Improved theoretical approximation for the ion drag force in collisionless plasma with strong ion-grain coupling

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We point out a deficiency in our previous analytic calculation of the ion drag force for conditions of the experiment by Nosenko *et al.* [Phys. Plasmas **14**, 103702 (2007)]. An inaccurate approximation is corrected and the ion drag force is recalculated. The improved model yields better overall agreement with the experimental results as compared to the original calculation. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3122049]

In a recent paper Nosenko *et al.*[1] measured the ion drag force acting on dust grains of different sizes in a low-pressure (collisionless) argon plasma. In this experiment the plasma was generated in a hot-filament direct current discharge with multidipole magnetic confinement. The grains (spherical particles) were dropped into the plasma and allowed to fall due to gravity. Their trajectories were deflected by the ion drag force associated with the momentum transfer from argon ions drifting in the axial ambipolar electric field naturally present in the discharge. Since other forces acting in the axial direction were negligibly small in this experiment, the ion drag force could be easily calculated from the angle of grain deflection.

Measurements of the plasma parameters made using a planar Langmuir probe allowed us to perform a detailed comparison with theory. Although a reasonable agreement between theory and experiment had been reported, it turned out later that one of the approximations used in the theoretical model was inappropriate. The purpose of this brief communication is to point out a flaw in our previous argumentation and report the recalculated theoretical values of the ion drag force. We demonstrate that the improved model yields an overall better agreement with the experimental results.

First, let us briefly remind the main features of the experiment described in Ref. 1, which determine the choice of the approximations employed in the theoretical model. The experiment was performed at very low pressures of argon in the range of 0.2–0.8 mTorr. At these pressures the ion mean free path is about three orders of magnitude longer than the ion Debye radius, which ensures that any collisional effects on either grain charging[2–5] or ion drag force[6–8] can be ignored. The ion drift velocities in the experiment are in the (sub)thermal range, which implies weak distortion of the potential isotropy around the grain and weak deviation of the ion velocity distribution from the shifted Maxwellian function.[9] The grains used in the experiment are rather big: 40 and 59 μm in diameter. Therefore, the charge of the grains, which is approximately proportional to their diameter, is rather high. This implies strong ion-grain coupling: the characteristic length of ion-grain interaction exceeds considerably the ion Debye radius and most of the ion scattering occurs with large angles in a wide grain vicinity.[9] The most natural way to describe the momentum transfer and the ion drag force in the considered parameter regime is the binary collision (BC) approach.[9]

An expression for the ion drag force derived in Ref. 1 using BC approach is

$$F_{\rm id} \approx \sqrt{2\pi} n m v_{T_i}^2 \rho_*^2 \left\{ \sqrt{\frac{\pi}{2}}(2+u_0^2+u_0^{-2})\,\mathrm{erf}\left(\frac{u_0}{\sqrt{2}}\right) + (u_0+u_0^{-1})\exp\left(-\frac{u_0^2}{2}\right)\right\}, \quad (1)$$

where n and m are the ion density and mass, $v_{T_i}=\sqrt{T_i/m}$ is the ion thermal velocity, $u_0=u/v_{T_i}$ is the normalized ion drift velocity, and ρ_* is a characteristic length scale of the ion-grain interaction. The latter can be expressed through the *effective* plasma screening length λ and scattering parameter β as

$$\rho_* \approx \lambda[\ln\beta + 1 - (2\ln\beta)^{-1}],$$

$$\beta = \frac{e|\phi_s|a}{\lambda T_i(1+u_0^2)},$$

where ϕ_s is the grain surface potential and a is the grain radius. Expression (1) was obtained as follows. The momentum transfer cross section for the attractive Debye–Hückel (Yukawa) potential $U(r)\propto \exp(-r/\lambda)/r$ in the limit of strong interaction obtained in Ref. 10 was integrated over the shifted Maxwellian distribution function of ions. The integration could be simplified by making use of the weak logarithmic dependence of the cross section on ion velocity. With a reasonable accuracy we replaced the ion velocity with an "average" velocity $v_{T_i}\sqrt{1+u_0^2}$ under the logarithm when integrating.

Most of the parameters required to evaluate the ion drag force from Eq. (1) are either measured in the experiment (e.g., n and T_e) or can be easily calculated from those. For instance, the ion drift velocity was calculated from the measured values of the electric field; the grain surface potential was calculated using the orbit motion limited approximation. The main uncertainty is the relation between the effective

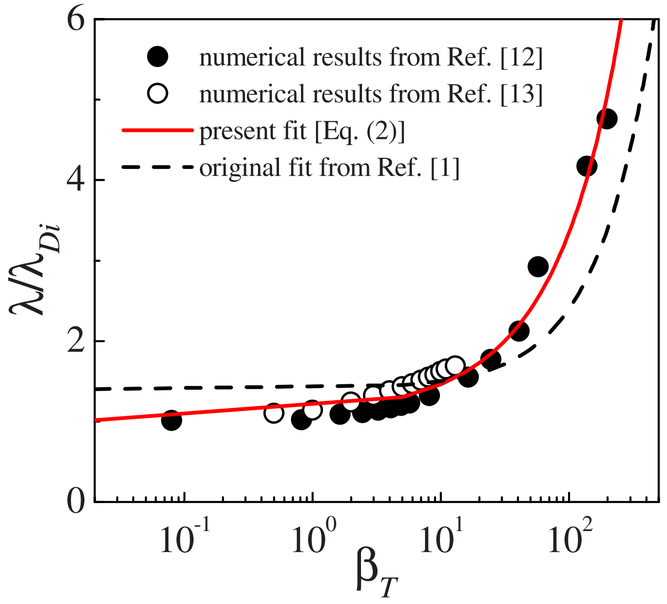

FIG. 1. (Color online) Ratio of the effective screening length to the ion Debye radius $\lambda/\lambda_{\mathrm{Di}}$ as a function of the thermal ion scattering (nonlinearity) parameter $\beta_T = (e|\phi_s|a/T_i\lambda_{\mathrm{Di}})$. Solid (open) circles correspond to numerical calculations of Ref. 12. (Ref. 13). In both cases λ is obtained from the best fit of the numerically calculated potential with the Debye–Hückel (Yukawa) expression. To obtain the dependence of $\lambda/\lambda_{\mathrm{Di}}$ on β_T from the data presented in Ref. 12 we assumed $\lambda_{\mathrm{Di}} = \sqrt{2\mathcal{E}_i/4\pi e^2 n_0}$, where $\mathcal{E}_i$ is the energy of monoenergetic ions, which yields the correct length scale for screening in the weakly ion-grain coupling (linear) regime and similarly assumed $\beta_T = (e|\phi_s|a/2\mathcal{E}_i\lambda_{\mathrm{Di}})$. The solid curve in the figure corresponds to the fit of Eq. (2). The dashed curve shows the original fit used in Ref. 1

screening length and the actual complex plasma parameters (e.g., grain charge, size, ion and electron Debye radii, etc). Two factors can influence the value of λ as follows:

(i) The effective screening length depends on the ion flow velocity. In Ref. 11 (see also Ref. 9) it was shown that at $u_0 \lesssim 1$ the flow-induced anisotropy of the potential is weak and the screening is determined by the linearized Debye radius $\lambda \simeq \lambda_{\mathrm{Di}}/\sqrt{1+(\lambda_{\mathrm{Di}}/\lambda_{\mathrm{De}})^2} \simeq \lambda_{\mathrm{Di}}$, where $\lambda_{\mathrm{Di}} = \sqrt{T_i/4\pi e^2 n}$ is the ion Debye radius. At highly superthermal ion flows $u_0 \gg 1$ the ion contribution to the screening vanishes and the effective screening length tends to the electron Debye radius $\lambda \simeq \lambda_{\mathrm{De}}$. Since in the considered experiment mostly (sub)thermal ion flow regime was studied, we neglected the effect of ion flow velocity on the effective screening length.

(ii) The effective screening length depends on the strength of ion-grain coupling,[1,12,13] and this is the main point of this brief Communication. In Ref. 1 we used a fit to the numerical results by Daugherty *et al.*[12] to evaluate λ, but unfortunately without proper adaptation. The point is that the numerical calculations in Ref. 12 were performed assuming *monoenergetic* ions. An appropriate matching between the case of monoenergetic and Maxwellian ions would be to substitute $T_i = 2\mathcal{E}_i$ in the expression for the ion Debye radius[14] (here $\mathcal{E}_i$ is the energy of monoenergetic ions). This yields the correct length scale for screening in

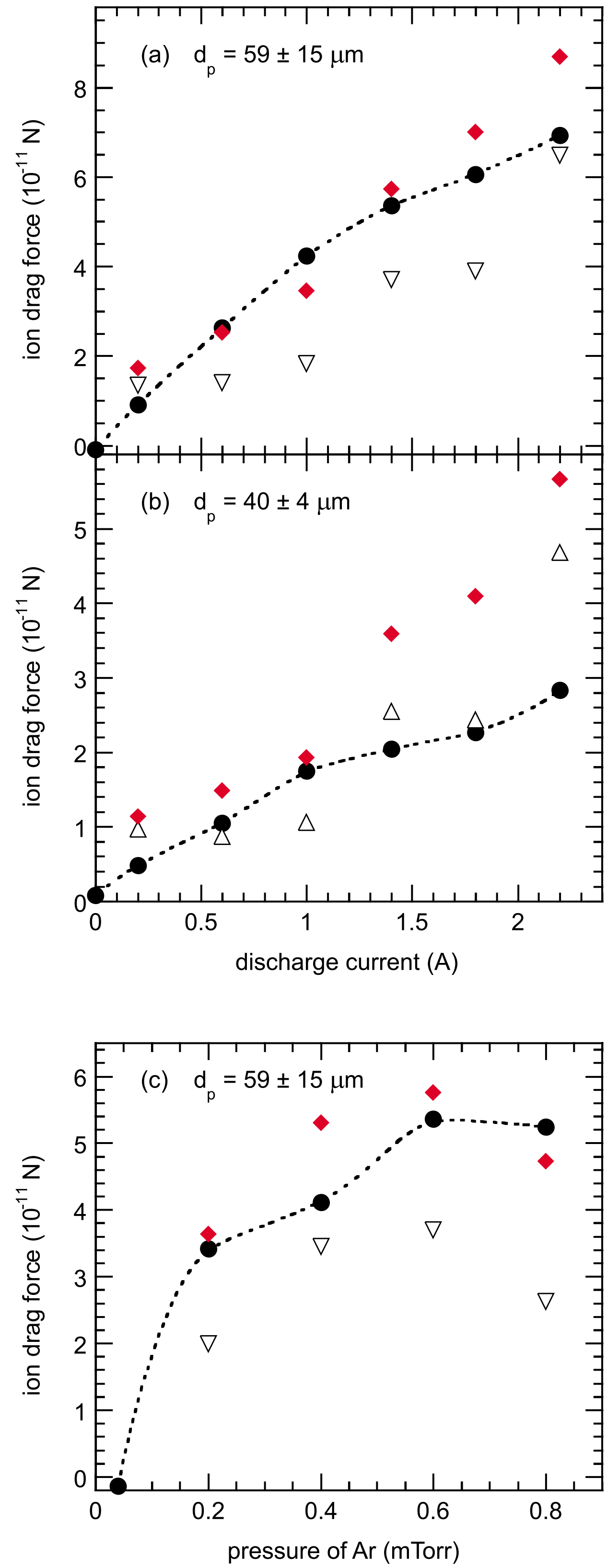

FIG. 2. (Color online) Ion drag force as a function of [(a) and (b)] discharge current and (c) discharge pressure. Experimental measurements in Ref. 1 for the grains of diameters [(a) and (c)] $d_p = 59 \pm 15$ μm and (b) $d_p = 40 \pm 4$ μm are shown by solid circles. Solid diamonds correspond to the improved analytic model of this paper. Open triangles correspond to the original analytic calculation of Ref. 1.

the weak ion-grain coupling (linear) regime, where $\lambda \simeq \lambda_{\mathrm{Di}}$. Similar substitution should be used in evaluating the scattering parameter β. This has been recently recognized in Ref. 15 but not in Ref. 1, where we used $T_i = \mathcal{E}_i$. The improved fit for the effective screening length is[15]

$$\lambda = \lambda_{\mathrm{Di}}(1 + 0.105\sqrt{\beta_T} + 0.013\beta_T), \quad (2)$$

where $\beta_T = (e|\phi_s|a/\lambda_{\mathrm{Di}}T_i)$ is the ion thermal scattering parameter. This new fit is shown in Fig. 1 along with the original fit used in Ref. 1.

The results of calculating the ion drag force using the approximation of Eq. (2) are shown in Fig. 2 along with the experimental results and original theoretical calculations. With this new fit the theoretical values of the ion drag force are somewhat higher and the overall agreement between the theory and experiment clearly improves. Note, however, that the agreement is considerably better for the large grains than for the small ones. A possible explanation for this is that the approximation used for the momentum transfer cross section was obtained for the limiting case of strong ion-grain coupling.[10] It should obviously work better for larger grains. In addition, in integrating this momentum transfer cross section over the shifted Maxwellian ion velocity distribution function, we neglected the weak logarithmic velocity dependence of the cross section. This works better for high β, i.e., again for larger grains.

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