Physics 132 Homework 3

Consider a set of N equivalent measurements of a quantity x (each measurement being labeled with a subscript x_i) which we assume contain random errors. Assume the usual normalized model for the probability distribution for each measurement:

$$f(x_i) = \frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(x_i-\mu)^2}{2\sigma^2})$$

Also, use the usual definition for the expectation value for a function $P(x_1, x_2, ..., x_N)$ which can depend on all of the measurements:

$$\langle P \rangle \equiv \int\int ... \int P(x_1, x_2, ..., x_N)\big(f(x_1)f(x_2)...f(x_N)\big)dx_1dx_2...dx_N$$

Show that:

$$\left\langle \sum_{i=1}^{N}(x_i - \overline{x})^2 \right\rangle = (N-1)\sigma^2$$

The average value of the measurements, $\overline{x}$, is also defined in the usual way:

$$\overline{x} \equiv \tfrac{1}{N}\sum_{i=1}^{N} x_i$$

Some help:
Don't be intimidated by all the integrals – notice that the expressions factorize into a product of N integrals over a single variable (do this with care, however!) and use the well known results:

$$\int f(x)dx = 1$$
$$\int xf(x)dx = \mu$$
$$\int (x-\mu)^2 f(x)dx = \sigma^2$$

From the these integrals you can quickly see also that:

$$\int x^2 f(x)dx = \sigma^2 + \mu^2$$