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Drift instability in a positive ion–negative ion plasma

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Abstract. Drift wave instability in a magnetized plasma composed of positive ions and negative ions is considered using linear kinetic theory in the local approximation. We consider the case where the mass (temperature) of the negative ions is much larger (smaller) than that of the positive ions, and where the gyroradii of the two ion species are comparable. Weak collisional effects are taken into account. Application to possible laboratory parameters is discussed.

1. Introduction

Recently, Kim and Merlino (2007) have shown that it is possible to generate an almost electron-free plasma (electron to ion density ratio $<10^{-4}$) in a thermal plasma using the electron-attaching gas of perfluorocarbon C_7F_{14}. The presence of the negative C_7F_{14} ions, which have a mass of 350 times the proton mass m_p, was determined by observing the fundamental electrostatic ion cyclotron (EIC) mode near the gyrofrequency of the negative ion along with its higher harmonics. The excitation of the modes was due to an electron current that was drawn along the magnetic field **B** (Kim and Merlino 2007). In addition to the negative ion EIC modes, positive ion EIC modes were also excited by the parallel electron current (Kim and Merlino 2007; see also Kim et al. 2008).

In the presence of ion density gradients perpendicular to **B**, and even in the absence of a parallel electron current, one might expect that a drift wave instability could occur driven by the free energy associated with the pressure gradient in the plasma. In this paper, we consider conditions for the excitation of drift waves via a universal type instability in an almost electron-free plasma composed of light positive ions and heavy negative ions, immersed in an external magnetic field. The motivation for the study is the recent report by Kim et al. (2012) on experimental observations of waves with frequency much lower than any gyrofrequency or plasma frequency in a plasma composed of positive and negative ions with scarce electrons (electron density $\sim10^{-3}$ negative ion density).

We consider a parameter regime where the gyroradii of the positive and negative ions are comparable. This can occur when the temperature of the light positive ion species is larger than that of the heavy negative ion species by a factor on the order of the ratio of the heavy to light ion masses. We neglect the effect of possible $\mathbf{E} \times \mathbf{B}$ drifts, due, for example, to the presence of an ambipolar electric field **E**, since this field would be expected to be small in this case owing to comparable diffusion rates of the two ion species.

There has been prior theoretical work on drift wave instabilities in plasmas containing negative ions or massive negatively charged dust grains (e.g. Shukla et al. 1991; Rosenberg and Krall 1996; Shukla and Rosenberg 2009; Saleem 2010; Knist et al. 2011). However, the only theory that we have found for a negative ion plasma with vanishingly small electron concentration appears to be for a pair-ion plasma in which there is no mass difference between the ion species (e.g., Saleem 2010). The effect of negative ions on the drift wave instability in a pair-ion electron plasma was found to be stabilizing, where the instability was found to be quenched in the limit of a pure pair-ion plasma with no electrons (Ali and Saleem 2010; Saleem 2010).

Section 2 gives the model, dispersion relation and some approximate analytic results. Section 3 gives numerical results for possible laboratory plasma parameters. Section 4 gives a brief summary and discussion.

2. Analysis

We consider a weakly ionized plasma composed of singly charged positive ions and singly charged negative ions. (Because we assume that the electron number density is orders of magnitude smaller than that of the positive ions, we neglect the electrons). The condition of equilibrium charge neutrality is then

$$n_+ \approx n_-, \tag{1}$$

where n is the number density and the subscripts $+,-$ refer to positive ions and negative ions, respectively. It is assumed that the mass of the negative ions, m_-, is much larger than that of the positive ions, m_+. We consider a slab geometry, with an external uniform magnetic field **B** in the **z** direction, and with the ion densities decreasing in the **x** direction. The density gradient scale length $L_n^{-1} = |\nabla n/n|$ is the same for both ion species. Thus, the diamagnetic drift velocity of the positive ions,

$$\mathbf{v}_+^* = -\frac{T_+}{eB}\frac{\nabla n_+}{n_+} \times \mathbf{z}, \tag{2}$$

is in the $-\mathbf{y}$ direction, while that of the negative ions is in the $+\mathbf{y}$ direction, with $\mathbf{v}_-^* = -(T_-/T_+)\mathbf{v}_+^*$. Thus, the diamagnetic drift frequency $\omega_j^* = \mathbf{k}\cdot\mathbf{v}_j^*$ is taken to be > 0 for a wave with k_y in the $-\mathbf{y}$ direction. We consider the frequency regime where $\omega \ll$ the negative ion gyrofrequency ω_{c-}, assume that the ions are described by Maxwellian velocity distributions, and retain collisions using a number-conserving Krook collision term (see e.g. Miyamoto 1989). In the local approximation, the dispersion relation for electrostatic waves is (e.g. Lindgren et al. 1976; Alexandrov et al. 1984; Miyamoto 1989):

$$1 + \sum_{j=+,-} \chi_j = 0, \tag{3}$$

where

$$\chi_j = \frac{1}{k^2\lambda_{Dj}^2}\left[1 + \left(1 - \frac{\omega_j^*}{\omega + i\nu_j}\right)\Gamma_0(b_j)\zeta_j Z(\zeta_j)\right] \times \left[1 + \frac{i\nu_j}{\omega + i\nu_j}\Gamma_0(b_j)\zeta_j Z(\zeta_j)\right]^{-1}. \tag{4}$$

Here, $\lambda_{Dj} = (T_j/4\pi n_j e^2)^{1/2}$, $b_j = k_y^2\rho_j^2$, where $\rho_j = v_j/\omega_{cj}$ is the gyroradius, v_j is the thermal speed, ω_{cj} is the gyrofrequency and ν_j is the collision frequency of ion species j. In addition, $\Gamma_0(x) = I_0(x)e^{-x}$ with I_0 as the modified Bessel function of order zero, and $Z(\zeta)$ is the plasma dispersion function (Fried and Conte 1961) with argument $\zeta_j = (\omega + i\nu_j)/\sqrt{2}k_z v_j$.

We give an approximate analytic limit of (3) in the kinetic regime for the positive ions where $\zeta_+ \ll 1$ and the non-resonant regime for the negative ions where $\zeta_- \gg 1$. Note that these regimes can occur simultaneously under conditions where $v_-/v_+ \ll 1$. However, for the parameters that will be considered in the next section, where $v_-/v_+ \sim 0.1$, this analytic limit is only marginally satisfied. Thus, we will retain higher order terms in the expansions of the plasma dispersion function than are usually considered for the universal drift wave instability. Furthermore, in order to obtain tractable analytic expressions, we will neglect collisional effects which are relatively small in the regime where $\nu_+, \nu_- \ll \omega$.

We also assume that $\omega = \omega_r + i\gamma$ with $\omega_r \gg |\gamma|$. For the positive ions, expanding the plasma dispersion function into real and imaginary parts as $Z(\zeta_+) \sim Z_r(\zeta_+) + i\sqrt{\pi}e^{-\zeta_+^2}$ (see Miyamoto 1989), the positive ion susceptibility becomes approximately

$$\chi_+ \sim \frac{1}{k^2\lambda_{D+}^2}\left[1 + \frac{(\omega - \omega_+^*)}{k_z v_+}\Gamma_0(b_+)\left(-\frac{\omega}{k_z v_+} + i\sqrt{\frac{\pi}{2}}e^{-\zeta_+^2}\right)\right]. \tag{5}$$

In contrast to the usual treatment of the universal instability in a standard electron–ion plasma (see e.g. Goldston and Rutherford 1995), in our case we cannot assume that the real part of $Z(\zeta_+)$ can be neglected. This is because, although ζ_+ is < 1, it is not necessarily $\ll 1$ when $\zeta_- \gg 1$. For the negative ion susceptibility, we expand the plasma dispersion function for large argument, so that

$$\chi_- \approx \frac{1}{k^2\lambda_{D-}^2}\left[1 - \frac{(\omega - \omega_-^*)}{\omega}D_- + iF_-\right], \tag{6}$$

where

$$D_- = \Gamma_0(b_-)\left(1 + \frac{1}{2\zeta_-^2}\right),$$

and

$$F_- = \sqrt{\frac{\pi}{2}}\frac{(\omega - \omega_-^*)}{k_z v_-}\Gamma_0(b_-)e^{-\zeta_-^2}.$$

We then use (5) and (6) in (3) along with $\omega_-^* = -(T_-/T_+)\omega_+^*$ to obtain the following approximate expression for the real part of the frequency:

$$\omega_r \approx \frac{\omega_+^* D_-}{1 + C_+ + \frac{T_+}{T_-}(1 - D_-)}, \tag{7}$$

where

$$C_+ = \Gamma_0(b_+)\frac{(\omega_+^* - \omega_r)\omega_r}{k_z^2 v_+^2}.$$

To obtain (7) we have also assumed that $k^2\lambda_{D+}^2 \ll (T_+/T_-)(1 - D_-)$. In the limit of a small negative ion Larmor radius, with $b_- \ll 1$, (7) becomes

$$\omega_r \approx \frac{\omega_+^*(1 - b_-)}{1 + C_+ + k_y^2\rho_s^2 - k_z^2 c_s^2/\omega^2}, \tag{8}$$

where $\rho_s = (T_+/T_-)^{1/2}\rho_-$ and $c_s = (T_+/m_-)^{1/2}$ is the sound speed in the system. Note that (8) is similar to the dispersion relation for the real frequency in the usual universal instability analysis for a standard electron–ion plasma, with the exception of the term C_+ (see Ichimaru 1973; Goldston and Rutherford 1995). In this model plasma, the (light) positive ions take the place of the electrons and the (heavy) negative ions take the place of the ions, so the electron diamagnetic frequency is replaced by ω_+^*, the electron temperature and the Larmor radius are replaced by T_+ and ρ_+, respectively, and the ion temperature and the Larmor radius are replaced by T_- and ρ_-, respectively. As in the standard electron–ion plasma, the real frequency tends to increase as k_z increases (see e.g. Goldston and Rutherford 1995). However, there is an additional factor C_+ in the denominator of (8) that reduces the real frequency for $\omega < \omega_+^*$. In addition, in contrast to the electron–ion plasma where the perpendicular wavelength of the drift wave is much larger than the electron Larmor radius, in this case, where $\rho_+ \sim \rho_-$, the perpendicular wavelength could be comparable to ρ_+.

The growth rate of this drift instability for $\zeta_+ \ll 1$ and $\zeta_- \gg 1$ is obtained as

$$\gamma \approx \sqrt{\frac{\pi}{2}} \frac{\omega_r^2}{\omega_+^* D_-} \left[\left(\frac{\omega_+^* - \omega}{k_z v_+} \right) \Gamma_0(b_+) e^{-\zeta_+^2} - \frac{T_+}{T_-} \left(\frac{\omega - \omega_-^*}{k_z v_-} \right) \Gamma_0(b_-) e^{-\zeta_-^2} \right]. \tag{9}$$

From (9) it can be seen that growth requires $\omega < \omega_+^*$. Note that (9) has the same form as the growth rate for the collisionless drift wave instability in a standard electron–ion plasma (e.g. Ichimaru 1973), with the replacements electron → positive ion, and ion → negative ion discussed above.

3. Numerical results

In this section, we show solutions of (3) for the following set of possible experimental parameters: $B = 0.3$ T, $n_+ = 1 \times 10^9$ cm^{-3} $= n_-$, $T_+ = 0.2$ eV, $T_- = 0.026$ eV, $m_+/m_p = 39$, $m_-/m_p = 350$. With these parameters, the ion gyrofrequencies are $\omega_{c+} \sim 7.3 \times 10^5$ rad s^{-1} and $\omega_{c-} \sim 8.2 \times 10^4$ rad s^{-1}. The ion gyroradii are comparable, with $\rho_+ \sim 0.95$ mm and $\rho_- \sim 1$ mm. (This implies that both ion species may diffuse across the magnetic field with comparable rates, in which case one might expect that any ambipolar field that might be set up would be relatively small.) Note that with these parameters, the ratio $v_+/v_- \sim 8.3$, so that the analytic result in the previous section is only marginally valid, and a numerical solution is needed. The gas pressure is assumed to be very low, on the order of $\sim 10^{-5}$ Torr, so that collisions with neutrals can be neglected. However, Coulomb collisions between positive and negative ions may play a role. Using expressions from Huba (2011) and Deutsch and Rauchle (1992), we obtain $\nu_+ \sim 1.4 \times 10^3$ s^{-1}, assuming that the K^+ ions are fast test ions colliding with the negative ions, and $\nu_- \sim 110$ s^{-1}, assuming the negative ions are slow test ions colliding with the K^+ ions. We consider ion density gradients of magnitude $L_n \sim 2$ cm. Thus, $\rho_+/L_n \sim 0.047$ (and $\rho_-/L_n \sim 0.05$).

The real frequency and growth rate (normalized to ω_{c-}) of this drift wave instability are shown in Fig. 1 for the above parameters. Results are shown for several values of the angle θ between **k** (with the y component in the direction of the positive ion diamagnetic drift velocity) and **B**. Note that the frequency increases as θ decreases, that is, as k_z increases, following the trend of the analytic results. For these parameters, growth appears to occur for $k\rho_- \sim k\rho_+ \lesssim 1$. Maximum growth occurs around $k\rho_s \sim 1$, where $\rho_s^2 = (T_+/T_-)\rho_-^2$, analogous to maximum growth of the universal instability in a standard electron–ion plasma (Angus and Krasheninnikov 2012). The growth rate decreases as k increases owing to several factors including the decrease of $\Gamma_0(b)$ with increasing k and the increase in collisionless damping by the negative ions as $\omega/k_z v_-$ becomes smaller and as the frequency approaches $k_z c_s$. Figure 2 shows the frequency and growth rate for a smaller gradient scale length of $L_n \sim 1$ cm, with the other parameters being the same as above. As expected, the growth rate increases as L_n decreases.

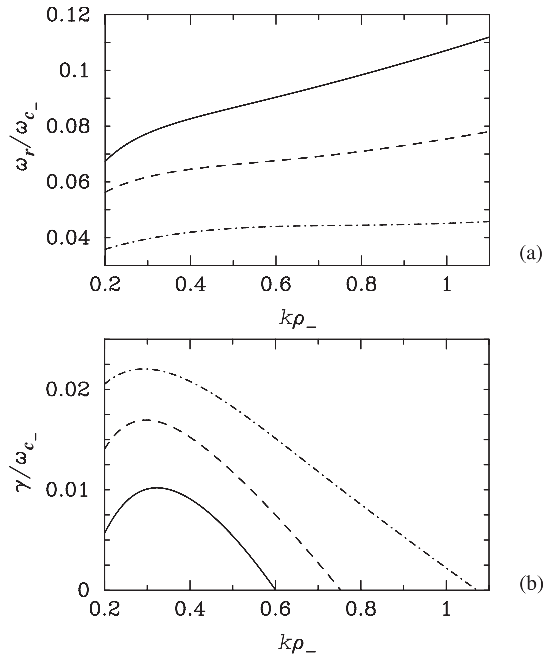

Figure 1. (a) Frequency ω_r and (b) growth rate γ normalized to ω_{c-} versus $k\rho_-$ obtained by solving (3). Parameters are: $m_+/m_p = 39$, $m_-/m_p = 350$, $T_+/T_- = 7.7$, $v_+/\omega_{c-} = 0.017$, $v_-/\omega_{c-} = 1.3 \times 10^{-3}$, $\omega_{p+}/\omega_{c+} = 9$, $\rho_+/L_n = 0.047$, $\theta = 89°$ (dot–dashed curves), $\theta = 88°$ (dashed curves), $\theta = 87°$ (solid curves).

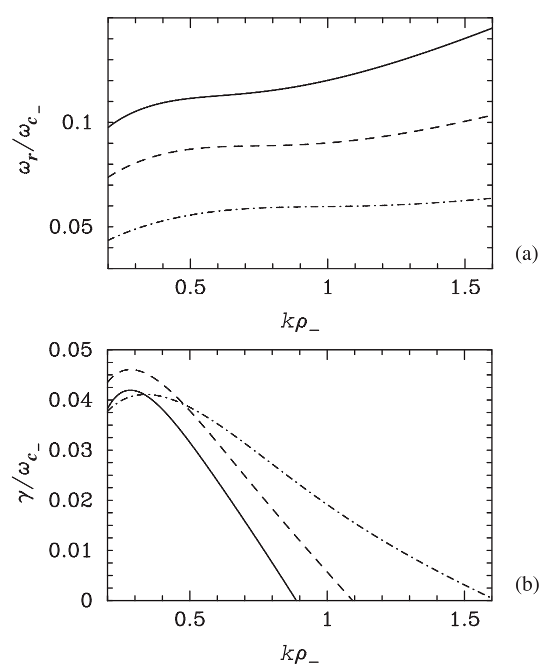

Figure 2. Frequency ω_r (a) and growth rate γ (b) normalized to ω_{c-} versus $k\rho_-$ obtained by solving (3). Parameters are the same as in Fig. 1, except that $\rho_+/L_n = 0.094$.

Although this simple analysis shows the possibility of wave growth, note, however, that kL_n is not $\gg 1$ and the local approximation used here may not apply very well. For example, for the parameters used, when $k\rho_- \sim 0.5$, we have that $kL_n \sim 10$ for Fig. 1 and ~ 5 for Fig. 2. Thus, future work should consider a non-local analysis to analyze the problem more rigorously.

4. Summary and discussion

A universal-type drift wave instability in a plasma composed of positive ions and negative ions was considered using a linear kinetic theory analysis in the local approximation. The positive ions have a much smaller mass and much higher temperature than the negative ions so that the universal instability limit could occur, with kinetic positive ions and non-resonant negative ions. A plasma with low collisionality was considered. Numerical results were presented for possible laboratory parameters.

This analysis was motivated by recently reported experimental observations of waves with frequencies much lower than the gyrofrequency of negative ions in a magnetized Q-machine plasma composed of potassium ions and negative ions of the perfluorocarbon C_7F_{14}, with an electron density of $\sim 10^{-3} n_-$ (Kim et al. 2012). Although the instability considered in this paper is in this frequency regime for the considered parameters, further work is needed to determine its relevance to the experimental results reported by Kim et al. (2012). Future theoretical work should consider a non-local analysis (e.g. Chen 1967; Davidson 1976), preferably in a cylindrical geometry appropriate for a Q-machine. It may be that effects of $\mathbf{E} \times \mathbf{B}$ and centrifugal drifts may also need to be considered if there is a radial electric field $\mathbf{E}$ (see e.g. Chu et al. 1969; Politzer 1971). In order for the magnitude of the $\mathbf{E} \times \mathbf{B}$ drift speed to be comparable to the positive ion diamagnetic drift speed for the parameters considered in Sec. 3 above, the electric field strength should be about 10–20 V m^{-1}. However, for the parameters considered in Sec. 3, one might expect any ambipolar field to be relatively weak, since the mobilities of the ion species across the magnetic field appear to be comparable. That is, the perpendicular mobility (see Krall and Trivelpiece 1973) $\mu_{\perp\alpha} \propto \nu_\alpha/(m_\alpha \omega_{c\alpha}^2) \propto m_\alpha \nu_\alpha$ for $\omega_{c\alpha} \gg \nu_\alpha$, so that $\mu_{\perp -}/\mu_{\perp +} \sim 1$ for the considered parameters.

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