

A note on dust wave excitation in a plasma with warm dust: Comparison with experiment

M. Rosenberg,¹ E. Thomas, Jr.,² and R. L. Merlino³

¹Department of Electrical and Computer Engineering, University of California San Diego, La Jolla, California 92093, USA

²Physics Department, Auburn University, Auburn, Alabama 36849, USA

³Department of Physics and Astronomy, The University of Iowa, Iowa City, Iowa 52242, USA

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Ion-dust streaming instability in a plasma containing dust grains with large thermal speeds is considered using kinetic theory. The results are compared with experimental measurements of the dispersion relation of dust acoustic waves performed at the University of Iowa and Auburn University. © 2008 American Institute of Physics. [DOI: 10.1063/1.2943218]

I. INTRODUCTION

Dust acoustic waves are very low frequency modes associated with the dynamics of charged dust in a plasma.^{1,2} These waves have phase speed much smaller than the ion thermal speed and frequencies typically below the dust plasma frequency which can be on the order of hertz to hundreds of hertz for typical lab dusty plasma parameters. When the phase speed of the wave is much larger than the thermal speed of the dust grains (i.e., cold dust), the dispersion relation of a dust acoustic wave (DAW) is given by¹

$$\omega = \frac{k\lambda_D\omega_{pd}}{(1+k^2\lambda_D^2)^{1/2}}. \quad (1)$$

Here, $\omega_{pd}=(4\pi Z_d^2 e^2 n_d/m_d)^{1/2}$ is the dust plasma frequency (Z_d , n_d , and m_d are the charge state, density, and mass of the dust, respectively), and $\lambda_D^{-1}=(\lambda_{Di}^{-2}+\lambda_{De}^{-2})^{1/2}$ is the linearized Debye length in the background plasma (λ_{Di} and λ_{De} are the ion and electron Debye lengths, respectively). Dc glow or rf laboratory dusty plasmas typically have $T_e\sim 2-3$ eV, while $T_i\sim$ room temperature (here, T_j is the temperature of particle species j , where $j=d,i,e$ denotes charged dust, ions, and electrons, respectively) so that generally, $\lambda_D\sim\lambda_{Di}$.

Dust acoustic waves have been studied in a number of experiments, typically in the long wavelength regime, where $k\lambda_D\ll 1$.³⁻¹³ Recently, measurements of the dispersion relation of dust waves in the shorter wavelength regime, where $k\lambda_D\lesssim 1$ have been reported.¹⁴ It was found that the mode frequency did not turn over and approach ω_{pd} as the wavenumber k increased, as would be predicted by the dust acoustic dispersion relation given in Eq. (1). It was therefore proposed that dust thermal effects were important for that experiment. Corroborating data on the velocity distribution of dust was obtained by Williams and Thomas¹⁵; their studies in dc glow discharge dusty plasmas show that the particle clouds can have kinetic temperatures in excess of 10 eV.

This note presents a comparison of measurements of the DAW dispersion relation performed at the University of Iowa and Auburn University with the theoretical dispersion relation derived from kinetic theory. In contrast to some previous works that compare fluid or kinetic

theories of ion-dust streaming instabilities with experimental data and that assume cold dust (e.g., Refs. 8, 9, 13, and 16-18), we take into account finite dust temperature.

II. ANALYSIS

The ratio of the phase speed of a dust wave to the dust thermal speed $v_d=(T_d/m_d)^{1/2}$ can be written as

$$\frac{\omega}{kv_d} = \frac{\omega}{\omega_{pd}} \frac{1}{k\lambda_{Di}} \alpha, \quad (2)$$

where

$$\alpha = \left(\frac{T_i Z_d^2 n_d}{T_d n_i} \right)^{1/2}, \quad (2')$$

which is basically the ratio of the dust acoustic speed, $\sim\lambda_{Di}\omega_{pd}$, to v_d . For the case when the dust has relatively low kinetic energy, with $T_d\sim T_i$, α is typically $\gg 1$ for typical values of Z_d and n_d/n_i in laboratory dusty plasmas. For example, assuming $Z_d\sim 2000$ and $n_d/n_i\sim 10^{-4}$ yields $\alpha\sim 20$ so that ω/kv_d can be large even for $k\lambda_{Di}\sim 1$, as can be seen from Eq. (2). This implies weak dust Landau damping for these modes.

However, if a situation occurs where the dust has high kinetic energy, so that $T_i/T_d\ll 1$, α can be on the order of unity, and thus $\omega/k\sim v_d$ for dust waves with $k\lambda_{Di}$ near unity. Such modes could undergo strong dust Landau damping. This would imply that there would be no turnover in the dispersion relation as predicted by Eq. (1), because when strong Landau damping occurs the magnitudes of both the real and imaginary parts of ω increase with k , with the imaginary part being negative (see, e.g., Ref. 19 for the analogous ion-acoustic waves in an electron-ion plasma). However, dust waves could still grow unstable if there is a strong driving mechanism such as ions streaming relative to the dust with speed larger than the ion thermal speed v_i . We note that an analogous electron-ion streaming instability (a type of Buneman instability²⁰) in a standard plasma was considered in Ref. 21 for the case $T_e=T_i$ and in Ref. 22 for $T_e\ll T_i$.

Here we consider an ion-dust streaming instability, in the regime where $T_d\gg T_e$, T_i/T_d , and where the ion drift speed u_{0i} is

$\geq v_i$. We assume that there is an electric field E_0 in the plasma that imparts a drift to the electrons and ions of magnitude

$$u_{0j} \approx \frac{Z_j e E_0}{m_j \nu_j}. \quad (3)$$

Here, Z_j , m_j , and ν_j are, respectively, the charge state, mass, and collision frequency of species j . In the following, we will consider the frame in which the dust is stationary, with $u_{0d}=0$, assuming that any dust drift is negligibly small.

Using drifting Maxwellians for the electron and ion velocity distributions, and a Maxwellian for the dust, and taking into account collisions, the linear dispersion relation for electrostatic waves with \mathbf{k} in the \mathbf{E}_0 direction is given by (see, e.g., Refs. 23–25)

$$1 + \sum_j \chi_j = 1 + \sum_j \frac{1}{k^2 \lambda_{Dj}^2} \frac{[1 + \zeta_j Z(\zeta_j)]}{[1 + (i\nu_j/\sqrt{2}kv_j)Z(\zeta_j)]} = 0, \quad (4)$$

where

$$\zeta_e = \frac{\omega + ku_{0e} + i\nu_e}{\sqrt{2}kv_e}, \quad (5a)$$

$$\zeta_i = \frac{\omega - ku_{0i} + i\nu_i}{\sqrt{2}kv_i}, \quad (5b)$$

$$\zeta_d = \frac{\omega + i\nu_d}{\sqrt{2}kv_d}. \quad (5c)$$

Here, $\lambda_{Dj} = (T_j/4\pi n_j Z_j^2 e^2)^{1/2}$, $\nu_j = (T_j/m_j)^{1/2}$ is the thermal speed, and $Z(\zeta)$ is the plasma dispersion function.²⁶ Collision rates are assumed to be due primarily to collisions with neutrals. For the electron-neutral and ion-neutral collision rates ν_e and ν_i we use $\nu_e \sim \sigma_{en} n_n \nu_e$ and $\nu_i \sim \sigma_{in} n_n \nu_i$, where n_n is the neutral density and σ_{in} (σ_{en}) is the cross section for collisions between ions (electrons) and neutrals.²⁷ For the dust-neutral collision rate ν_d we use the hard-sphere rate $\nu_d \sim 4R^2 n_n \nu_n (m_n/m_d)$, where R is the radius of the dust, and ν_n , n_n , and m_n are the thermal speed, number density, and mass of the neutrals, respectively.²⁸

A. Analytic results

Because we are interested in the regime where $u_{0i} \geq v_i$ and where dust Landau damping is important, $|\zeta_i|$ and $|\zeta_d|$ can be ≥ 1 , and there is no simple expansion possible of the plasma dispersion functions to provide analytic results for the dispersion relation given by Eq. (4). However, we consider some approximate limits to get an idea for the behavior of the wave frequency. Since we are considering waves with frequency around the dust plasma frequency, which is typically less than $\sim 10^3$ rad/s in laboratory dusty plasmas, we have $\omega \ll \nu_e$ and $\omega \ll \nu_i$ for neutral pressures on the order of 100 mTorr. For the electrons, we consider drift speeds $u_{0e} \ll \nu_e$, and wavelengths such that $\nu_e \ll kv_e$. Expanding the plasma dispersion function for small argument, the electron susceptibility in Eq. (4) becomes

$$\chi_e \approx \frac{1}{k^2 \lambda_{De}^2}. \quad (6)$$

For the ions, we consider the weakly nonresonant regime where $|\zeta_i| \geq 1$, and wavelengths such that $ku_{0i} > \nu_i \gg \omega$. In this case, the ion susceptibility in Eq. (4) becomes very roughly (see Ref. 29)

$$\chi_i \approx R - iD, \quad (7)$$

where

$$R \sim \frac{-\omega_{pi}^2}{(k^2 u_{0i}^2 + \nu_i^2)} \quad (7a)$$

and

$$D \sim \frac{1}{k^2 \lambda_{Di}^2} \sqrt{\frac{\pi}{2}} \frac{u_{0i}}{\nu_i} \exp\left(-\frac{u_{0i}^2}{2\nu_i^2}\right). \quad (7b)$$

To obtain Eq. (7b), it was assumed that $|\zeta_i| \leq 2$.²⁹ Finally, for the dust we consider the regime where $\omega > kv_d$ and assume weak collisions with $\nu_d \ll \omega$. Neglecting dust collisions for simplicity and expanding the plasma dispersion for large argument (not a good approximation in general for dust with high kinetic energy), the dust susceptibility in Eq. (4) becomes very roughly

$$\chi_d \sim -\frac{\omega_{pd}^2}{\omega^2} \left(1 + \frac{3k^2 v_d^2}{\omega^2}\right) + iF, \quad (8)$$

where

$$F = \epsilon \frac{\sqrt{\pi}}{k^2 \lambda_{Dd}^2} \zeta_d \exp(-\zeta_d^2), \quad (8')$$

where $\epsilon=0, 1$ for $\text{Im } \zeta_d > 0, =0$, respectively.²⁶

Using Eqs. (6)–(8) for the susceptibilities, Eq. (4) becomes

$$A - i(D - F) - \frac{\omega_{pd}^2}{\omega^2} \left(1 + \frac{3k^2 v_d^2}{\omega^2}\right) \approx 0. \quad (9)$$

Here,

$$A \approx 1 + \frac{1}{k^2 \lambda_{De}^2} + R. \quad (9')$$

Since $T_e/T_i \gg 1$ and $n_i/n_e > 1$ applies generally to rf or dc glow laboratory dusty plasmas, the main contribution to A in Eq. (9') comes from the ions. The term D is the driving term for the instability, while the term F represents dust Landau damping. In the following, we consider roughly the behavior of $\omega = \omega_r + i\gamma$ (ω_r and γ are the real and imaginary parts of the frequency, respectively) in two wavelength regimes: longer wavelengths near maximum growth where $\gamma \lesssim \omega_r$, and shorter wavelengths where $\gamma \ll \omega_r$.

Near maximum growth where $A \sim 0$ (which corresponds to a Buneman-instability-like resonance condition $ku_{0i} \sim \omega_{pi}$), we obtain from Eq. (9), neglecting the dust damping term F ,

$$\frac{\omega^2}{\omega_{pd}^2} \sim \frac{i + \sqrt{-1 + iCD}}{2D}, \quad (10)$$

where

$$C = \frac{12k^2 v_d^2}{\omega_{pd}^2} = \frac{12}{\alpha^2} k^2 \lambda_{Di}^2. \quad (11)$$

Note that $C \propto T_d$. When dust thermal effects are small (i.e., when $\alpha \gg 1$), and $CD \ll 1$, Eq. (10) yields $\omega_r \sim \gamma$,

$$\frac{\omega}{\omega_{pd}} \sim \frac{(1+i)}{\sqrt{2D}}. \quad (12)$$

However, when dust thermal effects are significant (i.e., when $\alpha \sim 1$) and $CD \gg 1$, Eq. (10) yields

$$\frac{\omega^2}{\omega_{pd}^2} \sim \frac{i}{2D} + \frac{(1+i)}{2D} \left(\frac{CD}{2} \right)^{1/2}. \quad (13)$$

Since $\omega^2 = \omega_r^2 - \gamma^2 + 2i\gamma\omega_r$, we see from Eq. (13) that $\omega_r^2 > \gamma^2$. Thus, dust thermal effects would tend to reduce the growth rate compared with the real frequency, with the instability becoming more kinetic (see Ref. 22). In addition, there is an increase in the real part of the frequency arising from the second term on the right-hand side of Eq. (13).

In the shorter wavelength regime, the collisionless dust damping term F reduces γ . For wavelengths shorter than where maximum growth occurs (so that $A > 0$) and when $|D-F| \ll A$, we can assume weak growth with $|\gamma| \ll \omega_r$. In this limit, the solution of Eq. (9) is roughly

$$\frac{\omega_r^2}{\omega_{pd}^2} \sim \frac{1}{2A} + \frac{1}{2A} \sqrt{1+CA}, \quad (14a)$$

$$\frac{\gamma}{\omega_r} \sim \frac{\text{Re}(D-F)}{4A}. \quad (14b)$$

Note that when dust thermal effects are significant with $CA > 1$, ω_r tends to increase with k , neglecting the smaller dependence of A on k . In addition, ω_r increases as T_d increases, since $C \propto T_d$. As $\text{Re}(D-F) \rightarrow 0$, the wave growth disappears. This can occur roughly when $\omega_r/kv_d \sim u_{0i}/v_i$, which can occur even for $k\lambda_{Di} < 1$, if T_d is very large.

B. Numerical results

We have numerically solved the kinetic dispersion relation (4) for two sets of lab parameters that are given in Table I, which roughly correspond to nominal parameters of the (i) Iowa and (ii) Auburn experiments. Both cases correspond to an argon plasma with electron temperature $T_e \sim 2-3$ eV and ion temperature $T_i \leq 0.05$ eV, so we have assumed that $\sigma_e \sim 5 \times 10^{-16} \text{ cm}^2$ (Ref. 30) and that roughly $\sigma_i \sim 5 \times 10^{-15} \text{ cm}^2$.³¹ In order to try to fit the experimental data, which extends to values of $k\lambda_{Di} > 1/2$, we chose a nominal value of $u_{0i}/v_i \sim 2$, so the Buneman instability type resonance condition $ku_{0i} \sim \omega_{pi}$ would cover this wavelength range. This nominal value of u_{0i}/v_i corresponds to reasonable values of electric fields in the experiments.

For the parameter set (i) in Table I, the ion Debye length is about $\lambda_{Di} \sim 7 \times 10^{-3} \text{ cm}$, the dust plasma frequency is

TABLE I. Two parameter sets.

	i	ii
P (mTorr)	~ 160	~ 70
m_i/m_p	40	40
n_i (cm^{-3})	$\sim 5 \times 10^8$	$\sim 7.5 \times 10^7$
T_e (eV)	~ 2.5	~ 3
T_i (eV)	~ 0.05	~ 0.025
T_d (eV)	$\sim 10-40$	$\sim 40-80$
R (μm)	~ 0.5	0.755
m_d/m_p	$\sim 6 \times 10^{11}$	$\sim 2.2 \times 10^{12}$
Z_d	~ 1700	~ 2600
n_d/n_i	$\sim 2 \times 10^{-4}$	$\sim 2 \times 10^{-4}$
ν_e (s^{-1})	$\sim 1.7 \times 10^8$	$\sim 8 \times 10^7$
ν_d (s^{-1})	~ 83	~ 24

about $\omega_{pd} \sim 916 \text{ rad/s}$, and the ion plasma frequency $\omega_{pi} \sim 4.7 \times 10^6 \text{ rad/s}$. We assume that $T_e/T_d = 0.1$, which implies the dust has a large temperature (kinetic energy) with $T_d \sim 25 \text{ eV}$. This yields $\alpha \sim 1.08$, so we expect dust Landau damping would be significant. We assume an electric field $E_0 \sim 400 \text{ V/m}$, which yields $u_{0i}/v_i \sim 2$ and $u_{0e}/v_e \sim 0.6$, and $\nu_i \sim 1.7 \times 10^6 \text{ s}^{-1}$. Thus, we have the following dimensionless collisional parameters: $\nu_e/\omega_{pi} \sim 36$, $\nu_i/\omega_{pi} \sim 0.35$, and $\nu_d/\omega_{pi} \sim 1.8 \times 10^{-5}$ (the latter corresponds to $\nu_d/\omega_{pd} \sim 0.09$). Figure 1 shows the results of solving Eq. (4) using the dimensionless parameters given above and those given in Table I for set (i). Note that there is stability for $k\lambda_{Di} \geq 0.65$, which corresponds to wavelengths smaller than about 0.7 mm. Figure 2 shows the results of solving Eq. (4) for the case when the dust is cold, using the same parameters as Fig. 1, but with $T_d = T_i$. It can be seen the ω_r has a very different behavior with k at shorter wavelengths, turning over and approaching ω_{pd} as $k\lambda_{Di}$ approaches unity. In addition, the growth rate in Fig. 1 is smaller than that in Fig. 2 due to dust Landau damping. In this regard, it is interesting to speculate that finite dust temperature might have affected the growth rate of an ion-dust streaming instability measured by Trottenberg *et al.*,⁸ which was reported to be smaller than kinetic theory predictions.

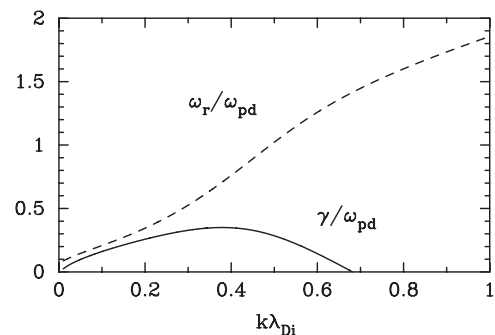


FIG. 1. Frequency (ω_r) and growth rate (γ) normalized to the dust plasma frequency ω_{pd} , obtained by solving Eq. (4). The parameters are $m_i/m_p = 40$, $T_e/T_i = 50$, $m_d/m_p = 6 \times 10^{11}$, $Z_d = 1700$, $n_d/n_i = 2 \times 10^{-4}$, $T_e/T_d = 0.1$, $\nu_i/\omega_{pi} = 0.35$, $\nu_e/\omega_{pi} = 36$, $\nu_d/\omega_{pi} = 1.8 \times 10^{-5}$, $u_{0i}/v_i = 2$, and $u_{0e}/v_e = 0.6$.

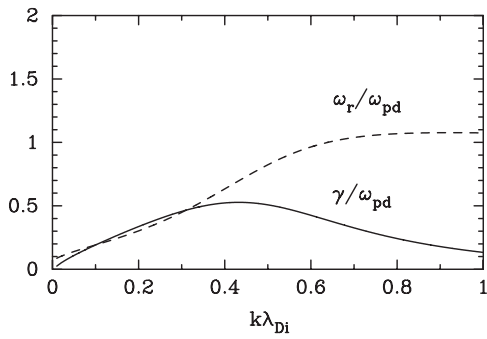


FIG. 2. Frequency (ω_r) and growth rate (γ) normalized to the dust plasma frequency ω_{pd} , obtained by solving Eq. (4). Parameters are the same as in Fig. 1, but with $T_e/T_d=50$.

For the parameter set (ii) in Table I, the ion Debye length is about $\lambda_{Di} \sim 0.014$ cm, the dust plasma frequency is about $\omega_{pd} \sim 280$ rad/s, and the ion plasma frequency $\omega_{pi} \sim 1.8 \times 10^6$ rad/s. Here we assume the dust kinetic energy is very large, with $T_e/T_d \sim 0.05$, which corresponds to a dust temperature $T_d \sim 60$ eV. This yields $\alpha \sim 0.75$, so again dust Landau damping should be significant. Taking $E_0 \sim 125$ V/m yields $u_{0i}/v_i \sim 2$ and $u_{0e}/v_e \sim 0.3$, and $v_i \sim 6 \times 10^5$ s $^{-1}$. The dimensionless collisional parameters are: $\nu_e/\omega_{pi} \sim 44$, $\nu_i/\omega_{pi} \sim 0.3$, and $\nu_d/\omega_{pi} \sim 1.3 \times 10^{-5}$ (the latter corresponding to $\nu_d/\omega_{pd} \sim 0.1$). Figure 3 shows the solution of Eq. (4) for the dimensionless parameters given above and for set (ii) in Table I. As can be seen, for this set of parameters there is no growth for wavelengths shorter than about $(2\pi\lambda_{Di})/0.5 \sim 1.7$ mm.

III. COMPARISON WITH EXPERIMENTS

The comparison of the theoretical dispersion relation (Fig. 1) with the Iowa and Auburn experimental data is shown in Fig. 4. Full details of the experiments at the University of Iowa are contained in Ref. 14. Briefly, the measurements were made in a dc glow discharge argon plasma in which kaolin powder (hydrated aluminum silicate) of nominal diameter 1 μ m was dispersed forming a dusty plasma in a fluidlike state. DAWs are spontaneously excited in this dusty plasma and by applying a sinusoidal modulation to the

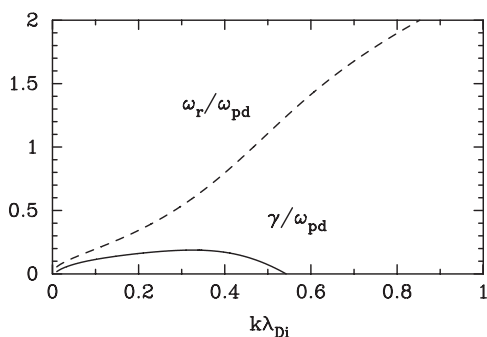


FIG. 3. Frequency (ω_r) and growth rate (γ) normalized to the dust plasma frequency ω_{pd} , obtained by solving Eq. (4). The parameters are $m_i/m_p=40$, $T_e/T_i=120$, $m_d/m_p=2.2 \times 10^{12}$, $Z_d=2600$, $n_d/n_i=2 \times 10^{-4}$, $T_e/T_d=0.05$, $\nu_i/\omega_{pi}=0.3$, $\nu_e/\omega_{pi}=44$, $\nu_d/\omega_{pi}=1.3 \times 10^{-5}$, $u_{0i}/v_i=2$, and $u_{0e}/v_e=0.3$.

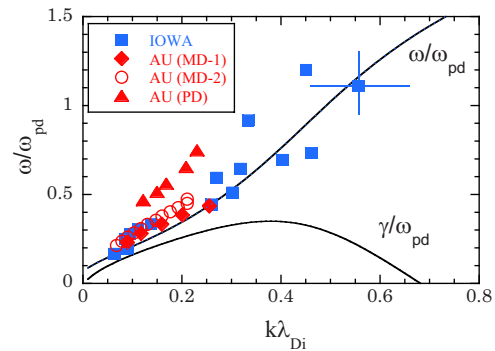


FIG. 4. (Color online) Comparison of experiment and theory. The solid and dashed lines are the normalized real frequency and growth rate, respectively, computed from kinetic theory, taken from Fig. 1. The dispersion relation measurements (ω_r/ω_{pd} vs $k\lambda_{Di}$) from the Iowa experiments. Diamonds (monodisperse dust cloud 1), open circles (monodisperse dust cloud 2), and triangles (polydisperse dust cloud) are dispersion relation measurements from the experiments at Auburn University. For clarity, typical error bars are shown on only one data point.

discharge current, the wave frequency could be controlled. The waves propagated horizontally in the Iowa experiment, and were imaged in scattered light from a green laser and recorded using a digital video camera. The wavelengths were determined by analyzing the light intensity profiles obtained from a 5 s sequence of single frame video images. The experiments at Auburn University are described in a recent paper.³² The Auburn experiments also utilize a dc glow discharge argon plasma, but the anode is oriented with its plane in the horizontal direction so that the DAWs propagate in the vertical (parallel to g) direction in this experiment. Three different dust clouds were investigated; two containing monodisperse (MD) 1.51 diameter silica microspheres, and another containing polydisperse (PD) silica microspheres with diameters in the range of 1.9–3.9 μ m. The two clouds of MD particles differed mainly in dust density. A naturally excited DAW was present in these dusty plasmas and the wave frequency could be controlled in the range of 7 to 100 Hz by applying a modulation to the discharge current.

The authors note that research on the thermal properties of the microparticle component of a dusty plasma has long been an integral part of dusty plasma research. Much of the early work was focused on studying the transitions of dusty plasma systems from a weakly coupled (disordered) configuration to a strongly coupled (ordered) state in rf generated plasmas.^{33–35} Often, this was achieved by adjusting the neutral pressure or applied rf power. More recent experiments have been performed to actively modify the kinetic temperature of the microparticles through laser heating^{36,37} or shock heating^{38,39} of the suspended microparticles in rf generated plasmas. Additionally, works by Williams and Thomas^{15,40} have provided measurements of the velocity space distribution function of microparticles in dc glow discharge dusty plasmas and have determined the dust kinetic temperature of those systems. A common theme of all of these experiments is that the observed dust kinetic temperature was found to be much greater than that of any other plasma component; i.e., $T_d \gg T_i, T_e$. The reason for the large dust kinetic energy is not known at present. We note however that theoretical models

for dust heating have been proposed in the past, including for example, instabilities that could result in dust heating (e.g., Refs. 34 and 41–43).

IV. SUMMARY

Measurements of the DAW dispersion relation performed at the University of Iowa and Auburn University were compared with the theoretical dispersion relation for an ion-dust streaming instability derived from kinetic theory. The essential point of the experiment/theory comparison (Fig. 4) is that it is necessary to take into account the effects of hot dust. The data for the real part of the dispersion relation *does not compare well* with the theoretical results in Fig. 2 in which the dust is taken as cold. It is important to note also that although the presence of hot dust leads to strong Landau damping of the DAWs, positive growth rates are predicted over the full range of observed waves due to the presence of the relatively strong ion drift.

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