Ion acoustic shock formation in a converging magnetic field geometry

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The effect of the magnetic field configuration on the formation of ion acoustic shocks has been investigated experimentally and through numerical solutions of the fluid equations. The tendency for compressive pulses to steepen into shocks is enhanced if the pulses travel into a configuration of converging magnetic field geometry.
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I. INTRODUCTION

It is well-known that large amplitude compressive pulses generated in an ordinary gas steepen and develop into shocks as they propagate. The possibility that compressional ion acoustic pulses *in a plasma* might steepen into shocks has also been investigated. Ion acoustic waves, which can be considered as Fourier components of a compressional pulse, are subject to Landau damping. This collisionless damping is particularly strong in plasmas having $T_e = T_i$. This was demonstrated by Andersen *et al.*[1] in experiments on shock formation in a Q machine. For the typical case in which $T_e = T_i$, pulses launched from a grid in the plasma were always observed to spread out as they propagated away from the grid. When, however the ratio T_e/T_i was increased by cooling the ions, shock formation was clearly observed. Theoretically, Montgomery[2] argued that if Landau damping could be neglected (for $T_e \gg T_i$) the ion acoustic pulses could be described by the two-fluid continuity and momentum equations. These equations are mathematically similar to the Euler equations for an ideal fluid, the solutions of which predict that any compressive pulse will steepen into a shock as it travels.

Shocks commonly occur in conjunction with strong plasma flows as, for example, in the case of shocks associated with the interaction of planets with the solar wind and the interaction of comets with the interstellar medium, or shocks associated with solar activity in the corona. They are important mechanisms for converting flow energy into thermal energy in plasmas and for accelerating charged particles. Geophysical and astrophysical plasmas are typically inhomogeneous and are often embedded in nonuniform magnetic fields, as in, e.g., the earth's polar cusp and the solar corona.

The present work was undertaken as a first step in understanding ion acoustic shock formation in nonuniform magnetic field geometries. Previous experiments were performed either in uniform magnetic fields[1] or in unmagnetized plasmas.[3] The present investigation was performed in two magnetic field configurations in which large amplitude plasma density pulses were launched along magnetic field lines with either positive (converging lines) or negative (diverging lines) field gradients. The results obtained in these cases are compared with those in a uniform magnetic field.

Following this Introduction, Sec. II provides the theoretical background for the experimental results which are presented in Sec. III. A summary of the conclusions is given in Sec. IV.

II. THEORY

The propagation of a plasma density pulse launched parallel to the magnetic field lines is analyzed using the two-fluid picture appropriate to ion acoustic phenomena. We consider the case in which the electrons and ions are strongly tied to the magnetic field lines so that a one-dimensional analysis is appropriate. The effect of the nonuniform magnetic field geometry is taken into account by including the term $(nv/A)(dA/dx)$ in the ion continuity equation. Here n is the ion density, v the ion fluid velocity, and $A = A(x)$ is the cross sectional area of the magnetic flux tube at some position x along the direction of propagation of the plasma density pulse. We take the plasma to be quasineutral and neglect the electron inertia. Since we are concerned with propagation parallel to B, we can ignore the presence of the B field. Of course, the magnetic field determines the geometry since, according to Gauss's law, $\alpha \equiv (1/A)(dA/dx) = -(1/B)(dB/dx)$. The plasma behavior is then described by the following two equations for ions:

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) + \alpha n v = 0, \tag{1}$$

$$n\frac{\partial v}{\partial t} + nv\frac{\partial v}{\partial x} + C_s^2\frac{\partial n}{\partial x} = 0, \tag{2}$$

where $C_s = [(kT_e + kT_i)/m_i]^{1/2}$ is the ion acoustic velocity, with $T_{e,i}$ the electron (ion) temperature and m_i the ion mass. In this picture, $\alpha > 0$ describes a diverging magnetic geometry, $\alpha < 0$ a converging magnetic geometry and $\alpha = 0$ a uniform magnetic field geometry.

A. Linear analysis

Before discussing the numerical solutions to the nonlinear fluid equations [(1) and (2)] for finite amplitude waves we consider the stability of small amplitude sinusoidal waves. Carrying out the standard linear perturbation analysis on (1) and (2) for perturbations of the type $\exp[i(Kx - \omega t)]$, propagating in a stationary $(v_0 = 0)$ and uniform zero-order

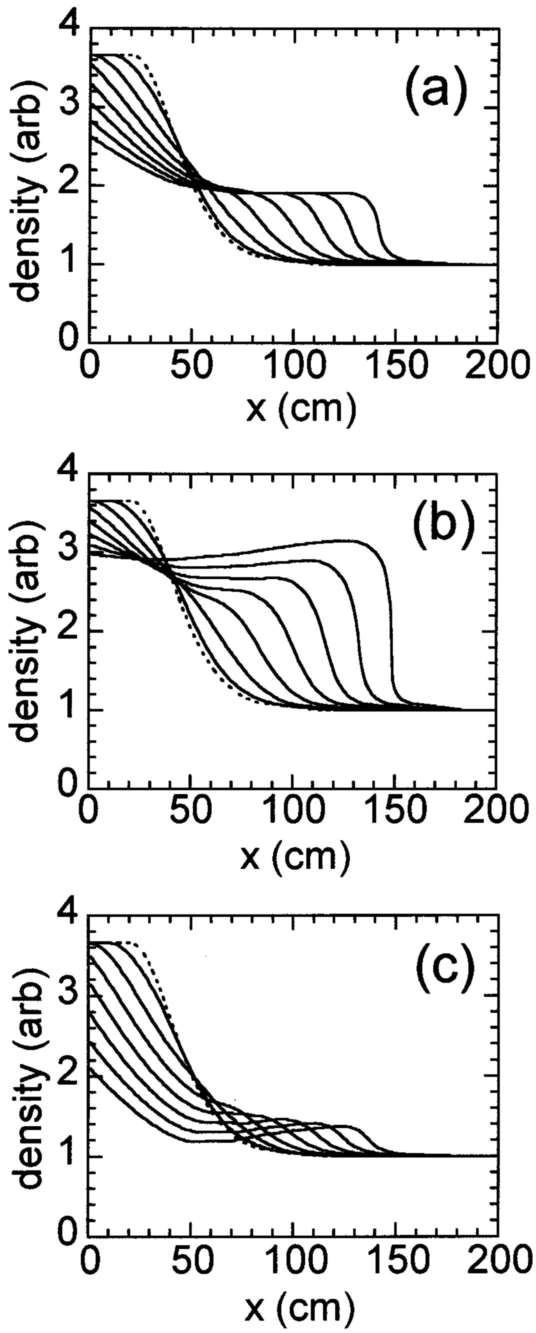

FIG. 1. Time evolution (in 0.1 ms steps) of an initial density pulse (dotted curve) obtained from a numerical integration of the fluid equations, for (a) uniform magnetic field, $\alpha=0$, (b) converging magnetic field, $\alpha<0$, and (c) diverging magnetic field, $\alpha>0$. For this calculation $C_s=1\times10^5$ cm/s.

plasma $(dn_0/dx=0)$, where K and ω are the wave number and angular frequency, respectively, we obtain the following dispersion relation:

$$\omega^2-K(K-i\alpha)C_s^2=0. \tag{3}$$

Assuming a real ω and complex $K(=K_r+iK_i)$, and setting the real and imaginary parts of (3) equal to zero yields

$$\omega^2=\left(K_r^2+\frac{\alpha^2}{4}\right)C_s^2, \tag{4a}$$

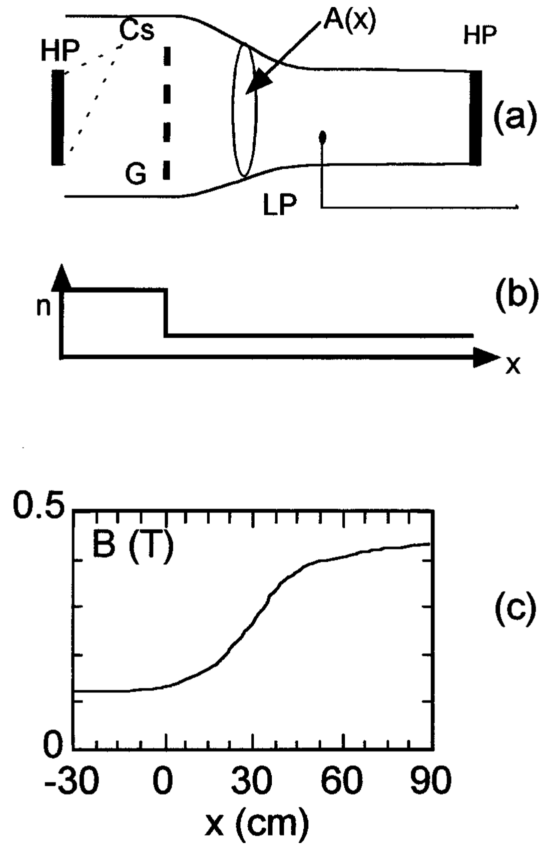

FIG. 2. Schematic of the experimental arrangement. (a) Double-ended Q machine device showing the two hot plates (HP), Cs atomic oven source, Langmuir probe (LP) and grid (G) for launching pulses. A converging magnetic flux tube with cross-sectional area $A(x)$ at location x is also shown. (b) Schematic axial plasma density distribution prior to the launching of the pulse. (c) Measured axial profile of the magnetic field strength for the converging field configuration. The grid used to launch the density pulses was located at $x=0$.

$$K_i=\frac{\alpha}{2}. \tag{4b}$$

For the case of waves propagating into a converging magnetic field geometry ($\alpha<0$), $K_i<0$ and thus the waves grow as they propagate. Physically, the wave amplitude must increase, since as the waves propagate into a region of converging magnetic field (smaller plasma cross sectional area) with $dn_0/dx=0$, the wave energy must be carried by fewer and fewer particles. It is then possible that the effect of the converging magnetic field may overcome the Landau damping allowing the waves to propagate even if $T_e=T_i$.

B. Numerical solutions to the fluid equations

Numerical integrations of the fluid equations have been performed for an initial step plasma density profile and with $C_s=1\times10^5$ cm/s. Examples of the solutions [density vs x for various times] corresponding to the cases of (a) uniform, $\alpha=0$, (b) converging, $\alpha<0$, and (c) diverging, $\alpha>0$, magnetic geometries are shown in Fig. 1. For cases (b) and (c) a constant value for α was assumed for simplicity. For $\alpha=0$, the steepening effect predicted by the analytical solution discussed by Montgomery[2] is observed. The time for the pulse

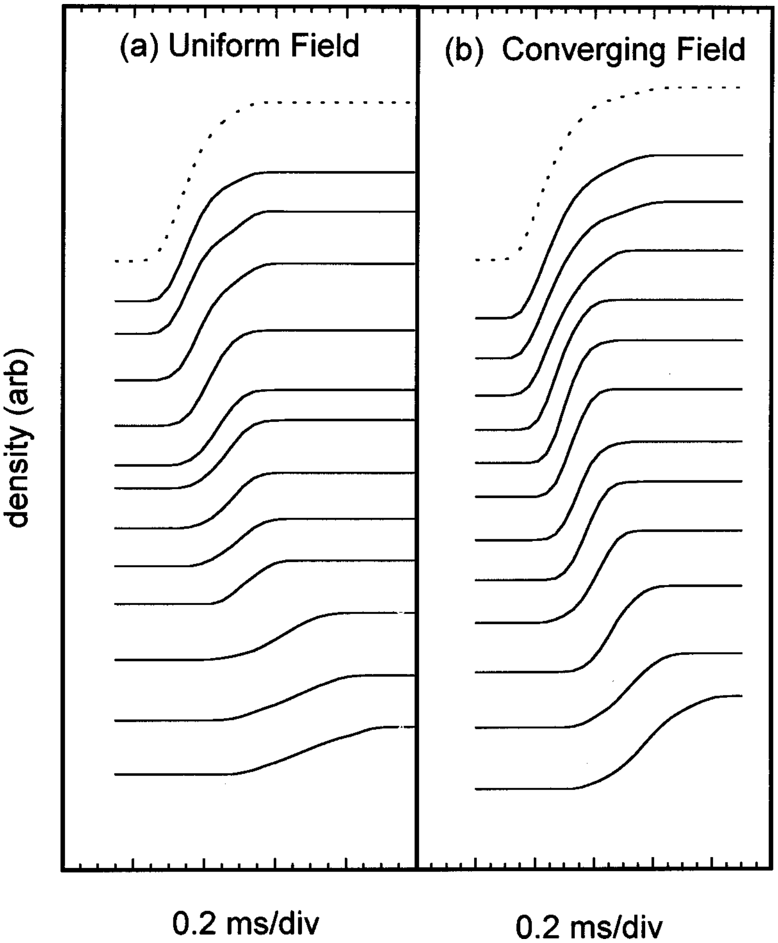

FIG. 3. Experimental curves of density vs time at various distances from the grid (1, 3, 6, 10, 15, 20, 25, 30, 35, 40, 50, 60, and 70 cm, respectively) which was located at $x=0$. (a) uniform magnetic field, (b) converging magnetic field. The curve corresponding to the first position after the grid (1 cm) is shown with a dashed line. The curves are vertically displaced for clarity.

to develop a vertical tangent in the plot of n vs x is also in good agreement with the analytic expression [Eq. (6)] of Montgomery,[2] $t_s=2/[(\gamma_e+1)|\partial v/\partial x|_{\max}]$, where γ_e is the specific heat ratio and $|\partial v/\partial x|_{\max}$ is to be evaluated at $t=0$. We can also observe that the effect of the converging magnetic field (b) is to enhance the steepening effect, whereas in the diverging field case (c) the steepening is impeded. In case (b) the pulse forms a vertical tangent (shock structure) at $t=0.7$ ms, earlier than in case (a). The pulse velocities are all $\sim 2\times 10^5$ cm/s$\sim 2\,C_s$.

Thus, the nonlinear *fluid* analysis of the propagation of ion acousticlike pulses into a converging magnetic field geometry suggests that in an actual case in which Landau damping is present, the steepening effect just discussed may be strong enough to overcome it.

III. EXPERIMENTAL SETUP AND RESULTS

The experiments were carried out in a double-ended Q machine device using the setup shown schematically in Fig. 2(a). A Cs^+/electron plasma was formed by contact ionization of cesium atoms on a 6 cm diameter hot tungsten plate (HP). The plasma was terminated at the opposite end by a second hot plate located 160 cm from the first one. The electron and ion temperatures were $T_e \approx T_i \approx 0.2$ eV, and the plasma density was in the range of 10^9-10^{10} cm^{-3}. The plasma was confined radially by a longitudinal magnetic field with a strength up to 0.5 T. The magnet coils could be configured to produce either a uniform magnetic field profile or diverging or converging magnetic field profiles. A plot of the magnetic field strength on axis for the case of the converging magnetic field is shown in Fig. 2(c). Converging magnetic field lines are also illustrated schematically in Fig. 2(a).

The plasma density pulses were produced by applying a

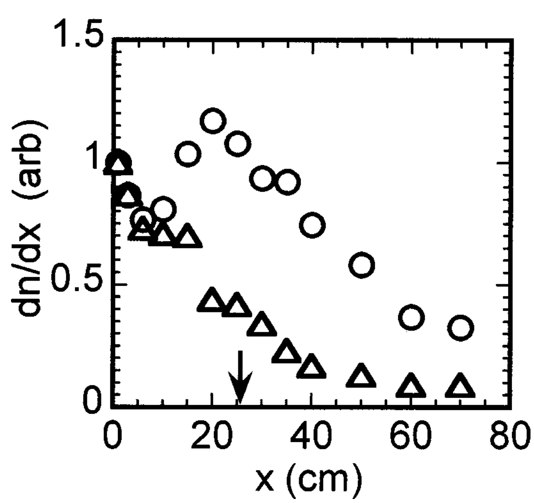

FIG. 4. Shock steepness, $\Delta n/\Delta x$ vs distance, x from the grid for the uniform field case (triangles) and converging field case (circles). The arrow indicates the position of the maximum magnetic field gradient.

voltage step to a grid (G) located about 30 cm from the hot plate/Cs source. The grid was normally biased at ~ -6 V with respect to the grounded HP and absorbed most of the ions resulting in an axial plasma density distribution of the type shown in Fig. 2(b). When the bias was suddenly ($\sim$0.1 μs) increased to ~ -2 volts ($\sim$the plasma space potential), a plasma density pulse was launched toward the other hot plate. This configuration is the electrical analog of the breaking of a membrane in a gasdynamic shock tube. The arrival of the density pulse at any position x from the grid was monitored by recording the ion saturation current to a negatively biased Langmuir probe.

Figure 3(a) shows n vs t curves recorded at various distances downstream of the grid, for a density pulse launched along a uniform magnetic field configuration. Under these conditions the pulse was observed to spread out as it propagated away from the grid. Although the numerical calculations (Fig. 1) showed a steepening in this case, this was not seen in the experiments since Landau damping (which was not included in the analysis) overcomes the tendency toward steepening.

The corresponding n vs t curves for a pulse launched into the converging magnetic field configuration [Fig. 2(c)] is shown in Fig. 3(b). In this case, the pulse was observed to steepen as it propagated through the region of strong magnetic field gradient.

Data (not shown) were also obtained for a pulse launched into a diverging magnetic field configuration. For this case the pulse spread even more quickly as compared to the uniform field case, with the pulse being almost completely ''washed out'' at a distance of 50 cm from the grid.

To further illustrate the effect of the converging magnetic field, we show in Fig. 4 plots of the shock steepness, $\Delta n/\Delta x$, of the n vs t curves at various distances from the grid for both the uniform and converging magnetic field cases. The $\Delta n/\Delta x$'s were obtained from the slopes, $\Delta n/\Delta t$ of the curves presented in Fig. 3 since the $\Delta n/\Delta x$'s are related to the $\Delta n/\Delta t$'s through $\Delta n/\Delta t = V_P(\Delta n/\Delta x)$, where V_P is the propagation speed of the pulse. Since V_P is approximately constant ($\sim 2-3\times 10^5$ cm/s) the $\Delta n/\Delta t$'s are a good approximation to the actual $\Delta n/\Delta x$'s. These plots clearly demonstrate the steepening effect associated with the converging magnetic field.

IV. SUMMARY AND CONCLUSIONS

In summary, numerical and experimental results have been presented to illustrate the effect of a nonuniform magnetic field on the evolution of plasma density pulses propagating along a magnetic field. Numerical solutions of the fully nonlinear fluid equations used to describe ion acoustic phenomena show that the tendency of a compressional pulse to steepen as it travels is enhanced for pulses travelling into a region of converging magnetic field and diminished for pulses travelling into a region of diverging magnetic field. These effects were borne out in the experimental investigations which showed that pulses launched into a converging magnetic geometry steepened into shocklike structures. The steepening effect was apparently of sufficient strength as to overcome the effects of Landau damping which would otherwise prevent the formation of ion acoustic shocks. We note in closing that ion acoustic shocks have also been recently observed in negative ion plasmas[4,5] and in dusty plasmas.[6,7]

ACKNOWLEDGMENTS

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