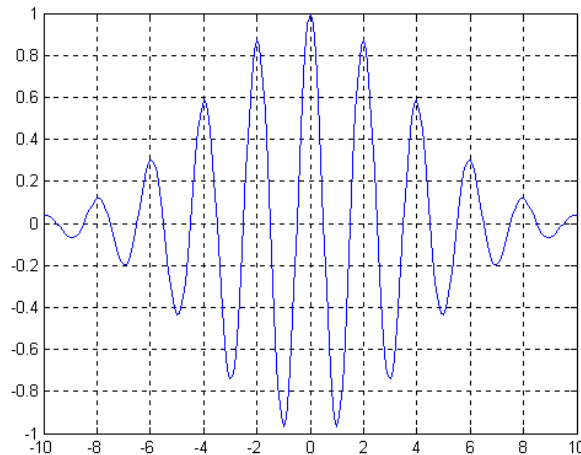


29:030 Physics IV
Exam II
 April 6, 2001

Closed book exam. Please show all work to get full credit. Point values for each problem are indicated. Please ask questions if you need clarification. If you don't show your work, or make written comments, then there is no way for me to give partial credit.

- 1) An electron microscope is to be used to resolve structures 1 nm in size.
 - a) What should be the de Broglie wavelength of the electrons?
 - b) What voltage is necessary to accelerate the electrons?

- 2) Based the wave function:



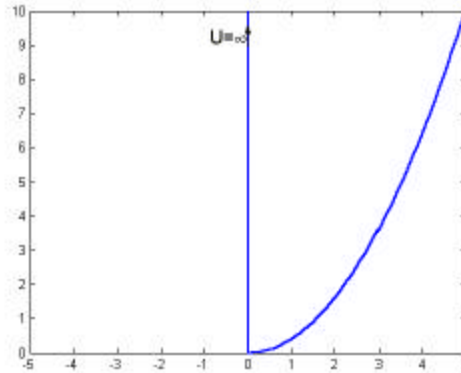
Time units are 10^{-14} sec

- a) Determine as best you can from the plot the angular frequency ω and the uncertainty $\Delta\omega$.
 - b) If this picture corresponds to the wave function of a passing electron $\Psi(x=0,t)$, what is the electron velocity if the potential energy $U=0$? (First find an equation for the velocity in terms of ω and other well known constants. Only plug in numbers if you have time)
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- 3) The wave functions and energies of the simple harmonic oscillator (solutions of the Schroedinger equation with a parabolic potential $U = \frac{1}{2}kx^2$) are tabulated below.

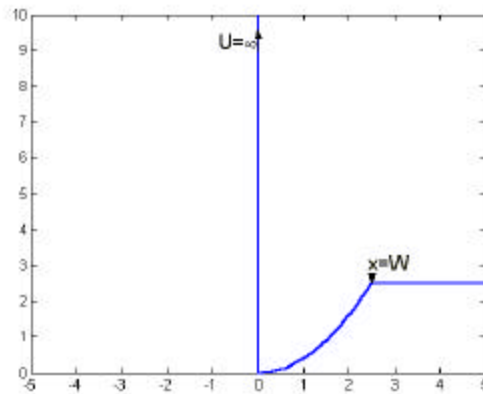
$$\Psi_n = (2^n n! \sqrt{\pi} a)^{-\frac{1}{2}} H_n(\sqrt{ax}) e^{-ax^2/2} \quad a = \frac{\sqrt{km}}{\hbar} \quad H_n(z) = (-1)^n e^{z^2} \frac{d^n}{dz^n} (e^{-z^2})$$

$$H_0 = 1, H_1 = 2z, H_2 = 4z^2 - 2, \dots \quad E_n = (n + \frac{1}{2}) \hbar \sqrt{\frac{k}{m}} \quad n=0, 1, 2, \dots$$

- a) What is the lowest energy and wave function for the potential well shown in the figure which is a half parabolic well [$U = \frac{1}{2} kx^2$, $x > 0$]?



- b) If the half parabola stops at a fixed level as shown in the next figure, estimate what values of curvature k and width W are necessary in order to have at least one bound state.



- 4) Consider the tunneling of particles of mass m through a potential barrier of width L and height U . The particles have energy $E \ll U$ and are incident from region 1.
- What form does the wave function have in the three regions?
 - Estimate what fraction of the beam gets through the barrier.

