- (d) What is the group velocity for these waves?

2. From the general form for linearized Ideal MHD,

$$\omega^2 \mathbf{U}_1 = (c_s^2 + v_A^2)(\mathbf{k} \cdot \mathbf{U}_1) \mathbf{k} - v_A^2 (\hat{\mathbf{b}} \cdot \mathbf{U}_1) (\hat{\mathbf{b}} \cdot \mathbf{k}) \mathbf{k} - v_A^2 (\hat{\mathbf{b}} \cdot \mathbf{k}) (\mathbf{k} \cdot \mathbf{U}_1) \hat{\mathbf{b}} + v_A^2 (\hat{\mathbf{b}} \cdot \mathbf{k})^2 \mathbf{U}_1,$$

solve for the MHD Dispersion Relation in the form $D(\omega, \mathbf{k}) = 0$ (this is the determinant set equal to zero).

3. One-Dimensional Solar Wind Model

Consider a simplified, steady-state (constant in time) model of the solar wind near the equatorial plane in which all quantities depend only on the spherical radius r . Assume the radial component of the solar wind velocity is a given function $v_r(r)$. Hint: Use spherical coordinates, and take $B_\theta = 0$.

- (a) Use the MHD continuity equation to derive the mass density ρ as a function of radius r in terms of the solar radius R_\odot , the density at the solar surface ρ_\odot , and the radial velocity at the solar surface $v_r(R_\odot)$.
- (b) Use divergence free condition on the magnetic field to determine the radial component of the magnetic field B_r as a function of radius r in terms of the solar radius R_\odot and the radial field at the solar surface $B_r(R_\odot)$. NOTE: This is an expression of the conservation of magnetic flux.
- (c) Use the azimuthal component of the Ideal MHD induction equation to derive an expression for B_ϕ as a function of radius r in terms of the angular rotational velocity at the solar surface Ω_\odot , the radial component of the magnetic field B_r , radial solar wind velocity v_r , and the azimuthal solar wind velocity v_ϕ . You may assume that the magnetic field at the solar surface is purely radial $B_\phi(R_\odot) = 0$. Hint: Use your results from part (b) to simplify the expression and express the v_ϕ at the solar surface as a function of Ω_\odot .