doi:10.1017/S0022377808007642

Instability of higher harmonic electrostatic ion cyclotron waves in a negative ion plasma

M. ROSENBERG[1] and R. L. MERLINO[2]

[1] Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, USA
(rosenber@ecepops.ucsd.edu)

[2] Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52242, USA

(Received 11 August 2008 and in revised form 3 September 2008)

Abstract. We present a kinetic theory analysis of the electrostatic ion cyclotron (EIC) instability in a plasma containing positive ions, electrons, and negative ions that are much more massive than the positive ions. Conditions are investigated for exciting the fundamental and the higher harmonic EIC waves associated with each ion species. We find that as the concentration of heavy negative ions increases, the wave frequencies increase, the unstable spectrum in general shifts to longer perpendicular wavelengths, and the growth of higher harmonic EIC waves tends to increase within certain parameter ranges. Applications to possible laboratory plasmas are discussed.

1. Introduction

The electrostatic ion cyclotron (EIC) instability is a low-frequency field-aligned current-driven instability that has one of the lowest threshold drift velocities among current-driven instabilities (Drummond and Rosenbluth 1962). In a plasma containing electrons and positive ions, the fundamental EIC wave has frequency of the order of the ion cyclotron frequency Ω_i, propagates nearly perpendicular to the magnetic field B, and has a small but finite wave number along B so that it can be destabilized by electrons drifting along B (see, e.g., Rasmussen and Schrittwieser 1991; Okuda et al. 1981).

The excitation of the fundamental EIC mode in a negative ion plasma has been studied experimentally (Song et al. 1989) and theoretically for a plasma containing heavy negative ions and light positive ions (D'Angelo and Merlino 1986; Chow and Rosenberg 1996a). In this type of plasma there can be an EIC wave associated with each ion component. Prior experiments on the EIC instability in negative ion plasmas indicate that the critical electron drift decreases as the density of negative ions increase (Song et al. 1989). It was found theoretically that the critical drift for the excitation of the fundamental EIC modes associated with either the positive or negative ions decreases as the relative density of negative ions increases (Chow and Rosenberg 1996a). The EIC instability in a dusty plasma containing electrons, light ions, and massive negatively charged dust grains was investigated both experimentally (Barkan et al. 1995) and theoretically (D'Angelo 1990; Chow and Rosenberg 1995, 1996b). Both theory and experiment show that as more negative charge is carried by the dust, the frequency and growth rate of the light-ion EIC mode increases. In addition, the critical drift decreases as the concentration of negatively charged dust increases (Chow and Rosenberg 1996b). The excitation of electrostatic dust cyclotron (EDC) modes by ions drifting along the magnetic field in a dusty plasma was investigated theoretically by D'Angelo (1998) and Rosenberg and Chow (1999). It was shown that the critical ion drift can decrease as the density of negatively charged dust increases. Conditions for exciting higher harmonic EDC modes by drifting ions was considered by Sorasio and Rosenberg (2001).

In this paper, we extend the analysis of the EIC instability (EICI) in a negative ion plasma by Chow and Rosenberg (1996a), by considering the conditions for exciting the higher harmonic EIC modes as well as the fundamental modes. We focus on the regime where the negative ions are much more massive than the positive ions, and where the negative ion density can be larger than that of the electrons. Such plasmas have been discussed recently in both laboratory and space contexts. For example, Merlino and Kim (2006) and Kim and Merlino (2006) considered the conditions under which dust grains injected into a negative ion plasma could become positively charged, when the electron density is small enough so the lighter positive ions are the more mobile species. Kim and Merlino (2007) reported on the generation of a negative ion plasma in the laboratory, and the detection of the fundamental and second harmonic EIC waves associated with each ion species. Very recently, Kim et al. (2008) reported on the generation of both the fundamental and higher harmonics associated with both the light positive ions (K^+) and heavy negative ions ($C_7F_{14}^-$) in a laboratory plasma at very low pressure. With regard to space plasmas, Rapp et al. (2005) have discussed the possible role of negative ions, in explaining their observations of positively charged nanoparticles in the mesosphere under night-time conditions.

The paper is organized as follows. The analysis is given in Sec. 2, and both analytic and numerical results are given in Sec. 3, with application to possible experimental laboratory plasma parameters. Section 4 gives a brief summary.

2. Analysis

We consider a plasma composed of electrons, singly charged positive ions, and singly charged negative ions. The ratio of the heavy-ion mass m_h to the light-ion mass m_l, $M_r = m_\mathrm{h}/m_\mathrm{l}$, is much greater than 1.

The condition of overall charge neutrality is given by

$$n_\mathrm{l} = n_\mathrm{e} + n_\mathrm{h}, \tag{1}$$

where n_α is the density of charged species α (the subscript $\alpha =$ l, e, h, denotes light positive ions, electrons, and heavy negative ions, respectively). This condition can be written as

$$\delta = \frac{n_\mathrm{l}}{n_\mathrm{e}} = 1 + \epsilon_\mathrm{h} \tag{2}$$

where $\epsilon_\mathrm{h} = n_\mathrm{h}/n_\mathrm{e}$.

The plasma is assumed to be homogeneous and immersed in a uniform magnetic field $B\mathbf{z}$. The electrons and negative ions have drifts in the $\mathbf{z}$-direction and the positive ions drift in the $-\mathbf{z}$-direction; the magnitude of these drifts is $u_{0\alpha}$. We neglect collisions in the following, and assume that the electrons and ions can be described by drifting Maxwellian distributions. Then the dispersion relation for electrostatic waves with perturbed $E_1 \sim \exp(i\mathbf{k}\cdot\mathbf{r}-\omega t)$ and frequency ω much less than the electron cyclotron frequency Ω_e, having wavevector components $k_\perp$ and k_z which are perpendicular and parallel to B, respectively, is given by (see, e.g., Kindel and Kennel 1971; Satyanarayana et al. 1985; Chow and Rosenberg 1996)

$$D(\omega, k) = 1 + \sum_\alpha \chi_\alpha = 0, \tag{3}$$

where

$$\chi_\mathrm{e} = \frac{1}{k^2\lambda_\mathrm{De}^2}[1 + \zeta_\mathrm{e}\Gamma_0(b_\mathrm{e})Z(\zeta_\mathrm{e})], \tag{4}$$

$$\chi_j = \frac{1}{k^2\lambda_{\mathrm{D}j}^2}\left[1 + \zeta_{j0}\sum_{m=-\infty}^{\infty}\Gamma_m(b_j)Z(\zeta_{jm})\right]. \tag{5}$$

Here

$$\zeta_\mathrm{e} = \frac{\omega - k_z u_{0\mathrm{e}}}{\sqrt{2}k_z v_\mathrm{e}}, \tag{6a}$$

$$\zeta_{jm} = \frac{\omega - \mathbf{k}\cdot\mathbf{u}_{0j} - m\Omega_j}{\sqrt{2}k_z v_j}, \tag{6b}$$

and the subscript $j = \mathrm{l,h}$. In the above equations, v_α, Ω_α, $\rho_\alpha = v_\alpha/\Omega_\alpha$, and $\lambda_{\mathrm{D}\alpha} = (T_\alpha/4\pi n_\alpha Z_\alpha^2 e^2)^{1/2}$ are the thermal speed, gyrofrequency, gyroradius, and Debye length of species α, respectively, $b_\alpha = k_\perp^2\rho_\alpha^2$, $\Gamma_m(x) = I_m(x)\exp(-x)$, with I_m the modified Bessel function of order m, $Z(\zeta)$ is the plasma dispersion function (Fried and Conte 1961), and $k^2 = k_\perp^2 + k_z^2$.

Assuming that the wave is either weakly damped or growing, the solution to the real part of the frequency ω_r is given by

$$D_\mathrm{r}(\omega_\mathrm{r}, k) = 0, \tag{7a}$$

where D_r is the real part of (3). The imaginary part of the frequency is then given by (e.g. Krall and Trivelpiece 1973)

$$\gamma = -\frac{D_\mathrm{i}(\omega_\mathrm{r}, k)}{\partial D_\mathrm{r}(\omega_\mathrm{r}, k)/\partial\omega_\mathrm{r}}, \tag{7b}$$

where D_i is the imaginary part of (3).

3. Results

3.1. Analytical results

In this section, we give analytical results following from (3) for certain regimes that have application to possible laboratory experimental parameters.

There is an EICI associated with each ion species. The frequencies of the fundamental harmonic EIC waves are slightly larger than their respective gyrofrequencies, and similarly for the higher harmonics. For the plasma we are considering, we refer to these instabilities as the light-ion EICI and the heavy-ion EICI. Generally, the EICI is driven by an electron current parallel to $\mathbf{B}$, with election drift speed larger than the parallel phase speed of the wave, i.e. $u_{0\mathrm{e}}/v_\mathrm{e} > \omega/k_z v_\mathrm{e}$. The instability can be damped by ion cyclotron damping (and by ion collisions if the ion collision frequency is not much smaller than the growth rate). In the negative ion plasma we are considering, there is the possibility that the drift of the light ions parallel to **B** could excite a heavy-ion EICI, with $\omega \sim \Omega_{\rm h}$ for the fundamental mode. In order for this to occur, it is required that $u_{0\rm l}/v_{\rm l} > \Omega_{\rm h}/k_z v_{\rm l}$, i.e. that the drift speed of the light ions is larger than the parallel phase speed of the heavy-ion EIC wave. The latter condition can be rewritten as

$$\frac{u_{0\rm l}}{v_{\rm l}} > \frac{1}{b_{\rm h}^{1/2}} \frac{k_\perp}{k_z} \frac{v_{\rm h}}{v_{\rm l}}.$$

Since $b_{\rm h}$ is of the order of unity, while $k_\perp/k_z \gg 1$ and $v_{\rm h}/v_{\rm l}$ is typically not smaller than about 0.1, this implies that the critical light-ion drift would be roughly of the order of the light-ion thermal speed. Thus, in the following we confine our attention to the regime where $u_{0\rm l}$ is much smaller than this value, i.e. where $u_{0\rm l} \ll \omega/k_z$, and set both $u_{0\rm l}$ and $u_{0\rm h}$ equal to zero for simplicity.

In the following, we consider the kinetic regime for the electrons, with $\zeta_{\rm e} \ll 1$, and the limit of small electron Larmor radius with $b_{\rm e} \ll 1$. The electron susceptibility (4) then becomes

$$\chi_{\rm e} \approx \frac{1}{k^2 \lambda_{\rm De}^2} \left[1 + i\sqrt{\frac{\pi}{2}} \frac{\omega - k_z u_{0\rm e}}{k_z v_{\rm e}} \right]. \tag{8}$$

3.1.1. Light-ion EICI.. For the light ions, we consider the phase velocity regime where $\zeta_{\rm l\mathit{m}} \gg 1$, where light-ion cyclotron damping is small. This implies that $(\omega - m\Omega_{\rm l}) \gg k_z v_{\rm l}$ for each mth harmonic. Then from (5) the light-ion susceptibility becomes approximately

$$\chi_{\rm l} \approx \frac{1}{k^2 \lambda_{\rm Dl}^2} \left\{ 1 - \sum_m \frac{\omega \Gamma_m(b_{\rm l})}{\omega - m\Omega_{\rm l}} + i\sqrt{\frac{\pi}{2}} \sum_m \frac{\omega \Gamma_m(b_{\rm l})}{k_z v_{\rm l}} e^{-\zeta_{\rm l\mathit{m}}^2} \right\}. \tag{9}$$

For the heavy ions, we make the following approximation in order to get a simple qualitative result. Since $M_r \gg 1$, we have that $\omega \gg \Omega_{\rm h}$. We assume that the heavy-ion response is the unmagnetized response, so that

$$\chi_{\rm h} \sim -\frac{k_{\rm Dh}^2}{2k^2} Z'(R_{\rm h})$$

where $R_{\rm h} = \omega/\sqrt{2}kv_{\rm h}$ (see Rosenbluth 1965; Kindel and Kennel 1971). The term $Z'(R_{\rm h})$ is estimated as follows. Using $\omega \sim m\Omega_{\rm l}$ and $k \sim k_\perp$, we have $R_{\rm h} \sim (m/b_{\rm l}^{1/2})(v_{\rm l}/v_{\rm h})$. Since generally $b_{\rm l} < m^2$ and $v_{\rm l} > v_{\rm h}$, we can expand Z' for large argument. Then $\chi_{\rm h}$ becomes approximately

$$\chi_{\rm h} \sim -\frac{\omega_{\rm ph}^2}{m^2 \Omega_{\rm l}^2},$$

where $\omega_{\rm ph}$ is the heavy ion plasma frequency.

Although the latter approximation is reasonable when $b_{\rm h} \gg 1$, it is not as good in the case of smaller $b_{\rm h}$. At any rate, if we compare $\chi_{\rm h}$ with the real part of $\chi_{\rm e}$ given in (8), we find that

$$\frac{\chi_{\rm h}}{\mathrm{Re}\ \chi_{\rm e}} \sim \frac{b_{\rm l}}{m^2} \frac{\epsilon_{\rm h}}{M_r} \frac{T_{\rm e}}{T_{\rm l}}.$$

Since this quantity is generally much less than 1, we neglect the contribution of $\chi_{\rm h}$ for the light-ion EICI. However, the heavy ions modify the dispersion relation of the light-ion EICI via the charge neutrality condition (1). This treatment is similar to that of the light-ion EICI in a dusty plasma (see Chow and Rosenberg 1995).

Using (8) and (9) for the electron and light-ion susceptibilities, we then use (7a) to obtain the real part of the frequency, ω_{r}. Writing $\omega_{\mathrm{r}} = m\Omega_{\mathrm{l}}(1+\Delta_{\mathrm{l}m})$, we obtain

$$\Delta_{\mathrm{l}m} = \frac{\delta\tau_{\mathrm{el}}\Gamma_m(b_{\mathrm{l}})}{1+k^2\lambda_{\mathrm{De}}^2+\delta\tau_{\mathrm{el}}[1-G(b_{\mathrm{l}})]}, \tag{10}$$

where $\tau_{\mathrm{el}} = T_{\mathrm{e}}/T_{\mathrm{l}}$, $\tau_{\mathrm{eh}} = T_{\mathrm{e}}/T_{\mathrm{h}}$, and

$$G(b_{\mathrm{l}}) = \Gamma_m(b_{\mathrm{l}}) + \sum_{n\neq m}\frac{\omega\Gamma_n(b_{\mathrm{l}})}{\omega - n\Omega_{\mathrm{l}}}. \tag{11}$$

What we can see from (10) is that as the density of heavy ions increases (i.e. as $\delta = 1+\epsilon_{\mathrm{h}}$ increases), the frequency of the mode increases, and the unstable mode shifts to larger perpendicular wavelengths, that is, smaller b_{l} (where Γ_m is smaller) in order that $\Delta_{\mathrm{l}m} < 1$. This behavior is similar to that of a standard electron–ion plasma having $\delta = 1$, as the ratio of the electron to ion temperatures increases (see Kindel and Kennel 1971; Rasmussen and Schrittwieser 1991). The increase in mode frequency is basically due to the increase in the ion acoustic speed. In our case, this is due to an increase in $\delta = n_i/n_{\mathrm{e}}$. (This can be compared with the standard electron–ion plasma where $n_i = n_{\mathrm{e}}$, where the ion acoustic speed increases as T_{e}/T_i increases). When the mode frequency increases, the ion cyclotron damping can decrease. This implies that it may be possible to excite higher-order harmonics at lower critical electron drifts.

For the fundamental harmonic with $m = 1$, (10) becomes approximately

$$\Delta_{l1} \approx \frac{\delta\tau_{\mathrm{el}}\Gamma_1(b_{\mathrm{l}})}{1+k^2\lambda_{\mathrm{De}}^2+\delta\tau_{\mathrm{el}}[1-\Gamma_1(b_{\mathrm{l}})-\Gamma_0(b_{\mathrm{l}})]}. \tag{12}$$

To obtain (12) we have retained only the contribution of the $m = 0, 1$ terms. For the higher harmonics with $m \geqslant 2$, when $\omega \sim m\Omega_{\mathrm{l}}$, $b_{\mathrm{l}} > 1$ and $b_{\mathrm{l}} < m^2$, the G functions (11) for the lower harmonics are close to one (Kindel and Kennel 1971). However, we cannot really make this approximation for the situation we are studying, because as the density of heavy ions increases, instability occurs at smaller values of b_{l}, and the mode frequency increases substantially. Thus we would need to retain the full expression in (10) with the G function in (11) determined numerically.

Marginal stability occurs when $D_{\mathrm{i}}(\omega_{\mathrm{r}}, k) = 0$ from (7b). From (8) and (9) we find that the critical electron drift at marginal stability is given by

$$\begin{aligned}\frac{u_{0\mathrm{e,c}}}{v_{\mathrm{l}}} \approx m(1+\Delta_{\mathrm{l}m})\xi_{\mathrm{l}}\Bigg[1+\delta\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{l}}}\right)^{3/2}\left(\frac{m_{\mathrm{l}}}{m_{\mathrm{e}}}\right)^{1/2} \\ \times\left\{\Gamma_m(b_{\mathrm{l}})\exp\left(-\frac{m^2\Delta_{\mathrm{l}m}^2\xi_{\mathrm{l}}^2}{2}\right)+\Gamma_{m+1}(b_{\mathrm{l}})\exp\left(-\frac{(m\Delta_{\mathrm{l}m}-1)^2\xi_{\mathrm{l}}^2}{2}\right)\right\}\Bigg],\end{aligned} \tag{13}$$

where

$$\xi_{\mathrm{l}} = \frac{1}{b_{\mathrm{l}}^{1/2}}\frac{k_\perp}{k_z},$$

The first term in brackets in (13) reflects the requirement that $u_{0\mathrm{e}} > \omega/k_z$ for instability, while the second term corresponds to light-ion cyclotron damping, where we have retained contributions both from the m and $m+1$ harmonics, since the latter can be significant when the frequency increases as the negative ion concentration ϵ_{h} increases. In order for ion cyclotron damping to be small, the corresponding exponential terms in (13) should be small; for damping by the m harmonic, this implies the condition $\zeta_{\mathrm{l}} \gg 1/m\Delta_{\mathrm{l}m}$. Now, as ϵ_{h} increases, $\Delta_{\mathrm{l}m}$ increases as can be seen from (10), so it is easier to satisfy the latter condition on ζ_{l}. In addition, b_{l} tends to decrease as the negative ion density increases which results in the following effect. Since the critical drift is proportional to ζ_{l}, as b_{l} decreases maximum growth of the instability tends to shift to smaller values of $k_\perp/k_z$ that still satisfy the condition $\zeta_{\mathrm{l}} \gg 1/m\Delta_{\mathrm{l}m}$ for small ion cyclotron damping. Thus in general we expect that higher harmonics of the light-ion EIC could be more easily excited as ϵ_{h} increases, with the unstable waves tending to propagate at smaller angles with respect to B, and having longer perpendicular wavelengths. However, a practical consideration is that if ϵ_{h} gets too large, the unstable wavenumbers shift to such small values of b_{l} that the wavelengths may not fit in the plasma. Now, as ϵ_{h} increases further, the mode frequency can approach the $m+1$ harmonic, at least for fixed b_{l}. Then, to avoid cyclotron damping from the $m+1$ harmonic, $k_\perp/k_z$ increases; thus ζ_{l} increases as does the critical drift (which is proportional to ζ_{l}). Thus we may expect a minimum in the behavior of the critical drift as a function of ϵ_{h}, at least for fixed b_{l}.

We estimate the growth rate when $u_{0\mathrm{e}} \gg u_{0\mathrm{e,c}}$. Neglecting ion cyclotron damping terms, (7b) yields, using (8) and (9),

$$\frac{\gamma}{\Omega_{\mathrm{l}}} \approx \frac{1}{\delta\tau_{\mathrm{el}}} \frac{m\Delta_{\mathrm{l}m}^2}{\Gamma_m(b_{\mathrm{l}})} \sqrt{\frac{\pi}{2}} \left(\frac{u_{0\mathrm{e}}}{v_{\mathrm{e}}} - \frac{\omega_{\mathrm{r}}}{k_z v_{\mathrm{e}}} \right). \tag{14}$$

To obtain (14), we have assumed that $\partial D_r(\omega_{\mathrm{r}}, k)/\partial\omega_{\mathrm{r}}$ is determined from the mth harmonic term contribution to χ_{l}. (This may not be so good an approximation as ϵ_{h} increases and the mode frequencies increase.)

3.1.2. Heavy-ion EICI.. Here we consider the excitation of the fundamental and higher harmonics of the heavy-ion EIC mode. Similarly to the case of the light-ion EICI, we assume that $(\omega - m\Omega_{\mathrm{h}}) \gg k_z v_{\mathrm{h}}$ for each mth harmonic. Then from (5) the heavy negative ion susceptibility has the same form as (9) for the light ions, that is

$$\chi_{\mathrm{h}} \approx \frac{1}{k^2\lambda_{\mathrm{Dh}}^2} \left\{ 1 - \sum_m \frac{\omega\Gamma_m(b_{\mathrm{h}})}{\omega - m\Omega_{\mathrm{h}}} + i\sqrt{\frac{\pi}{2}} \sum_m \frac{\omega\Gamma_m(b_{\mathrm{h}})}{k_z v_{\mathrm{h}}} e^{-\zeta_{\mathrm{h}m}^2} \right\}. \tag{15}$$

Because the frequency of the heavy-ion EIC mode is less than Ω_{l}, we retain only the $m = 0$ term in the real part of the light-ion susceptibility in (5), which becomes

$$\chi_{\mathrm{l}} \sim \frac{1}{k^2\lambda_{\mathrm{Dl}}^2} \left(1 - \Gamma_0(b_{\mathrm{l}}) + i\sqrt{\frac{\pi}{2}} \frac{\omega}{k_z v_{\mathrm{l}}} \Gamma_0(b_{\mathrm{l}}) e^{-\zeta_{\mathrm{l}0}^2} \right). \tag{16}$$

Using (8), (15), and (16) we can use (7a) to obtain the real part of the frequency $\omega_{\mathrm{r}} = m\Omega_{\mathrm{h}}(1 + \Delta_{\mathrm{h}m})$. This yields

$$\Delta_{\mathrm{h}m} \approx \frac{\epsilon_{\mathrm{h}}\tau_{\mathrm{eh}}\Gamma_m(b_{\mathrm{h}})}{1 + k^2\lambda_{\mathrm{De}}^2 + \epsilon_{\mathrm{h}}\tau_{\mathrm{eh}}[1 - G(b_{\mathrm{h}})] + \delta\tau_{\mathrm{el}}[1 - \Gamma_0(b_{\mathrm{l}})]}, \tag{17}$$

where

$$G(b_{\mathrm{h}}) = \Gamma_m(b_{\mathrm{h}}) + \sum_{n \neq m} \frac{\omega\Gamma_n(b_{\mathrm{h}})}{\omega - n\Omega_{\mathrm{h}}}. \tag{18}$$

From (17), we can see that as ϵ_{h} and $\delta(= 1+\epsilon_{\mathrm{h}})$ increase, b_{h} and b_{l} should decrease in order to keep $\Delta_{\mathrm{h}m} < 1$. This trend is similar to the case of the light-ion EICI, although it can be more dramatic because of the additional term in the denominator of (17) due to light ions. For the fundamental heavy-ion harmonic, (17) becomes

$$\Delta_{\mathrm{h}1} \approx \frac{\epsilon_{\mathrm{h}}\tau_{\mathrm{eh}}\Gamma_1(b_{\mathrm{h}})}{1+k^2\lambda_{\mathrm{De}}^2+\epsilon_{\mathrm{h}}\tau_{\mathrm{eh}}[1-\Gamma_1(b_{\mathrm{h}})-\Gamma_0(b_{\mathrm{h}})]+\delta\tau_{\mathrm{el}}[1-\Gamma_0(b_{\mathrm{l}})]}. \tag{19}$$

To obtain (19), we retained only the $m=1,0$ harmonics for the heavy ions and the $m=0$ harmonic for the light ions.

The critical electron drift at marginal stability can be obtained in a similar way as the critical drift for the light-ion EICI (viz., (13)). Thus we have

$$\begin{aligned}\frac{u_{0\mathrm{e,c}}}{v_{\mathrm{h}}} \approx m(1+\Delta_{\mathrm{h}m})\xi_{\mathrm{h}}\Bigg[1+\epsilon_{\mathrm{h}}\left(\frac{T_{\mathrm{e}}}{T_{\mathrm{h}}}\right)^{3/2}\left(\frac{m_{\mathrm{h}}}{m_{\mathrm{e}}}\right)^{1/2} \\ \times\left\{\Gamma_m(b_{\mathrm{h}})\exp\left(-\frac{m^2\Delta_{\mathrm{h}m}^2\xi_{\mathrm{h}}^2}{2}\right)+\Gamma_{m+1}(b_{\mathrm{h}})\exp\left(-\frac{(m\Delta_{\mathrm{l}m}-1)^2\xi_{\mathrm{h}}^2}{2}\right)\right. \\ \left.+\frac{\delta}{\epsilon_{\mathrm{h}}}\left(\frac{T_{\mathrm{h}}}{T_{\mathrm{l}}}\right)^{3/2}\frac{\Gamma_0(b_{\mathrm{l}})}{\sqrt{M_r}}\exp\left(-\frac{m^2(1+\Delta_{\mathrm{h}m})^2\xi_{\mathrm{h}}^2}{2}\frac{v_{\mathrm{h}}^2}{v_{\mathrm{l}}^2}\right)\right\}\Bigg],\end{aligned} \tag{20}$$

where

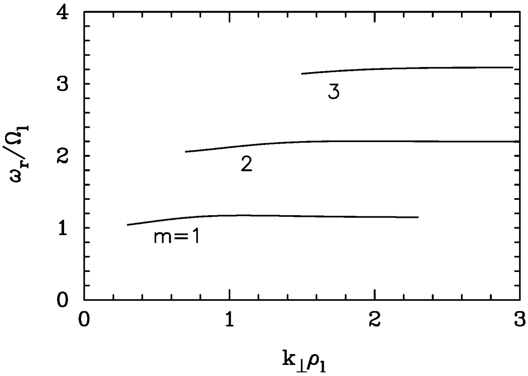
$$\xi_{\mathrm{h}} = \frac{1}{b_{\mathrm{h}}^{1/2}}\frac{k_\perp}{k_z}.$$

The condition $u_{0\mathrm{e}} > \omega/k_z$, which is reflected in the first term in the brackets in (20), is easier to satisfy compared with the corresponding condition for the light-ion mode, basically because the frequency of the heavy-ion mode is smaller. The second and third terms in the brackets in (20) correspond to heavy-ion cyclotron damping (due to the m and $m+1$ harmonics), while the last term corresponds to light-ion cyclotron damping. In order for the ion cyclotron damping terms to be small, the corresponding exponential terms should be small. This implies the condition $\zeta_{\mathrm{h}} \gg 1/m\Delta_{\mathrm{h}m}$ to minimize the m harmonic contribution to heavy-ion cyclotron damping. To minimize light-ion cyclotron damping requires roughly the condition $\zeta_{\mathrm{h}} \gg v_{\mathrm{l}}/mv_{\mathrm{h}}$, which can actually be more constraining than the former condition, since generally $v_{\mathrm{l}} \gg v_{\mathrm{h}}$. Thus we might expect a narrower range of unstable wavenumbers compared with the light-ion EICI, due to the light-ion cyclotron damping of the heavy-ion modes. In addition, as ϵ_{h} increases the mode frequency increases and so the $m+1$ harmonic can contribute to heavy-ion cyclotron damping, further limiting the range of unstable wavenumbers.

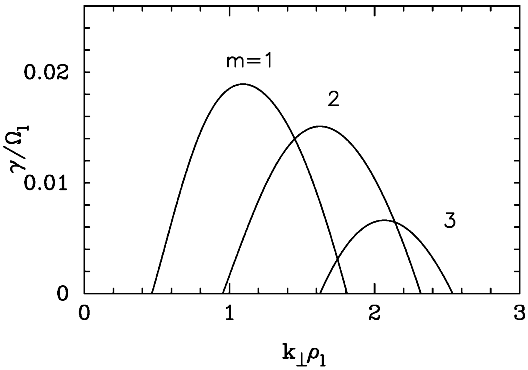
We can obtain the growth rate for the heavy-ion EICI in the limit $u_{0\mathrm{e}} \gg u_{0\mathrm{e,c}}$, in the same way we obtained the growth rate (14) for the light-ion EICI. Neglecting ion cyclotron damping terms, (7b) yields, using (8) and (15),

$$\frac{\gamma}{\Omega_{\mathrm{h}}} \approx \frac{1}{\epsilon_{\mathrm{h}}\tau_{\mathrm{eh}}}\frac{m\Delta_{\mathrm{h}m}^2}{\Gamma_m(b_{\mathrm{h}})}\sqrt{\frac{\pi}{2}}\left(\frac{u_{0\mathrm{e}}}{v_{\mathrm{e}}}-\frac{\omega_{\mathrm{r}}}{k_z v_{\mathrm{e}}}\right). \tag{21}$$

To obtain (21), we have assumed that $\partial D_r(\omega_{\mathrm{r}},k)/\partial\omega_{\mathrm{r}}$ is determined from the mth harmonic term contribution to χ_{h}. Note that the latter is not always a good approximation. For example, when the mode frequencies increase substantially so that $\Delta_{\mathrm{h}m} > 1/2$, the $m+1$ harmonic can contribute to $\partial D_{\mathrm{r}}/\partial\omega_{\mathrm{r}}$.



(a)



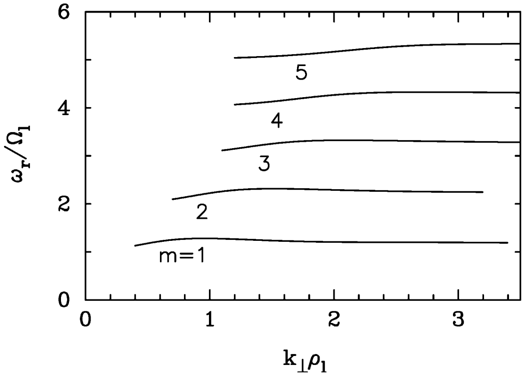
(b)

Figure 1. (a) Real ω_r and (b) imaginary γ parts of ω (normalized to Ω_l) for the light-ion EICI versus $k_\perp \rho_l$ obtained by solving (3). Parameters are: $m_l/m_p = 39$, $m_h/m_p = 350$, $T_e/T_l = 1$, $T_e/T_h = 8$, $\omega_{pl}/\Omega_l = 9$, $\theta = 88°$ where $\tan\theta = k_\perp/k_z$, and fixed $u_{0e}/v_e = 0.25$. Here $\epsilon_h = n_h/n_e = 0$.

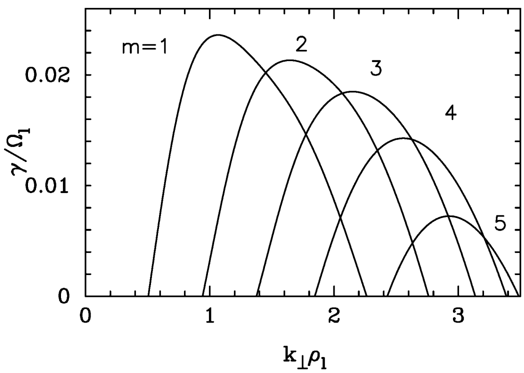
3.2. Numerical results

In this section, we show solutions of (3) for parameters that may be representative of laboratory negative ion plasmas discussed by Kim and Merlino (2007). We consider a plasma in which the light ions are singly ionized potassium K^+ and the heavy ions are $C_7F_{14}^-$. Thus the ratio of the heavy-ion to light-ion mass is $M_r = 350/39 \approx 9$. We assume that the electron temperature is $T_e \sim 0.2$ eV, and that $\tau_{el} = T_e/T_l \sim 1$, and that the temperature of the heavy negative ions is near room temperature, so that $\tau_{eh} = T_e/T_h \sim 8$. It is further assumed that the plasma is immersed in a magnetic field of strength $B \sim 0.3$ T. In this case, the light-ion gyroradius $\rho_l \sim 0.095$ cm and the heavy-ion gyroradius is about $\rho_h \sim 0.1$ cm. We take a light-ion density of $n_l \sim 1 \times 10^9$ cm^{-3}, so that $\omega_{pl}/\Omega_l \sim 9$, where ω_{pl} is the plasma frequency of the light ions. We note that, in the type of plasma we are considering, $u_{0e}/v_e \sim u_{0l}/v_l$. Thus if we confine our attention to the parameter regime where $u_{0e}/v_e < 1$ we also have $u_{0l}/v_l < 1$. In this case, we would expect that the heavy-ion EIC mode could not become excited by drifting light ions (see the discussion at the beginning of Sec. 3.1).

Figures 1 and 2 show the frequency and growth rate of the light-ion EICI versus $k_\perp \rho_l$, for the following parameters: $M_r \approx 350/39$, $T_e = T_l = 8T_h$, $\epsilon_h = 1$, $\omega_{pl}/\Omega_l = 9$,



(a)



(b)

Figure 2. (a) Real ω_r and (b) imaginary γ parts of ω (normalized to Ω_l) for the light-ion EICI versus $k_\perp\rho_l$ obtained by solving (3). Parameters are the same as in Fig. 1, except that $\epsilon_h = 1$. The curves correspond to different harmonics.

electron drift speed $u_{0e}/v_e = 0.25$, and $\theta = 88°$, where θ is the angle between **k** and **B** ($\tan\theta = k_\perp/k_z$, so $\theta = 88°$ corresponds to $k_\perp/k_z \approx 28.6$). Figure 1 shows results when there are no negative ions present, with $\epsilon_h = 0$. As can be seen, harmonics up to the third harmonic can be excited for this set of parameters. Figure 2 shows results for the light-ion EICI when there are negative ions present, with $\epsilon_h = 1$. The addition of negative ions leads to the appearance of higher harmonics beyond the third harmonic, an increase in the frequency of the modes, and increased growth rates. In addition, as discussed in Sec. 3.1.1, as ϵ_h increases, the range of unstable $k_\perp/k_z$ can increase. This is illustrated in Fig. 3 which shows growth rates for the fundamental mode at fixed $k_\perp\rho_l = 1$ as a function of θ, where $\tan\theta = k_\perp/k_z$. Figure 4 shows the approximate critical drift for exciting the fundamental light-ion EIC mode for fixed $k_\perp\rho_l = 0.3$ as a function of ϵ_h and the corresponding mode frequency. As can be seen, the mode frequency increases while the critical drift decreases as ϵ_h increases up to about $\epsilon_h \sim 20$ beyond which there is a slow turnaround to larger critical drifts, reflecting the increase of ion cyclotron damping due to the $m + 1$ harmonic.

Figure 5 shows the frequency and growth rate for the light-ion EICI for larger $\epsilon_h = 10$, and at a smaller value of $\theta = 85°$ (corresponding to $k_\perp/k_z \approx 11.4$). As can be seen, as ϵ_h increases even higher harmonics can be excited. In addition, the modes shift to smaller k, and the frequencies increase.

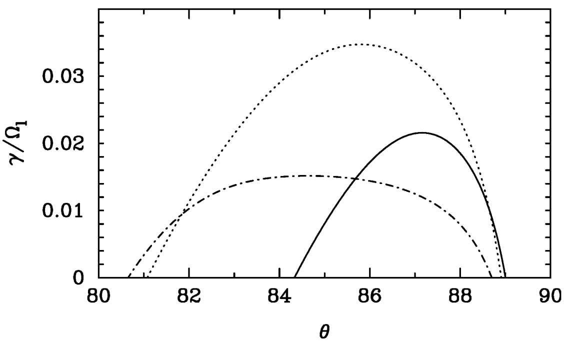


Figure 3. Imaginary γ parts of ω (normalized to Ω_l) for the light-ion EICI for the fundamental harmonic versus θ where $\tan\theta = k_\perp/k_z$, for fixed $k_\perp\rho_l = 1$, obtained by solving (3). Other parameters are the same as in Fig. 1, except that the curves correspond to different values of ϵ_h: $\epsilon_h = 0$ (solid curve), $\epsilon_h = 1$ (dotted curve), $\epsilon_h = 10$ (dash-dotted curve).

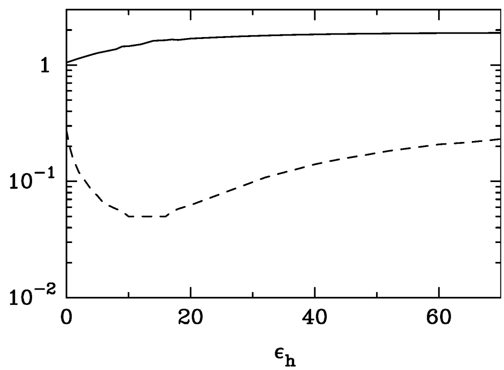


Figure 4. Normalized critical electron drift $u_{0e,c}/v_e$ (dashed curve) and corresponding normalized real frequency ω_r/Ω_l (solid curve) for the fundamental light-ion EICI versus ϵ_h obtained using (3). Other parameters are the same as in Fig. 1, except that $b_l^{1/2} = 0.3$ is fixed and θ is varied.

Figure 6 shows the frequency and growth rate of the heavy-ion EICI versus $k_\perp\rho_h$, for the same parameters as in Fig. 1 but with $\epsilon_h = 1$. As ϵ_h increases, the mode frequency increases and the mode shifts to smaller $k_\perp$. These effects are illustrated in Figs 7. Figure 7 uses the same parameters as in Fig. 6, but with an increased $\epsilon_h = 10$. As can be seen, as ϵ_h increases the mode frequencies increase, the unstable modes shift to smaller wavenumbers (at least for fixed $k_\perp/k_z$), and the growth rates decrease for the higher harmonic modes. Figure 8 shows growth rates for the fundamental mode at fixed $k_\perp\rho_h = 0.5$ as a function of θ, where $\tan\theta = k_\perp/k_z$. The decreased range of unstable $k_\perp/k_z$ as ϵ_h increases, as compared with the light-ion EICI, may arise for several reasons. First, the heavy-ion cyclotron damping term in (20) is proportional to both ϵ_h and τ_{eh}, the latter already being large ($\sim$8) for our choice of parameters. Secondly, the mode frequency increases substantially as ϵ_h increases, so that $k_\perp/k_z$ has to increase to minimize damping by the $m+1$ cyclotron harmonic. Finally, there is also damping by the light ions. Figure 9 shows the approximate critical drift and corresponding mode frequency

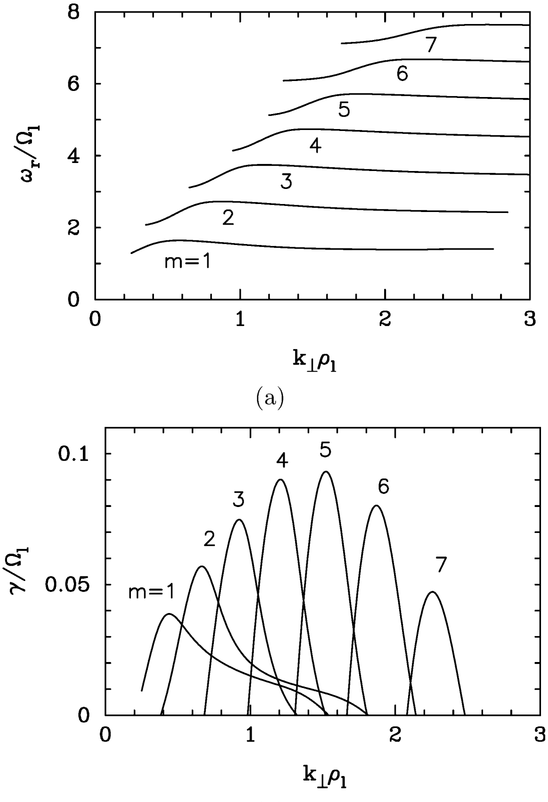


Figure 5. (a) Real ω_r and (b) imaginary γ parts of ω (normalized to Ω_l) for the light-ion EICI versus $k_\perp\rho_l$ obtained by solving (3). Parameters are the same as in Fig. 1, except that $\theta = 85°$ and $\epsilon_h = 10$. The curves correspond to different harmonics.

for the fundamental heavy-ion mode for fixed $k_\perp\rho_h = 0.3$ as a function of ϵ_h. As can be seen, the mode frequency increases while the critical drift decreases as ϵ_h increases up to about $\epsilon_h \sim 8$ beyond which there is a slow turnaround to larger critical drifts, reflecting the increase of ion cyclotron damping due to the $m+1$ harmonic as the mode frequency increases, similar to the light-ion EICI.

4. Summary and discussion

We have investigated the EICI in a magnetized plasma containing electrons, heavy negative ions, and light positive ions, using linear kinetic theory. We have considered the excitation of the fundamental mode and higher harmonic modes associated with each ion species, focusing on the regime where the negative ion density is larger than the electron density. It was found that, in general, as the density of negative ions increases, the wave frequencies increase and the unstable spectrum shifts to longer wavelengths. In addition, instability can occur over a wider range of $k_\perp/k_z$, where $k_\perp$ and k_z are the components of the wavevector perpendicular and parallel, respectively, to the magnetic field. For fixed perpendicular wavelength,

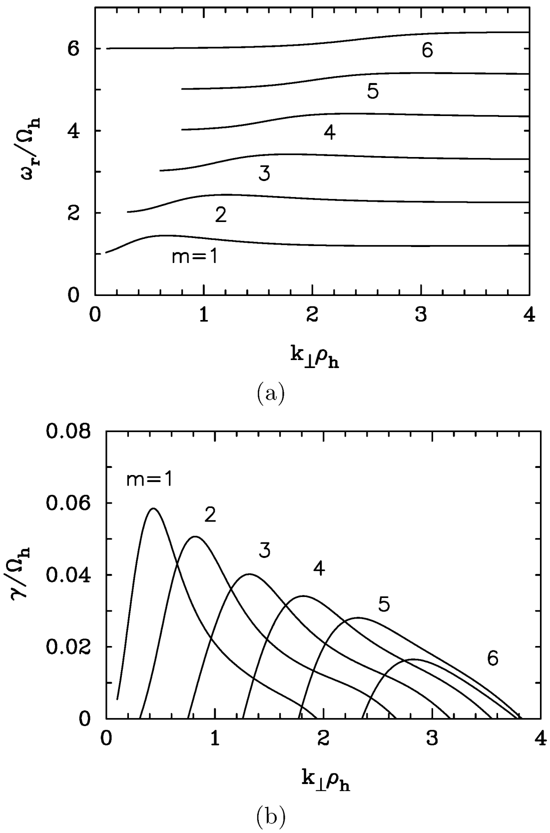
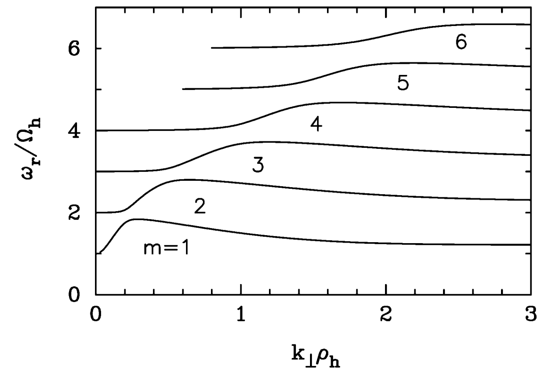


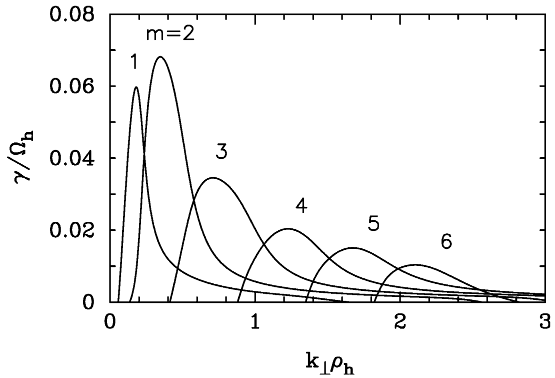
Figure 6. (a) Real ω_r and (b) imaginary γ parts of ω (normalized to Ω_h) for the heavy-ion EICI versus $k_\perp\rho_h$ obtained by solving (3). Parameters are the same as in Fig. 1, except that $\epsilon_h = 1$. The curves correspond to different harmonics.

it appears that the critical drift for both the light-ion and heavy-ion EICI of the $m = 1$ harmonic decreases as the density of negative ions increases, up to a point where the frequency approaches the second harmonic. In general, it appears that it is easier to excite the fundamental and higher harmonic EIC modes of both ion species in the presence of a large (but not too large) density of negative ions. The results seem to have qualitative similarity with the recent experimental results of Kim et al. (2008). Future work will compare the theoretical and experimental results in more detail. We note that this may involve including collisional damping effects as well.

However, we point out that when ϵ_h gets very large, there is a relatively small amount of energy in the streaming electrons, so that one might expect that the energy density of any excited waves to also be relatively small. It should also be noted that for very large ϵ_h one might expect the instability of the higher frequency 'fast' ion-acoustic wave to occur as well. For large δ, the phase speed of the ion acoustic wave can be much greater than v_l even in a plasma with $T_e = T_l$, so the wave can be driven unstable by relatively small electron drifts since ion Landau damping would be negligible (see, e.g., Rosenberg and Merlino 2007).



(a)



(b)

Figure 7. (a) Real $\omega_{\rm r}$ and (b) imaginary γ parts of ω (normalized to $\Omega_{\rm h}$) for the heavy-ion EICI versus $k_{\perp}\rho_{\rm h}$ obtained by solving (3). Parameters are the same as in Fig. 6, except that $\epsilon_{\rm h} = 10$. The curves correspond to different harmonics.

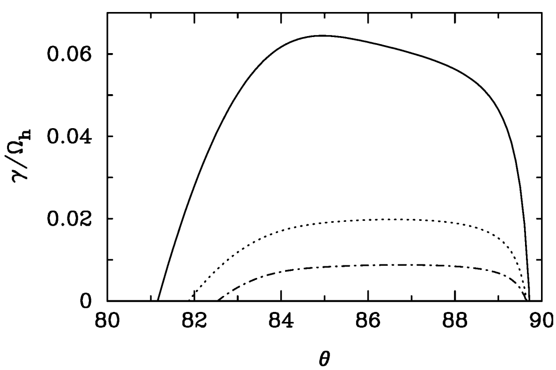


Figure 8. Imaginary γ parts of ω (normalized to $\Omega_{\rm h}$) for the heavy-ion EICI for the fundamental harmonic versus θ where $\tan\theta = k_{\perp}/k_z$, for fixed $k_{\perp}\rho_{\rm h} = 0.5$, obtained by solving (3). Other parameters are the same as in Fig. 1, except that the curves correspond to different values of $\epsilon_{\rm h}$: $\epsilon_{\rm h} = 1$ (solid curve), $\epsilon_{\rm h} = 5$ (dotted curve), $\epsilon_{\rm h} = 10$ (dash-dotted curve).

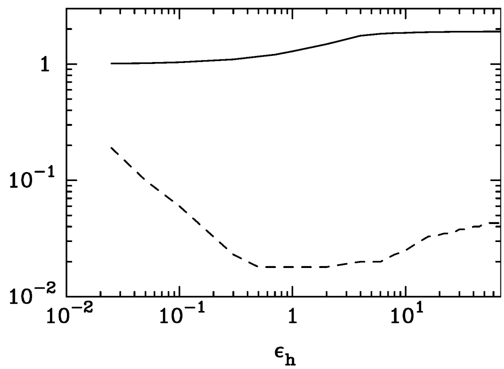


Figure 9. Normalized critical electron drift $u_{0e,c}/v_e$ (dashed curve) and corresponding normalized real frequency ω_r/Ω_h (solid curve) for the fundamental heavy-ion EICI versus ϵ_h obtained using (3). Other parameters are the same as in Fig. 1, except that $b_h^{1/2} = 0.3$ is fixed and θ is varied.

Acknowledgements

This work was partially supported by the DOE grants DE-FG02-04ER54804 (MR) and DE-FG02-04ER54795 (RLM).

References

Barkan, A., D'Angelo, N. and Merlino, R. L., 1995 *Planet. Space Sci.* **43**, 905.

Chow, V. W. and Rosenberg, M. 1995 *Planet. Space Sci.* **43**, 613.

Chow, V. W. and Rosenberg, M. 1996a *Phys. Plasmas* **3**, 1202–1211.

Chow, V. W. and Rosenberg, M. 1996b *Planet. Space Sci.* **44**, 465.

D'Angelo, N. and Merlino, R. L. 1986 *IEEE Trans. Plasma Sci.* **14**, 285.

D'Angelo, N. 1990 *Planet. Space Sci.* **38**, 1143.

D'Angelo, N. 1998 *Planet. Space Sci.* **46**, 1671.

Drummond, W. E. and Rosenbluth, M. N. 1962 *Phys. Fluids* **5**, 1507.

Fried, B. D. and Conte, S. 1961 *The Plasma Dispersion Function*. Academic Press, NY.

Kim, S.-H. and Merlino, R. L. 2006 *Phys. Plasmas* **13**, 052118.

Kim, S.-H. and Merlino, R. L. 2007 *Phys. Rev.* E **76**, 035401.

Kim, S.-H., Heinrich, J. R. and Merlino, R. L. 2008 Electrostatic ion-cyclotron waves in a plasma with heavy negative ions. *Planet. Space Sci.* doi:10.1016/j.pss.2008.07.020.

Kindel, J. M. and Kennel, C. F. 1971 *J. Geophys. Res.* **76**, 3055.

Krall, N. A. and Trivelpiece, A. W. 1973 *Principles of Plasma Physics*, McGraw-Hill, NY.

Merlino, R. L. and Kim, S.-H. 2006 *Appl. Phys. Lett.* **89**, 091501.

Okuda, H., Cheng, C. Z. and Lee, W. W. 1981 *Phys. Plasmas* **24**, 1060–1068.

Rapp, M. et al. 2005 *Geophys. Res. Lett.* **32**, L23821.

Rasmussen, J. J. and Schrittwieser, R. W. 1991 *IEEE Trans. Plasma Sci.* **19**, 457–501.

Rosenberg, M. and Chow, V. W. 1999 *J. Plasma Phys.* **61**, 51.

Rosenberg, M. and Merlino, R. L. 2007 *Planet. Space Sci.* **55**, 1464.

Rosenbluth, M. N. 1965 Microinstabilities. *Plasma Physics*. Vienna: IAEA, pp. 485–514.

Satyanarayana, P., Chaturvedi, P. K., Keskinen, M. J., Huba, J. D. and Ossakow, S. L. 1985 *J. Geophys. Res.* **90**, 2209.

Song, B., Suszcynsky, D., D'Angelo, N. and Merlino, R. L. 1989 *Phys. Fluids* B **1**, 2316.

Sorasio, G. and Rosenberg, M. 2001 *J. Plasma Phys.* **65**, 319.