Planet. Space Sci., Vol. 38, No. 9, pp. 1143–1146, 1990
Printed in Great Britain.

0032–0633/90 $3.00+0.00

LOW-FREQUENCY ELECTROSTATIC WAVES IN DUSTY PLASMAS

N. D'ANGELO
Department of Physics and Astronomy, The University of Iowa, Iowa City, IA 52242-1479, U.S.A.

(*Received in final form* 24 *April* 1990)

Abstract—A dispersion relation for low-frequency electrostatic waves in a magnetized, homogeneous dusty plasma is derived. The only possible modes are EIC and ion-acoustic modes. The connection to previous results on wave modes in plasmas containing negative ions or in plasmas with two positive ion species is discussed.

1. INTRODUCTION

Plasma is the most common state of matter in the universe, dust also being a frequent occurrence. Thus, it is not too surprising that in several situations of astrophysical interest one should be led to a study of "dusty plasmas" (Spitzer, 1978). For example, dust occurs in nebulas, in planetary magnetospheres (e.g. the rings of Saturn), and in comet tails. A review of dusty plasmas in the solar system has been published by Goertz (1989).

Dust particles can be charged through different processes, such as collection of charged particles from the surrounding plasma, photoionization, secondary electron emission, sputtering by energetic ions, etc. When collection of charged particles from the surrounding plasma is the dominant charging process, the situation of a dust grain is rather similar to that of an electrically floating Langmuir probe in a laboratory plasma, which charges up negatively to a potential of $\sim 2(\kappa T_e/e)$. In this case, viewing a spherical dust particle of radius a as a spherical capacitor of capacitance $C \simeq 4\pi\varepsilon_0 a$, the negative charge on the grain is readily estimated as $Q \approx C \cdot 2\kappa T_e/e$. For instance, for dust grains of radius $a \simeq 10^{-7}$ m, immersed in a plasma in which the electron temperature is $\kappa T_e \simeq 1$ eV, we obtain $Q \simeq 2\times10^{-17}$ C, i.e. ~ 125 elementary charges. This estimate assumes that the dust is sufficiently "diluted", so that the effects of near dust neighbors can be neglected, and also that the grain size, a, is much smaller than the plasma Debye length (see e.g. Goertz, 1989, or Whipple *et al.*, 1985).

Waves in dusty plasmas have been briefly considered by Goertz (1989), who provides a reference to a previous work by Bliokh and Yaraskenko (1985) on electrostatic waves in Saturn's rings.

In the present paper, low-frequency electrostatic waves in a dusty, homogeneous plasma immersed in a static and uniform magnetic field are considered. Section 2 presents the results of a simple analysis based on fluid theory. Section 3 contains the conclusions and a discussion of the connection of these results to those obtained previously in laboratory work in a Q-machine, either in plasmas with two positive ion species (Suszcynsky *et al.*, 1989) or in plasmas with large percentages of negative ions (Song *et al.*, 1989).

2. ELECTROSTATIC WAVES IN DUSTY PLASMAS

We consider a uniform plasma consisting of singly-charged positive ions of mass m_i and density n_i, of electrons of mass m_e and density n_e, and of negatively-charged dust grains. Although both the charge and the mass of the dust grains vary from one grain to another, we assume that the dust grains all have the same mass, m_d, and the same negative charge $-Ze$. The density of the grains is n_d. T_i, T_e, and T_d are the temperatures of the three components of the dusty plasma, which is assumed to be immersed in a static and uniform magnetic field, $\mathbf{B}$.

The three plasma components are described by their continuity and momentum equations

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{v}_j) = 0 \tag{1}$$

$$n_j m_j \frac{\partial \mathbf{v}_j}{\partial t} + n_j m_j \mathbf{v}_j \cdot \nabla \mathbf{v}_j + \kappa T_j \nabla n_j + q_j n_j \nabla \varphi - q_j n_j \mathbf{v}_j \times \mathbf{B} = 0 \tag{2}$$

with $j =$ i, e, d. The charge q_j is taken as $+$e for the positive ions, $-$e for the electrons, and $-Ze$ for the negatively-charged dust grains. In addition, we confine ourselves to a study of low-frequency waves, for which the electron inertia is negligible and assume the electron motion to be entirely along $\mathbf{B}$ (i.e. $\mathbf{v}_e \times \mathbf{B} = 0$).

This implies the condition, $\kappa T_e \nabla n_e = e n_e \nabla \varphi$, of Boltzmann equilibrium for the electrons. An additional equation employed is the one expressing charge neutrality, namely

$$n_i = Z n_d + n_e. \tag{3}$$

Equation (3) is appropriate not only for a zero-order steady state, but also for a perturbed state in which only low-frequency waves are allowed.

Equations (1)–(3) are linearized around a zero-order state in which $\partial/\partial t = \partial/\partial x = 0$, $\mathbf{v}_{j0} = 0$, and $n_{d0} = \varepsilon n_{i0}$, $n_{e0} = (1-\varepsilon Z)n_{i0}$, $Z\varepsilon$ thus representing the fraction of negative charge density attached to the dust grains. With the z-axis along the direction of the **B** field, and assuming that the first-order quantities vary as $e^{i(K_x x + K_z z - \omega t)}$, where $\mathbf{K} = (K_x, 0, K_z)$ is the wave vector and ω the angular frequency, one finds the following three relations between the first-order densities and the first-order potential:

$$\left\{-\omega^2 + \frac{\kappa T_i}{m_i}\left[\frac{\omega^2 K_x^2}{\omega^2-\omega_{ci}^2} + K_z^2\right]\right\} n_{i1} + \frac{e n_{i0}}{m_i}\left\{\frac{\omega^2 K_x^2}{\omega^2-\omega_{ci}^2} + K_z^2\right\}\varphi_1 = 0 \tag{4}$$

$$n_{e1} = \frac{e n_{e0}}{\kappa T_e}\varphi_1 \tag{5}$$

$$\left\{-\omega^2 + \frac{\kappa T_d}{m_d}\left[\frac{\omega^2 K_x^2}{\omega^2-\omega_{cd}^2} + K_z^2\right]\right\} n_{d1} - \frac{Z e n_{d0}}{m_d}\left\{\frac{\omega^2 K_x^2}{\omega^2-\omega_{cd}^2} + K_z^2\right\}\varphi_1 = 0, \tag{6}$$

where $\omega_{ci} = eB/m_i$ and $\omega_{cd} = ZeB/m_d$ are the positive ion and dust gyrofrequencies, respectively. Combining equations (4)–(6) with

$$n_{i1} - Z n_{d1} - n_{e1} = 0 \tag{7}$$

which expresses the condition of charge neutrality in first-order, one obtains with a little algebra the following dispersion relation:

$$\left(\frac{\xi^2}{\xi^2-1}d_x^2 + d_z^2\right)\left\{\xi^2 - \left[\frac{\xi^2}{\xi^2-(Z^2/\mu^2)}d_x^2 + d_z^2\right]\frac{\gamma}{\mu}\right\} + \frac{\varepsilon Z^2}{\mu}\left[\frac{\xi^2}{\xi^2-(Z^2/\mu^2)}d_x^2 + d_z^2\right] \cdot \left[\xi^2 - \left(\frac{\xi^2}{\xi^2-1}d_x^2 + d_z^2\right)\right] - \tau(1-\varepsilon Z)\left\{\xi^2 - \left[\frac{\xi^2}{\xi^2-1}d_x^2 + d_z^2\right]\right\} \cdot \left\{\xi^2 - \left[\frac{\xi^2}{\xi^2-(Z^2/\mu^2)}d_x^2 + d_z^2\right]\frac{\gamma}{\mu}\right\} = 0, \tag{8}$$

where $\xi = \omega/\omega_{ci}$, $\mu = m_d/m_i$, $d_x = K_x\rho_i$, $d_z = K_z\rho_i$, $\rho_i = (\kappa T_i/m_i)^{1/2}/\omega_{ci}$, $\gamma = T_d/T_i$, $\tau = T_i/T_e$.

Equation (8) provides four positive roots for ξ, two of which correspond to the positive ion and dust EIC modes, the other two roots to ion-acoustic modes. Three of the four positive roots of equation (8), as functions of ε, are shown in Fig. 1, for $\mu = 1\times10^8$, $Z = 100$, $\gamma = T_d/T_i = 0.1$, $\tau = T_i/T_e = 0.2$, $d_x = 0.4$, and $d_z = 0.04$. The choice $\mu \simeq 1\times10^8$ is appropriate to the case of dust grains of $a \approx 0.1$ μm and density of ~ 0.1 g cm^{-3}, when the positive ions are H^+ ions. If the electron temperature is ~ 1 eV, the charge on the grains $Q \sim (4\pi\varepsilon_0 a)\times 2\kappa T_e/e$ is approximately equal to 100 elementary charges (see Section 1). The maximum value of ε in Fig. 1 is $\varepsilon = 1\times10^{-2}$, since with $Z = 100$, all electrons are removed from the plasma and attached to the dust grains at this value of ε.

In addition to the three modes shown in Fig. 1, a

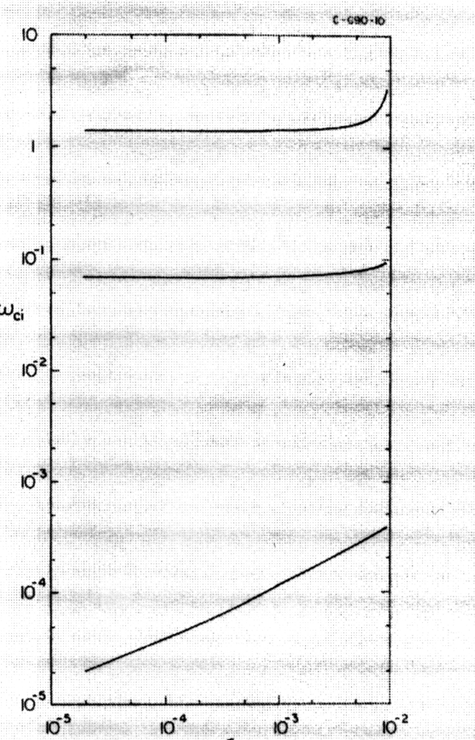

FIG. 1. WAVE FREQUENCIES (IN TERMS OF THE POSITIVE ION GYROFREQUENCY) VS ε, THE CONCENTRATION OF NEGATIVELY-CHARGED DUST GRAINS.
$\mu = m_d/m_i = 1\times10^8$, $Z = 100$, $\gamma = T_d/T_i = 0.1$, $\tau = T_i/T_e = 0.2$, $d_x = K_x\rho_i = 0.4$, and $d_z = K_z\rho_i = 0.04$.

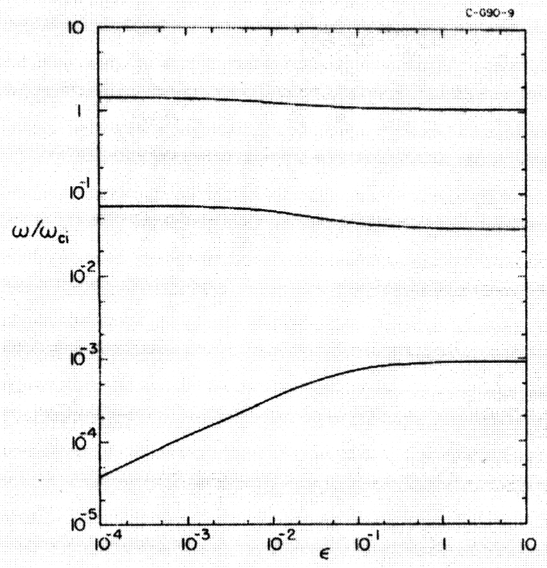

FIG. 2. SAME AS FIG. 1, BUT FOR POSITIVELY-CHARGED DUST GRAINS ($Z = -100$).
$\mu = 1\times10^8$, $\gamma = 0.1$, $\tau = 0.2$, $d_x = 0.4$, and $d_z = 0.04$.

fourth one is provided by equation (8), with $\omega/\omega_{ci} = Z/\mu = 1.0\times10^{-6}$ for all values of ε. This last mode is a dust EIC mode, while the mode with $\omega/\omega_{ci} \gtrsim 1.4$ is the positive ion EIC mode. The two other modes, with intermediate frequencies, are ion-acoustic modes.

If we wish to consider the case in which the dust particles have an overall positive charge (as might be the case when copious amounts of photoelectrons are ejected from the dust grains), we only need to change the sign of Z from positive (for negatively-charged grains) to negative (for positively-charged grains). Figure 2 shows an example of the possible modes for $\mu = 1\times10^8$, $Z = -100$, $\gamma = 0.1$, $\tau = 0.2$, $d_x = 0.4$, and $d_z = 0.04$. An additional mode with $\omega/\omega_{ci} = 1.0\times10^{-6}$ for all εs is not shown in the figure.

The above choice of the parameters μ, Z, γ, τ, d_x, and d_z was made for the purpose of illustrating by a numerical example the general nature of the roots of equation (8). At the same time it seemed desirable that this set of parameters be sufficiently representative of some particular dusty plasma occurring in nature. As an example, the charged dust in the tail of comet Giacobini-Zinner (Horanyi and Mendis, 1986) has grain sizes, a, in the range from ~ 0.03 to ~ 1 μm and, probably, a density for the matter in the grains on the order of 0.1 g cm^{-3}, as might be expected for a "fluffy" material somewhat lighter than water. A T_e of a few electron volts is the electron temperature in the solar wind, in which, also, $\tau = T_i/T_e \approx 0.1$–1. The choice $d_x = K_x\rho_i = 0.4$ was largely motivated by the fact that EIC waves are often observed, either in the laboratory or in space, with $d_x \lesssim 1$. In addition, the d_z/d_x ratio for these waves is typically of the order of 0.1, thus suggesting a $d_z = 0.04$.

The question may be asked why a treatment based on fluid theory should be appropriate for situations such as those we discuss here. Fluid theory has been shown repeatedly to be quite adequate for the description of many plasma wave modes (see e.g. Suszcynsky *et al.*, 1989; Song *et al.*, 1989, for recent examples). It has, of course, the limitation of being unable to account for wave–particle interactions (such as Landau damping). This, on the other hand, is more than compensated in very many situations, by the great simplification that a fluid treatment affords, compared with kinetic theory.

3. DISCUSSION AND CONCLUSIONS

A dispersion relation for low-frequency electrostatic waves in dusty plasmas has been derived. The plasma is homogeneous and is immersed in a steady and uniform magnetic field. The only modes possible are then two ion-acoustic and two EIC modes, associated with the positive ions and the dust grains. For negatively-charged dust grains, the present results are entirely similar to those obtained for plasmas consisting of positive ions, negative ions, and electrons, apart from the effects related to the much larger mass and charge of the dust grains compared with the negative ions. The ion-acoustic modes in the presence of negative ions were examined by D'Angelo *et al.* (1966) and studied in the laboratory by Wong *et al.* (1975). The EIC modes have been studied recently, both theoretically and experimentally, by Song *et al.* (1989). It was found that their frequencies increase with increasing ε, the concentration of negative ions. The frequency increase is accompanied by a decrease of the electron critical drift velocity for excitation of either mode, a result which may be understood as a consequence of both Landau and cyclotron damping decrease with increasing ε. A similar effect had been predicted by D'Angelo (1967) for the critical drift for ion-acoustic waves in a plasma with negative ions. If we look at the results of the present calculations for dusty plasmas, with negatively-charged dust grains, given in Fig. 1, we observe a steady increase of the wave frequencies of the positive ion EIC mode and of the two acoustic modes. Thus, although the present calculations do not allow for any zero-order drift of the electrons (which would provide a wave growth), it may still be surmised that an increase of the concentration of the negatively-charged dust should make the plasma more unstable.

The results for positively-charged dust grains, Fig. 2, are similar to those obtained by Suszcynsky *et al.* (1989) for EIC waves in plasmas containing two positive ion species. Both calculations and laboratory observations gave the result that the frequency of an EIC mode approaches its own gyrofrequency as the concentration of the other ion is increased. This is borne out by Fig. 2, where the frequency of the light ion EIC mode approaches the ion gyrofrequency as the positive dust concentration increases. In analogy to the results of Suszcynsky *et al.* (1989), it is to be expected that at large ε the light ion EIC mode becomes more difficult to excite.

Acknowledgements—Work supported by ONR and NASA.

REFERENCES

Bliokh, P. V. and Yaraskenko, V. V. (1985) Electrostatic waves in Saturn's rings. *Soviet Astr. Engl. Transl.* **29**, 330.

D'Angelo, N. (1967) Negative ions in the night ionosphere and the continuous sub-ELF emissions. *J. geophys. Res.* **72**, 1541.

D'Angelo, N., von Goeler, S. and Ohe, T. (1966) Propagation and damping of ion waves in a plasma with negative ions. *Phys. Fluids* **9**, 1605.

Goertz, C. K. (1989) Dusty plasmas in the solar system. *Rev. Geophys.* **27**, 271.

Horanyi, M. and Mendis, D. A. (1986) The dynamics of charged dust in the tail of comet Giacobini-Zinner. *J. geophys. Res.* **91**, 355.

Song, B., Suszcynsky, D., D'Angelo, N. and Merlino, R. L. (1989) Electrostatic ion-cyclotron waves in a plasma with negative ions. *Phys. Fluids B* **1**, 2316.

Spitzer, L. (1978) *Physical Processes in the Interstellar in the Medium*. John Wiley, New York.

Suszcynsky, D., D'Angelo, N. and Merlino, R. L. (1989) An experimental study of electrostatic cyclotron waves in a two-component plasma. *J. geophys. Res.* **94**, 8966.

Whipple, E. C., Northrup, T. G. and Mendis, D. A. (1985) The electrostatics of a dusty plasma. *J. geophys. Res.* **90**, 7405.

Wong, A. Y., Mamas, D. L. and Arnush, D. (1975) Negative ion plasmas. *Phys. Fluids* **18**, 1489.

Pergamon

Planet. Space Sci., Vol. 44, No. 10, p. 1219, 1996
Elsevier Science Ltd
Printed in Great Britain
0032–0633/96 $15.00+0.00

Corrigendum

N. D'Angelo, Low-frequency electrostatic waves in dusty plasmas. *Planet. Space Sci.* **38,** 1143, 1990.

The root $\xi = 10^{-6}$, independent of ε, mentioned in connection with Fig. 1 and not shown in the figure, is incorrect. It should be $\xi \simeq 10^{-7}$, also very weakly dependent on ε, and it corresponds to the DA (dust acoustic) wave. The root shown in Fig. 1 with $10^{-5} \lesssim \xi \lesssim 4 \times 10^{-4}$ is the EDC (electrostatic dust cyclotron) wave. Similar remarks apply to Fig. 2.

I would like to thank A. Barkan for pointing out the need for this revision.