The charge on dust particles in a plasma

1.—The calculation of the charge on dust grains in a plasma depends on whether or not the dust grains can be considered “isolated” or not. A single grain in a plasma is, of course, isolated. When more and more grains become immersed in a plasma, the proximity of one grain to another affects the charge on the grains. This effect, known as the close-packing effect, is important when $\Delta/\lambda_D \sim 1$, where Δ is the average interparticle spacing ($\Delta \sim n_d^{-1/3}$, n_d is the number density of dust particles in the plasma) and λ_D is the plasma Debye length. This can also be formulated in terms of the so-called Havnes parameter $P_H = Zn_d/n_i$, where Z is the charge number on the dust and n_i is the ion density. If $P_H \sim 1$, generally the grains cannot be considered as isolated and the close packing effects must be taken into account. If the grain density is relatively high, the electron and ion densities will not be equal due to the charges on the grains.
In the analysis that follows, we DO NOT take into account collisions between the plasma electrons and ions.

2.—To compute the charge in either case (isolated or close-packed) we start with the plasma charging equations – the electron and ion currents to the particle. Let a be the radius of the dust grain, T_e and T_i are the electron and ion temperatures, m_e and m_i the electron and ion masses, n_e and n_i are the electron and ion densities. The particle charges to a potential V_s (the grain surface potential), which under typical conditions (electron/ion plasma no photoelectric charging), V_s will be negative.

electron current: $$I_e = -en_e\sqrt{\frac{kT_e}{m_e}}e^{eV_s/kT_e}\,4\pi a^2 \qquad (1)$$

ion current: $$I_i = en_i\sqrt{\frac{kT_i}{m_i}}\left(1-\frac{eV_s}{kT_i}\right)4\pi a^2 \qquad (2)$$

The surface potential V_s is determined by the floating grin condition $\Sigma I(V_s) = 0$, or

$$I_e + I_i = 0 \qquad (3)$$

The surface potential is then found from the solution of the equation

$$n_e\sqrt{\frac{T_e}{T_i}}\sqrt{\frac{m_i}{m_e}}e^{eV_s/kT_e}+n_i\left(1-\frac{eV_s}{kT_i}\right)=0 \tag{4}$$

Once V_s is known the particle charge Q, is then determined from

$$Q=eZ=4\pi\varepsilon_o aV_s \tag{5}$$

2(a).—*Isolated grains– Z_o.* Set $n_e = n_i$ in Eq. (4) to find V_s.

$$\left(\frac{T_e}{T_i}\right)^{\frac{1}{2}}\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}e^{eV_s/kT_e}+\left(1-\frac{eV_s}{kT_i}\right)=0 \tag{6}$$

The charge number Z_o can be found directly using Eq. (5), $V_s=\frac{eZ_o}{4\pi\varepsilon_o a}$, in Eq. (6)

$$\left(\frac{T_e}{T_i}\right)^{\frac{1}{2}}\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}e^{\frac{eZ_o}{4\pi\varepsilon_o a(kT_e/e)}}+1-\frac{eZ_o}{4\pi\varepsilon_o a(kT_i/e)}=0 \tag{7a}$$

or $$\left(\frac{T_e}{T_i}\right)^{\frac{1}{2}}\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}}e^{\frac{eZ_o}{4\pi\varepsilon_o aT_e}}+1-\frac{eZ_o}{4\pi\varepsilon_o aT_i}=0 \tag{7b}$$

where, in Eq 7(b) T_e and T_i are expressed in voltage units.

2(b).— *Close-packed grains, charge number Z.–* To take into account the close-packing effect, we use the condition of charge neutrality to relate the ion and electron densities in Eq. 4,

$$n_i + Zn_d = n_e \tag{8}$$

Define

$$P \equiv \frac{n_d}{n_i} \qquad (9)$$

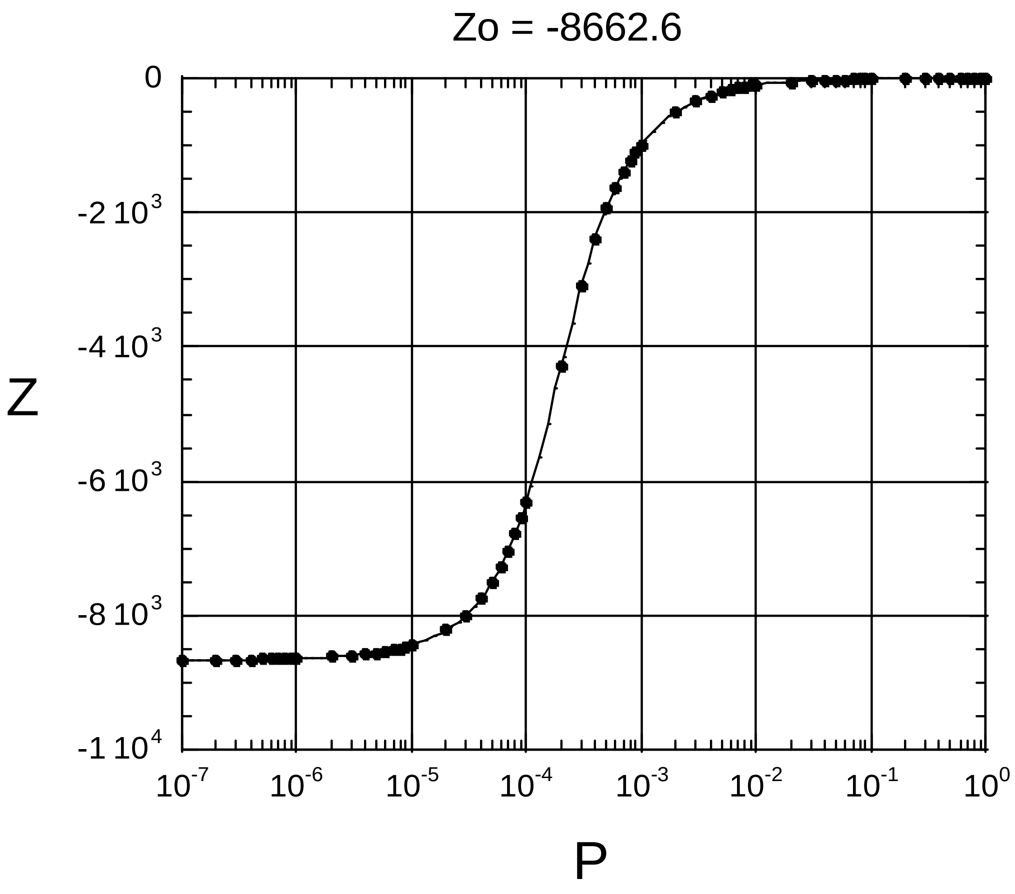
then Eq. 4 can be expressed as

$$-(1+PZ)\left(\frac{T_e}{T_i}\right)^{\frac{1}{2}}\left(\frac{m_i}{m_e}\right)^{\frac{1}{2}} e^{\frac{eZ}{4\pi\varepsilon_o a T_e}} + 1 - \frac{eZ}{4\pi\varepsilon_o a T_i} = 0 \qquad (10)$$

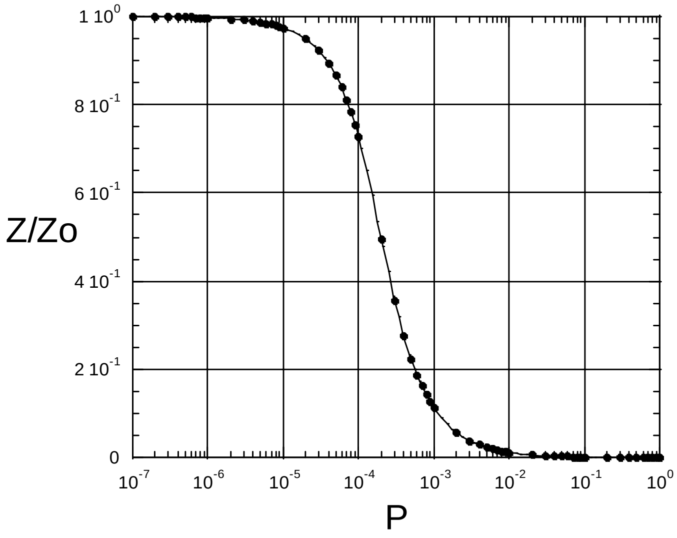
We recognize in Eq. 10, that PZ = P_H is the Havnes parameter. To find the charge Z from Eq. 10, one needs to input the ion mass, the temperatures, T_e, T_i, (volts) the grain radius a (meters) , and $P = n_d / n_i$.

3.—Some results for A = 40 (argon), T_e = 2 eV, T_i = 0.025 eV, a = 2.5 μm

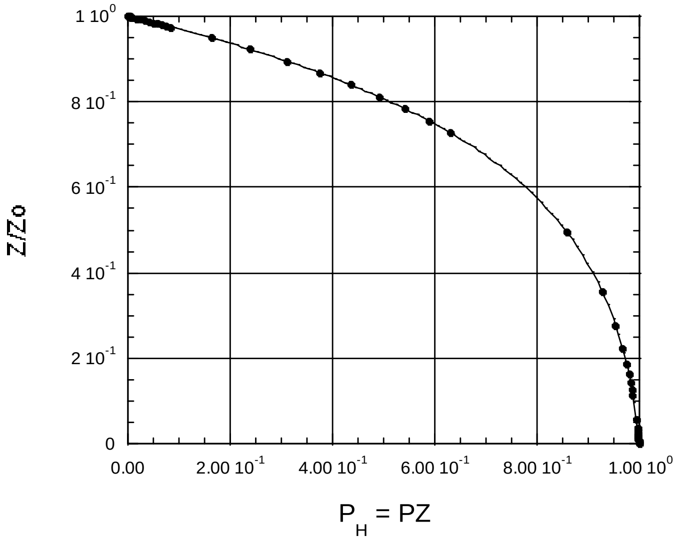
(a) Z vs. P



(b) Normalized Z/Zo vs P



(c) Z/Zo vs Havnes parameter ZP



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