29:194 Homework #6

Reading: Gurnett & Bhattacharjee, Sec 5.1–5.4 (p.137–155), Sec 5.6–5.7 (p.162–169)
Supplemental Reading (optional): Chen, Sec 5.6 (p.176–184), Sec 7.1–7.3 (p.225–240), Sec 3.1–3.3 (p.53–68)

Due at the beginning of class, Thursday, October 21, 2010.

1. How does the mean free path for electron-ion collisions $\lambda_{m(e-i)}$ depend on the electron temperature T_e?

2. Show, for a plasma species s with a Maxwellian velocity distribution
$$f_{s_m}(\mathbf{x}, \mathbf{v}, t) = \frac{n_s(\mathbf{x}, t)}{\pi^{3/2} v_{ts}^3} \exp\left[\frac{-m_s|\mathbf{v} - \mathbf{U}_s(\mathbf{x}, t)|^2}{2kT_s(\mathbf{x}, t)}\right]$$
that the first velocity moment gives the result $n_s(\mathbf{x}, t)\mathbf{U}_s(\mathbf{x}, t)$.

3. Show, for a plasma species s with a Maxwellian velocity distribution (same as above), that the second velocity moment
$$\int d^3\mathbf{v} \frac{1}{2} m_s v^2 f_{s_m}(\mathbf{x}, \mathbf{v}, t)$$
gives the result $\frac{3}{2} n_s k T_s + \frac{1}{2} n_s m_s |\mathbf{U}_s|^2$.

4. A particle species s has a distribution function of the form
$$f_s(\mathbf{x}, \mathbf{v}, t) = n_s \left(\frac{m_s}{2\pi k T_s}\right)^{3/2} \exp\left[\frac{-(\frac{1}{2} m_s v^2 + q_s\phi)}{kT_s}\right],$$
where $\phi = \phi(\mathbf{x})$ is a constant electrostatic potential, $\mathbf{E} = -\nabla\phi$, and density $n_s = n_{0s}$ and temperature $T_s = T_{0s}$ are both uniform in space and constant in time. Explicitly show that this distribution function satisfies the Vlasov equation with $\mathbf{B} = 0$,
$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \nabla f_s + \frac{q_s \mathbf{E}}{m_s} \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

5. Determining how the plasma parameters of a given system depend upon the independent variables is a powerful technique called scaling theory. Here we will construct a scaling theory for the near-earth solar wind, determining how the temperature varies with the heliocentric radius, $T \propto r^{\alpha}$, where we want to determine the exponent α.

   A simple steady-state model of the near-earth solar wind specifies a solar wind with a constant radial velocity $\mathbf{v} = v_0 \hat{\mathbf{r}}$ and a density that scales with radial distance from the sun as $n = n_0 (r_0/r)^2$ where $r_0 = 1$ AU is the heliocentric distance of the earth.

   (a) Assuming the solar wind is a fully ionized plasma of protons and electrons, but that the effects of the magnetic field are negligible, calculate the predicted scaling of the plasma temperature as a function of radius using the Adiabatic Equation of State. You may make use of the relation $p = nkT$.

   (b) The solar wind, however, is a weakly collisional magnetized plasma. Therefore, the temperatures parallel and perpendicular to the magnetic field will evolve independently for adiabatic changes. Near the earth, the interplanetary magnetic field forms the Parker spiral, with the radial and azimuthal components of the magnetic field related by

   $$\frac{B_\phi}{B_r} \simeq \frac{-r\Omega_0}{v_r},$$

   where $\Omega_0 = 2.85 \times 10^{-6}$ rad/s is the equatorial angular velocity of the solar surface and we assume $B_\theta = 0$. The conservation of radial magnetic flux in a spherically symmetric system implies that the radial magnetic field scales as $B_r = B_0 (r_0/r)^2$. Using the Double Adiabatic Equations of State, find the predicted scaling of the perpendicular and parallel temperatures $T_\perp$ and $T_\parallel$ with the radius *in the limit* that $r\Omega_0/v_0 \ll 1$. You may use the relations $p_\perp = nkT_\perp$ and $p_\parallel = nkT_\parallel$.

   (c) Find the scaling of $T_\perp$ and $T_\parallel$ with r in the opposite limit $r\Omega_0/v_0 \gg 1$.