

29:194 Homework #7

Reading: Gurnett & Bhattacharjee, Sec 6.1–6.4 (p.175–194)

Supplemental Reading(optional): Chen, Sec 5.7 (p 184–186), Sec 4.1-4.3 (p.79–86)

Due at the beginning of class, Thursday, October 28, 2010.

1. Beginning with the Adiabatic Equation of State, derive the following alternative equation for the evolution of the pressure

$$\frac{dp}{dt} = -\gamma p \nabla \cdot \mathbf{U}$$

2. Magnetic Pressure vs. Magnetic Tension

- (a) For a magnetic field of the form $\mathbf{B} = B_0 y / L \hat{\mathbf{z}}$, calculate the force density due to the magnetic pressure term and magnetic tension term in the simplified form of the Momentum Equation.
- (b) Calculate the magnetic tension and magnetic pressure force densities for a wave-like magnetic field of form $\mathbf{B} = B_0 \hat{\mathbf{z}} + B_1 \sin(k_{\parallel} z) \hat{\mathbf{y}}$. You may take $B_1 \ll B_0$ and drop terms that are second-order in B_1 .

3. A dipole magnetic field has the form

$$\mathbf{B} = \frac{\mu_0 M}{4\pi} \frac{1}{r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

We can represent \mathbf{B} in Clebsch form $\mathbf{B} = \nabla \psi \times \nabla \phi$, where ϕ is the azimuthal coordinate and ψ is a scalar function $\psi(r, \theta)$.

- (a) Find $\psi(r, \theta)$.
- (b) Use this result to easily calculate the equation for a magnetic field line in the form $r = r(\theta)$.
HINTS: Choose an appropriate way to label the field line *and* look back at problem 3 on HW#3.

4. Consider the Clebsch representation of a magnetic field $\mathbf{B} = \nabla \alpha \times \nabla \beta$. Show that

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B})$$

is satisfied if the Clebsch variables satisfy

$$\frac{\partial \alpha}{\partial t} + \mathbf{U} \cdot \nabla \alpha = 0 \quad \text{and} \quad \frac{\partial \beta}{\partial t} + \mathbf{U} \cdot \nabla \beta = 0.$$

5. Star Formation

Suppose the interstellar medium has a number density of 10^6 m^{-3} and a straight, uniform magnetic field of magnitude $B = 3 \times 10^{-10} \text{ T}$.

- (a) Calculate the field strength in a star if the flux remains “frozen-in” and the star forms by a spherical collapse to the radius and mass of the sun.
- (b) Compare the force densities at the surface of the star due to the magnetic pressure and to gravity, where the Momentum Equation with the gravitational force added is

$$\rho \frac{\partial \mathbf{U}}{\partial t} + \rho \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla \left(p + \frac{B^2}{2\mu_0} \right) + \frac{(\mathbf{B} \cdot \nabla) \mathbf{B}}{\mu_0} - \rho \nabla \Phi_G$$

where the gravitational potential energy is $\Phi_G = -GM/r$ outside of a star of mass M .