DUSTY PLASMAS IN THE SOLAR SYSTEM

C. K. Goertz
Department of Physics and Astronomy
University of Iowa, Iowa City

Abstract. The processes that lead to charging of dust grains in a plasma are briefly reviewed. Whereas for single grains the results have been long known, the reduction of the average charge on a grain by "Debye screening" has only recently been discovered. This reduction can be important in the Jovian ring and in the rings of Uranus. The emerging field of gravitoelectrodynamics which deals with the motion of charged grains in a planetary magnetosphere is then reviewed. Important mechanisms for distributing grains in radial distance are due to stochastic fluctuations of the grain charge and a systematic variation due to motion through plasma gradients. The electrostatic levitation model for the formation of spokes is discussed, and it is shown that the radial transport of dust contained in the spokes may be responsible for the rich radial structure in Saturn's rings. Finally, collective effects in dusty plasmas are discussed which affect various waves, such as density waves in planetary rings and low-frequency plasma waves. The possibility of charged grains forming a Coulomb lattice is briefly described.

1. INTRODUCTION

Dust is quite common throughout the universe and represents much of the solid matter in it. On the other hand, the gaseous component of matter is often ionized (at least partially), and thus the dust coexists with plasma and forms a "dusty plasma." The evolution of the solar system from its solar nebula stage to its present form has passed through a stage during which almost all solid matter was in the form of dust which we will define here as solid objects smaller than 1 cm. Larger objects such as meteorites, planetesimals, and planets presumably evolved through coagulation of dust. Although the properties of the early solar nebula are not well known, it is very likely that the gaseous component was also partially ionized. Thus the early solar nebula must be thought of as a dusty plasma.

In many cases one observes that the dust is not uniformly distributed in space. For example, comets possess long dust tails and trails. Dust in planetary magnetospheres is concentrated in rings whose radial width is often quite small. Some rings show radial and azimuthal (longitudinal) optical depth modulations. Much of this paper will deal with explanations for these structures which rely on electric and magnetic forces acting on dust particles. This is not to say that all structures are electromagnetically induced, but we now have good reasons to believe that these forces are important and need to be considered if one wants to understand the spatial distribution of dust.

For the purposes of this paper we define a dusty plasma as an ensemble of dust particles immersed in a (perhaps partially ionized) plasma consisting of electrons, ions, and neutrals. Dusty plasmas occur in many astrophysical contexts, but this paper will only deal with dusty plasmas in the solar system. Usually, the strongest force acting on dust particles in the vicinity of stars or planets is gravity or radiation pressure. However, in a dusty plasma the dust particles are, in general, charged and will therefore also be affected by electric and magnetic fields often in subtle and surprising ways. In addition, the dust particles may change the properties of the plasma itself and change the dispersion relation of various, usually low frequency, plasma waves.

Dust particles can be charged by various means, for example, by photoionization or absorption of charged particles. If such charged dust particles exist in a plasma, i.e., in a conducting fluid, the interaction between the particles and externally applied electric and magnetic fields (e.g., a planetary magnetic field) is modified by the presence of the plasma. For example, a negatively charged dust, or for that matter any negatively charged particle, will be surrounded by a plasma which is not charge neutral everywhere but has a positive charge density near the grain because it tends to attract the positive ions and repel the negative electrons. This positive charge density partially screens out the negative dust charge and reduces the strength of the interaction. This "debye screening" effect is a unique consequence of the fact that the dust is immersed in a plasma.

Several effects which will be discussed below are a direct consequence of the fact that dust particles in a dusty plasma are charged. For example, if two dust particles carry the same sign of charge, they repel each other, and the cross section for collisions (e.g., sticking collisions necessary for coagulation) is reduced. On the other hand, it is possible that some dust particles carry opposite signs of charge, in which case these particles will attract each other and coagulate much faster than if they were uncharged. The distribution of dust in the vicinity of comets or in planetary magnetospheres where plasmas exist is significantly affected by the electromagnetic forces acting on the charged dust particles which may cause changes of the dust particles' angular momentum and hence lead to orbital decay. This electromagnetically induced radial transport of small dust particles is important for the maintenance of the ephemeral (optically thin) rings of Saturn and Jupiter. Finally, the electromagnetically induced angular momentum transport can significantly affect the evolution even of the optically thick main rings of Saturn whose visual appearance reflects the distribution of big (meter size) particles which are not directly influenced by electromagnetic forces.

The fundamental quantity one needs to know in order to assess the importance of electromagnetic forces on dust particles is their charge to mass ratio. The charge is the product of the dust grain's surface potential and its capacitance. The surface potential is relatively easy, but by no means trivial, to calculate for a single stationary dust grain in a plasma by balancing the currents due to plasma ions, electrons, photoelectrons, and secondaries. However, when the number of dust grains becomes large, the potential at any grain's surface includes contributions from the charges on all other grains within a "Debye sphere," and thus the charge on the grain itself could be quite small. The Debye sphere characterizes the region throughout which the electric field of a point charge placed in a plasma is felt in this plasma. Beyond the Debye sphere the electric field is significantly reduced because of the fact that the plasma inside the Debye sphere is not neutral but carries a net charge opposite in sign to that of the point charge. Thus grains which are separated by more than a Debye length do not interact electrostatically with each other. These questions will be discussed in section 2.

Many of our applications will deal with dust in planetary rings which all exist inside the planets' magnetospheres and which thus must be regarded as examples of dusty plasmas. The question arises, of course, of whether the fact that ring particles are charged has any observable or important dynamic consequences. Traditional theories of rings (for reviews, see *Greenberg and Brahic* [1984]) neglect the charge effects completely, usually without any detailed justification. When one compares the gravita-

tional force on a typical ring particle with the electromagnetic force in a corotating magnetosphere, one finds that it is, indeed, often much larger and that the motion of such a particle is to a very high degree of accuracy described by the normal Kepler motion. However, subtle deviations from Keplerian orbits are important in rings. Electromagnetic forces acting on charged dust particles can provide for these deviations. The study of the motion of charged dust particles in a planetary magnetosphere is the subject of "gravitoelectrodynamics." This will be discussed in section 3.

Because the motion of charged dust particles in a planetary magnetosphere is not exactly Keplerian, radial transport of dust can occur, at times very rapidly. Such a transport can cause a redistribution of angular momentum in rings which has important dynamical consequences for the evolution of planetary rings. These will be described in section 4 as well as the electrostatic levitation model for the formation of the "spokes" in Saturn's rings. It will be shown that the spokes may be responsible for the surprisingly rich radial structure of the optically thick B ring.

When the dust number density becomes very large and the intergrain distance is small, the grains may interact with each other via electrostatic Coulomb forces, and collective effects may occur. Dust particles may then not move independently of each other but in a coherent fashion. The number density of the dust particles may be modulated coherently by a wave propagating through the dust medium. In addition, dust grains may change the dispersion properties of low-frequency plasma waves. The possibility also exists that the Coulomb interaction energy exceeds the thermal energy of the dust particles. In that case one expects dust grains to exhibit long-range order and form liquidlike or even lattice structures. This will be investigated in section 5.

Dusty plasmas have not been studied extensively, and many questions must remain open at this time. In particular, the relevance of the effects discussed below to natural phenomena such as planetary rings or cometary dust tails is not clear, simply because the plasma properties and the grain characteristics are not well known. Our models are also rather simple, assuming, for example, spherical grains of radius a. Usually, all grains are assumed to have the same size. Some of the charging currents are neglected, etc. Several outstanding questions and suggestions for future theoretical and observational work will be discussed in the final section of this paper. The field of dusty plasmas is quite young, and a critical reader may doubt the wisdom of writing a review paper at this time. However, the field is evolving quite rapidly, and exciting discoveries have been made which even space scientists are not aware of. This review paper cannot possibly cover all aspects, but it is hoped that readers not

familiar with this field may find the questions challenging and exciting, at least, and may even be motivated to contribute to this field in the future.

2. THE CHARGE OF A DUST GRAIN IN A PLASMA

2.1. Isolated Grains

There are several processes which cause the charge on a grain to be nonzero, and the calculation of the equilibrium charge on a grain can become quite complex if all processes are included. For grains in a plasma with a temperature T_e for electrons (mass m_e) and T_i for ions (mass m_i) the fact that the flux of electrons having a thermal velocity of $c_e = (kT_e/m_e)^{1/2}$ (k is the usual Boltzmann constant) is larger than that of the heavier ions which have a smaller thermal velocity $c_i = (kT_i/m_i)^{1/2}$ will make the grain charge Q and its surface potential ϕ_s negative. The density of electrons and ions far away from the dusty plasma where the plasma is charge neutral is $n_0 (n_{0e} = n_{0i})$. On the other hand, the primary electrons can, if they are energetic enough, release secondary electrons, which causes the surface potential to become positive. The absorption of plasma ions also tends to make the surface potential positive. The currents of primary electrons and ions are, of course, affected by the surface potential of the grain (of radius a) itself and depend on the relative velocity w between the plasma and the grain. If the surface potential is negative, electrons are repelled, and the current to the grains carried by them is reduced. On the other hand, ions are attracted, and their current is increased. For positive ϕ_s the situation is reversed. The currents can be written as

$$J_e = J_{oe} e^{e\phi_s/kT_e} \qquad \phi_s < 0$$

$$J_e = J_{oe}(1 + e\phi_s/kT_e) \qquad \phi_s > 0$$

$$(1)$$

$$J_i = J_{oi} e^{z_i e\phi_s/kT_i} \qquad \phi_s > 0$$

$$(2)$$

$$J_i = J_{oi}(1 - z_i e\phi_s/kT_i) \qquad \phi_s < 0$$

where e is the magnitude of the electron charge, and $z_i e$ is the ion charge (in what follows we will assume singly charged ions, i.e., $z_i = 1$). The current carried by particle species a (electron or ion) for $\phi_s = 0$ is given by

$$J_{0\alpha} = n_\alpha q_\alpha (kT_\alpha/m_\alpha)^{1/2} \pi a^2 f_\alpha(w)$$

where n_α is the number density of the species and f_α is a rather complicated function of the relative velocity w between the plasma and the grains described by Whipple [1981]. In general the ion current intercepted by the grains

increases with $|w|$. Thus the charge on a moving grain tends to be more positive than on a grain at rest with respect to the plasma. However, this is not necessarily always the case, as discussed by Whipple, to whom we refer the reader for more details. The electron current is not affected by the slow relative drift between the dust and the plasmas because $w \ll c_e$ where c_e is the thermal speed of the electrons. Becasue the ions are much heavier than the electrons ($m_i \gg m_e$), the ion current is, for $\phi_s = 0$, much smaller than the electron current density, and the grain becomes negatively charged. This reduces the electron current and increases the ion current.

The flux of secondaries depends on the energy E of the plasma electrons and the surface potential. If the grain is positive, some of the secondary electrons may be recaptured and not contribute to the charging of the grain. The temperature of the secondaries, T_s, is of the order of a few 10^4 K. The net charging current of the secondaries depends also on the material properties of the grains characterized by the secondary yield parameter δ and has been discussed in detail by Meyer-Vernet [1982], who has derived expressions for the secondary current using the yield function

$$\delta(E) = 7.4\delta_m E/E_m \exp[-2(E/E_m)^{1/2}]$$

where E_m is a characteristic energy at which the release of a secondary electron peaks. Integrating this over a Maxwellian primary electron distribution yields

$$J_{sec} = 3.7\delta_m J_e = F_5(E_m/4kT_e) \qquad \phi_s < 0$$

$$(3)$$

$$J_{sec} = 3.7\delta_m J_e = e^{\{(-e\phi_s/k)[(1/T_s)-(1/T_e)]\}}$$

$$\cdot F_{5,B}(E_m/4kT_e) \qquad \phi_s > 0$$

where

$$F_5(x) = x^2 \int_0^\infty t^5 \exp[-(xt^2 + t)] \, dt$$

$$F_{5,B}(x) = x^2 \int_B^\infty t^5 \exp[-(xt^2 + t)] \, dt$$

$$\tilde{B} = (e\phi_s/kT_s)$$

Typical values for the material constant δ_m range between 0.5 and 30 and for E_m between 0.1 and 2 keV. The emitted electrons have a Maxwellian distribution with $kT_s \approx 1 - 5$ eV.

The absorption of solar UV radiation releases photoelectrons and hence constitutes a positive charging current. Its magnitude depends on the material properties of the grain, i.e., its photoemission efficiency, and on the

grain's surface potential, which may, if positive, recapture a fraction of the photoelectrons. The spectrum of photoelectrons released is often assumed to be a Maxwellian with a temperature T_p which corresponds to an energy $e\phi^* = KT_p$ of about 1 eV. Then the net charging current due to photoemission is given by

$$J_v = \pi a^2 K \qquad \phi_s \leq 0$$
$$J_v = \pi a^2 K e^{-e\phi_s/kT_p} \qquad \phi_s > 0 \tag{4}$$

where $K = \eta(2.5 \times 10^{14})D^{-2}$ photoelectrons per square meter per second is the photoelectron flux at a distance D (in astronomical units) from the Sun and η is the photoemission efficiency ($\eta \approx 1$ for metals, and $\eta = 0.1$ for dielectrics).

There are several other charging currents due to thermal emission, field emission, sputtering, and proton-induced secondaries. In most solar system applications, these have been neglected, although it is clear that energetic radiation belt protons could very well be important and should be considered. Very small grains may also have a very large surface electric field $E_s = \phi_s/4\pi\varepsilon_0 a$, and field emission may become important. We will come back to this point below.

The charge Q on a grain changes because of these currents according to the equation

$$\frac{dQ}{dt} = \sum_\alpha J_\alpha \tag{5}$$

The charge itself is related to the surface potential by

$$Q = C\phi_s \tag{6}$$

where the capacitance C of a spherical grain in a plasma is given by Whipple et al. [1985]:

$$C \approx 4\pi\varepsilon_0 a\, e^{-a/\lambda_D} \tag{7}$$

The exponential factor in (7) reflects the Debye screening of the grain's charge by the charge density in the plasma surrounding the charge. For example, if Q is negative, the positive plasma ions will have a slightly higher density than the plasma electrons near the grain. The positive space charge in the plasma surrounding the negative grain causes the electric field to fall off with an exponential decrease rather than the usual $1/r^2$ decrease. The scale length for the exponential falloff is the Debye length $\lambda_D = (kT_e\varepsilon_0/n_e e^2)^{1/2}$.

It is obvious that the assumption of spherical grains is a great simplification which is most likely not satisfied. However, a realistic treatment of the charge distribution on nonspherical grains is very complicated and requires numerical simulations.

The equilibrium surface potential is, obviously, given by the requirement that $\Sigma J_\alpha = 0$. A very simple but important special case applies to small grains ($a \ll \lambda_D$) in the distant planets' magnetospheres where the photoelectron current is small. If we neglect secondaries, the negative surface potential of a stationary grain is then simply given by

$$[1 - (e\phi_s/kT_i)] = (m_i/m_e)^{1/2} e^{+e\phi_s/kT_e} \tag{8}$$

For $m_i = m_H$ (protons) and $T_i = T_e = T$ the solution is the well-known Spitzer [1978] result, $\phi_s = -2.51kT/e$. The grain is negatively charged. This is easily understood by noting that for $\phi_s = 0$ the electron current far exceeds the ion current, and the grain charge and its surface potential would become negative. As ϕ_s becomes negative, electrons are repelled, the electron current is reduced, and the ion current is increased. The two currents become equal when $\phi_s = -2.51kT/e$. At this value the net current to the grain is zero.

For the more general case, including all charging currents, the solution must be obtained numerically. The potential and hence the charge can be either positive or negative. Whereas in most cases one has only one solution for the surface potential, the zero-current requirement may also allow for multiple (one or three) solutions. This can be seen by considering the special case of zero photoelectron current but finite primary ion and electron currents and secondary currents. Figure 1 shows the primary electron and ion currents as a function of surface potential and the net current (ΣJ_α). There are three values of ϕ_s for which $\Sigma J_\alpha = 0$. The two extreme values correspond to stable equilibria, whereas the intermediate equilibrium is unstable.

Figure 2 shows the variation of surface potential as a function of plasma temperature ($T_e = T_i = T$) for different values of the secondary yield parameter δ_m. For $\delta_m > 5$, three solutions are possible for a certain temperature range ($T_1 < T < T_2$). If the plasma is cold ($T < T_1$), the grain is negative because the flux of secondaries is small. In a hot plasma ($T < T_2$) the grain becomes positive because the flux of secondaries exceeds the flux of primaries. This behavior suggests that in a plasma whose temperature fluctuates, some grains may be positive, whereas others are negative.

Consider, for example, an initially cold plasma. All grains are then negative. As the temperature increases above T_2, the equilibarium charge of all grains should become positive. However, the charge will not change instantaneously but will evolve according to (5). Large grains will respond rapidly and acquire a positive charge quickly. Small grains collect a smaller current and thus take longer to reach the new equilibrium value. If the temperature is reduced again before they reach the positive
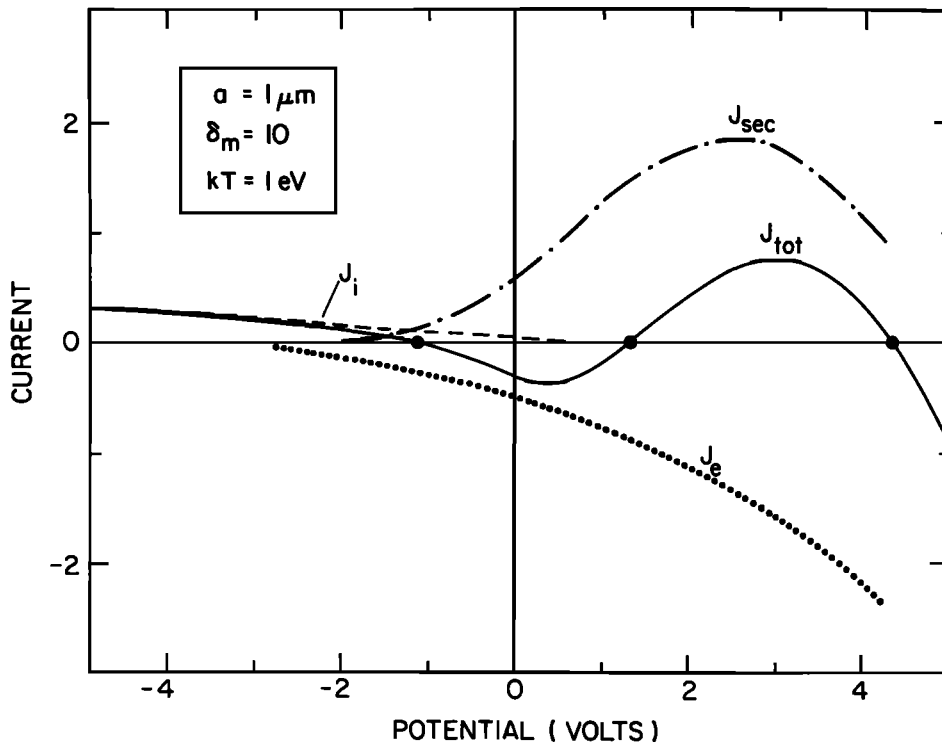
Figure 1. The charging current density due to plasma electrons (dotted curve) plasma ions (dashed curve), and secondaries (dashed-dotted curve) as a function of the grain's surface potential ϕ_s. The solid curve is the net charging current density including secondaries for $\delta_m = 10$ and $kT = 1$ eV. The equilibrium surface potential is given by the zeros of the net charging current.
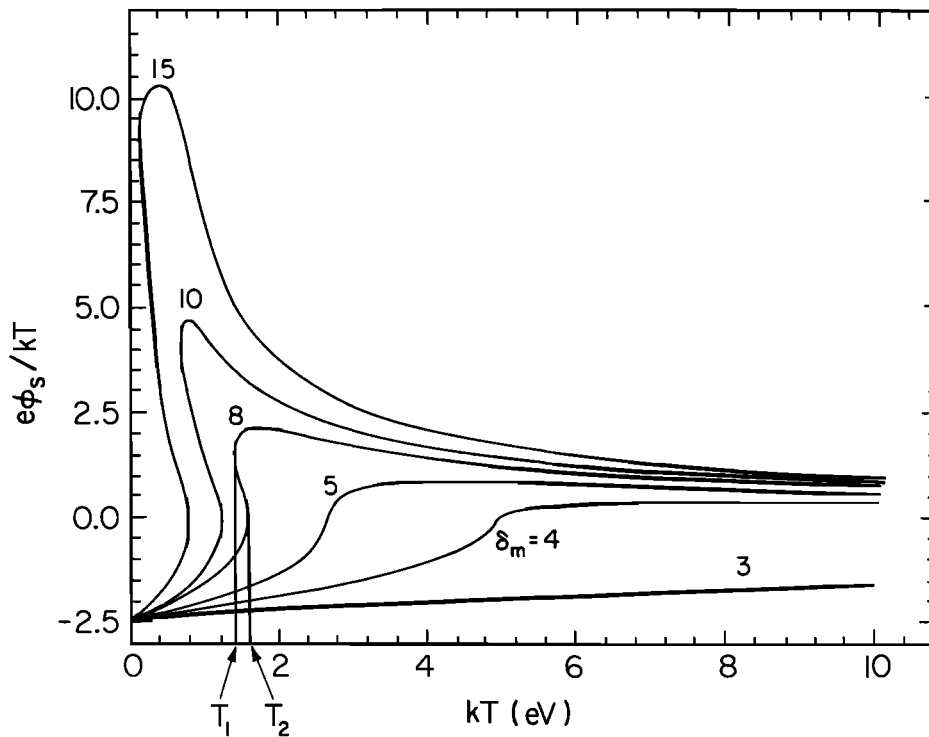

Figure 2. The equilibrium surface potential ϕ_s as a function of plasma temperature T for different values of the secondary yield parameter δ_m.

equilibrium value, their charge will decrease and remain negative. The big particles, however, can remain positive until T is decreased below T_1. One can solve the charging equations numerically. The results (M. Horanyi and C. K. Goertz, Coagulation of dust particles in a dusty plasma, unpublished manuscript, 1989) (hereinafter Horanyi and Goertz, 1989) are shown in Figure 3. In each figure we start with uncharged grains of different radii (varying from 1 μm to 6 μm) and keep $kT = kT = kT_0 = 1$ eV constant for 5 min. By that time all grains have obtained the same negative surface potential. After this time we allow the temperature to vary as

$$kT = kT_0 + k\Delta T \sin(\omega t) \qquad (9)$$

In Figure 3a we have $k\Delta T = 0.2$ eV and $\omega = 0.1$ s^{-1} (the period is 1 min). Clearly, the fluctuations in the surface potential (and hence charge) are larger for the big particles. If $k\Delta T$ is increased to 0.3 eV ($\omega = 0.1$ s^{-1}), the 6 μm particles obtain a positive charge and fluctuate about a positive equilibrium value, whereas the smaller grains remain negative as shown in Figure 3b. For $k\Delta T = 0.4$ eV (Figure 3c) the big grains' charge fluctuates greatly but has a positive time average. The small particles retain an average negative value.

The possibility that different size particles have opposite charge may be very important for coagulation of dust grains into bigger particles. Two grains with the same sign of charge have a smaller collision cross section thatn two particles of opposite charge. The evolution of the size distribution of dust grains could then be quite different from the standard coagulation model. Horanyi and Goertz (1989) show that the distribution becomes bimodal with a deep minimum at a certain critical size which depends on the material properties and the temperature fluctuation characteristics. They also find that the growth of bigger particles is enhanced because of this effect.

The charge on very small grains ($a < 0.1$ μm) is limited by field emission and electrostatic disruption [Öpik, 1956] which should occur if the surface field exceeds a critical value. The critical surface electric field ($E_s = \phi_s/4\pi\varepsilon_0 a$) for ion and electron field emission is ~5 ×10^{10} V/m and 10^9 V/m, respectively [Müller, 1956]. Electrostatic disruption occurs if the electrostatic energy density exceeds the tensile strength T_t (newtons per square meter) of the particle, that is if

$$E > E_d = 3.36 \times 10^5 T_t^{1/2} \qquad (10)$$

For fluffy aggregates, T_t is about 10^4 N/m^2 and E_d is 3 × 10^7 V/m; for ice T_t is between 10^5 and 10^7 N/m^2 and E_d varies between 10^8 and 10^9 V/m. For silicates the critical field strength varies between 3 × 10^8 and 3 × 10^9 V/m. For metals, $E_d \approx 1.5 \times 10^{10}$ V/m (for references, see Grün et al. [1984]). Electrostatic disruption for particles with rough
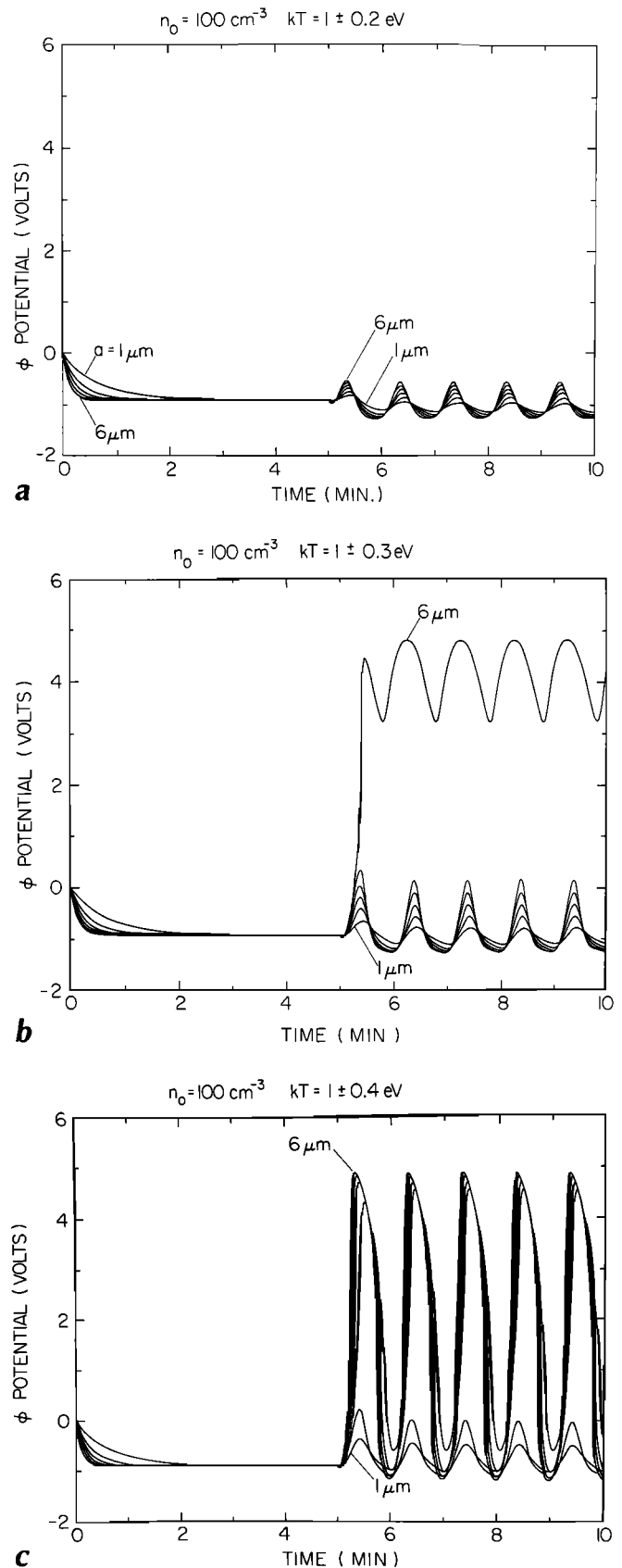
Figure 3. The evolution of the surface potential ϕ_s of grains with radius a varying from 1 μm to 6 μm. In all three panels, initially, $\phi_s = 0$. The plasma temperature is constant ($kT = 1$ eV) for 5 min, after which the temperature oscillates with a period of 1 min. The amplitude of the temperature oscillation is ΔkT equal to (a) ±0.2 eV, (b) ±0.3 eV, or (c) ±0.4 eV.

surfaces has been treated by *Hill and Mendis* [1981]. The electrostatic stress tends to erode bumps on the surface and render the grains more spherical. For 1-μm grains these effects limit the surface potential to about 10^3 V (for electron field emission) or 30 V if $T_t \simeq 10^4$ N/m^2.

2.2. Ensemble of Dust Grains

The potential in the vicinity of a singly-charged grain is given by the solution of Poisson's equation:

$$\Delta^2 \phi = (e/\varepsilon_0) [n_e(\phi) - n_i(\phi)] \qquad (11)$$

For a Boltzmann distribution of the plasma electrons and singly charged ions we have

$$n_e(\phi) = n_0 e^{+e\phi/kT_e}$$
$$\qquad\qquad\qquad (12)$$
$$n_i(\phi) = n_0 e^{-e\phi/kT_i}$$

The density n_0 is the electron density far away from the dust where the plasma potential is zero and the plasma is charge neutral. The surface electric field is given by

$$E_s = Q/4\pi\varepsilon_0 a^2 \qquad (13)$$

An analytic solution of (11) and (12) for $e\phi/kT \ll 1$ was discussed by *Whipple et al.* [1985], who show that for $a/\lambda D \ll 1$ the potential falls off as

$$\phi(r) = \phi_s(a/r)e^{-(r-a)/\lambda D} \qquad (14)$$

The exponential factor represents the Debye screening by the plasma with λ_D being the plasma Debye length. Now consider an ensemble of grains each carrying a charge Q_i. Then

$$\nabla^2 \phi = (e/\varepsilon_0)[n_e(\phi) - n_i(\phi)] + \Sigma Q_i f_i(r) \qquad (15)$$

where $f(r)$ is zero outside the grain and $f(r) = 3/4\pi a^3$ inside the grain assuming that the charge Q is uniformly distributed throughout the grain. For small grains ($a \ll \lambda_D$) we can write $f_i(r) = (3/4\pi a_i^3)\delta(r_i)$ where r_i is the position of the ith grain.

Figure 4 shows a steady state one-dimensional numerical solution of (5) and (15) for a series of 10 equally spaced infinite plane sheets of grains. Only charging due to plasma electrons and ions is considered. While such a one-dimensional solution is very unrealistic, it illustrates the effects of placing grains close to each other. If the distance between the grains is larger than the Debye length, the maximum potential between the grains is almost zero. The surface charge density on each sheet is σ $= E_s/\varepsilon_0 \simeq \phi_s/\lambda_D \varepsilon_0$. When the sheets are placed closer to each other, the potential between the grains is smaller, and the average charge density on each sheet is reduced. The
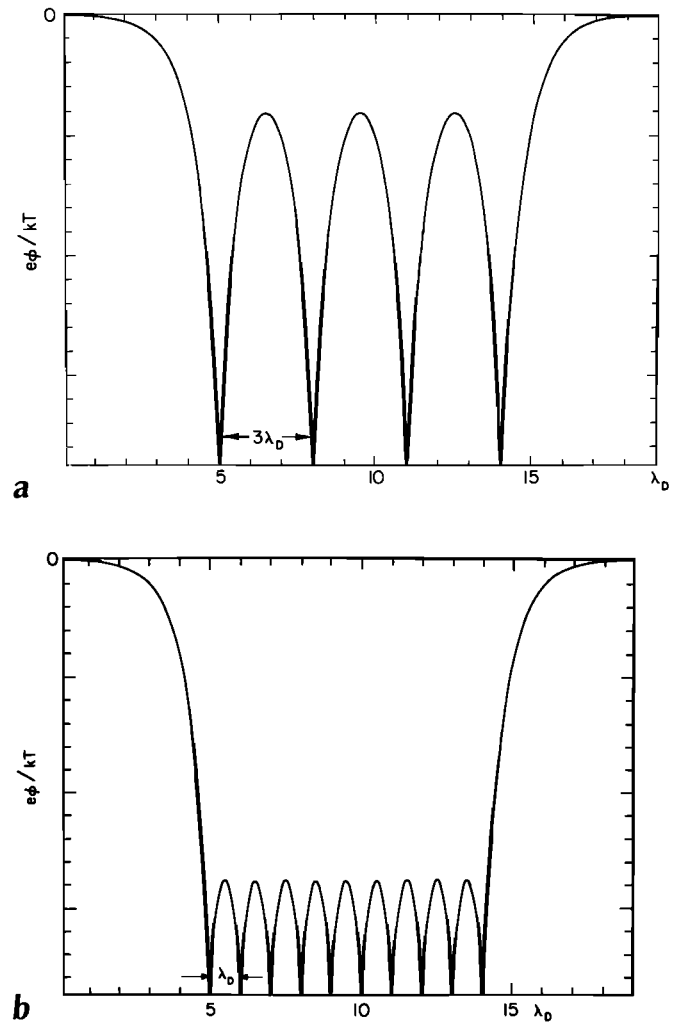

Figure 4. Steady state solution of the one-dimensional Poisson equation for grain sheets placed at regular intervals. The distance between grains is (a) $3\lambda_D$ and (b) $1\lambda_D$.

same effect occurs for grains in the three-dimensional case. When the intergrain distance d is smaller than the Debye length, the average charge on each grain is reduced, and the maximum potential between the grains is negative compared to the potential at infinity which we assume to be zero. The surface potential of each grain is still given by (8). However, the charge on each grain is now (for $a \ll \lambda_D$)

$$Q = (\phi_s - V_p)4\pi\varepsilon_0 a = U4\pi\varepsilon_0 a \qquad (16)$$

where V_p is the maximum plasma potential between the grains, i.e., where $\nabla\phi = 0$. The average charge on the grains is thus smaller than that of an isolated grain at the same surface potential (see also *Goertz and Ip* [1984]).

We can also calculate the average charge $\langle Q \rangle$ and the average plasma potential V_p by requiring overall charge neutrality

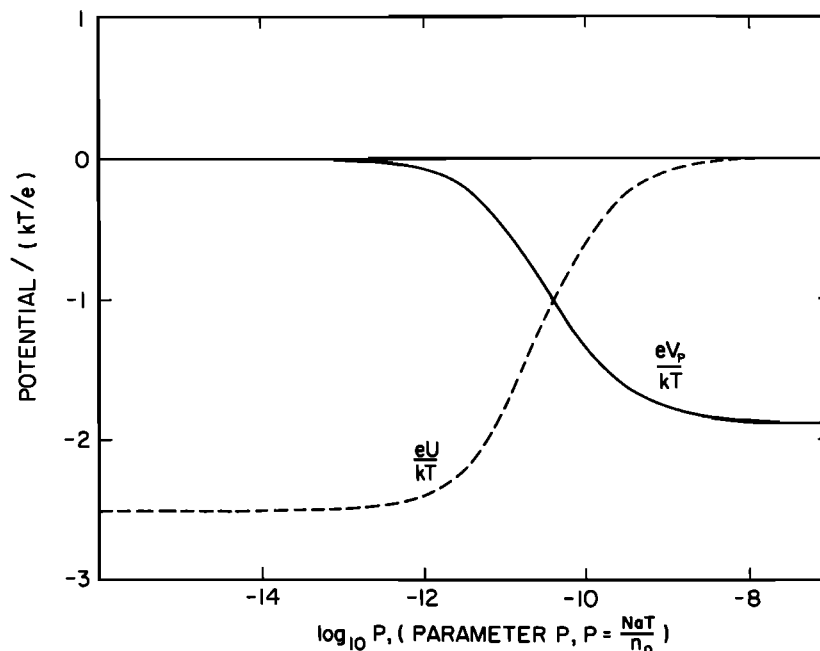$$e[n_e(V_p) - n_i(V_p)] - N\langle Q \rangle = 0 \qquad (17)$$

Figure 5. The variation of plasma potential V_p and grain potential U (proportional to the grain charge) as a function P, i.e., dust density. T is the plasma temperature in electron volts, a is the grain radius in meters, and N and n_p are the dust and plasma density, respectively.

where N is the density of grains. We also require zero current to the grain:

$$\Sigma J_\alpha = 0 \tag{18}$$

The set of equations (1), (2), and (16) through (18) completely specifies the problem.

Havnes et al. [1987] have shown that Q (or U) and V_p only depend on the parameter

$$P = aNT/n_0 \tag{19}$$

The variations of V_p and U with P are shown in Figure 5. The expected result is that when P is small (i.e., small dust density N), U approaches the classical single-grain value of $-2.5kT_e/e$, and the plasma potential is zero. If the density N and hence P are increased, the magnitude of the average charge (proportional to U) is reduced, and the plasma potential becomes negative. This collective effect (Debye screening) is most important when $P \approx P_c = 10^{-9}$ if T is the thermal energy of the plasma electrons (kT_e) expressed in electron volts. At this value of P the charge density carried by the grains (i.e., NQ) is comparable to the charge density of the electrons $(n_0 e)$, and hence most of the electrons are trapped on the grains. In that case the dusty plasma consists mainly of positive ions and negatively charged massive grains. If P becomes larger than 10^{-8}, the average charge on a grain is very small, and collective electrostatic effects are negligible. If P is less

than 10^{-10}, the grains can be treated as isolated grains, and collective effects are also small. The parameter P for various magnetospheric environments is shown in Table 1.

3. THE MOTION OF CHARGED DUST GRAINS

3.1. Forces on Charged Grains

The equation of motion of a grain is given by

$$m \frac{d\mathbf{v}}{dt} = Q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \mathbf{F}_G + \mathbf{F}_c - \pi a^2 \mathbf{q} \tag{20}$$

where $\mathbf{F}_G$ is the gravitational force, and $\mathbf{q}$ is the radiation pressure.

The Coulomb drag $\mathbf{F}_c$ to the relative motion $\mathbf{w}$ between the grain $(\mathbf{v})$ and the plasma $(\mathbf{v}_p)$ was calculated by *Morfill et al.* [1980]. However, T. G. Northrop (private communication, 1988) has questioned the accuracy of their expressions.

The radiation pressure is directed away from the Sun and given by

$$q = f_{pr} S_{Sun}/c D^2 \tag{21}$$

where f_{pr} is the scattering efficiency for radiation related to the albedo of the dust particle, S_{Sun} is the solar constant, D is the distance from the Sun (in astronomical units), and c is the speed of light.

TABLE 1. Nominal Values for Various Planetary Ring Environments

Ring	n_p, m^{-3}	kT/e, eV	τ	H, m	a, m	N	P
AB	10^5	2	2	10^2	5	2.5×10^{-2}	3×10^{-8}
AB	10^5	2	2	10^2	1	6.3×10^{-1}	2×10^{-7}
F		10^7	10^2	0.1	10^3	10^{-2}	0.32 3×10^{-8}
F		10^7	10^2	0.1	10^3	10^{-6}	3.2×10^7 3×10^{-4}
Spokes	10^5	2	0.1	3×10^4	10^{-6}	10^6	2×10^{-5}
Spokes	10^8	2	0.1	3×10^4	10^{-6}	10^6	2×10^{-8}
Uranus α	5×10^7	30	0.35	10^2	5×10^{-2}	0.446	10^{-8}
Uranus α	2×10^8	30	0.35	10^2	5×10^{-2}	0.446	3×10^{-9}
Uranus ε	5×10^7	30	1.8	10^2	0.2	0.14	2×10^{-8}
Uranus ε	2×10^8	30	1.8	10^2	0.2	0.14	4×10^{-9}
JR	10^8	10^2	10^{-5}	3×10^4	2×10^{-6}	26.5	3×10^{-10}
G	10^7	10^2	3×10^{-4}	3×10^5	3×10^{-4}	3.5×10^{-3}	10^{-11}
G	10^7	10^2	10^{-6}	10^6	10^{-6}	3.1×10^{-1}	3×10^{-12}
E	10^7	10^2	10^{-6}	10^6	10^{-6}	0.32	3×10^{-12}
E	10^7	10^2	10^{-7}	3×10^7	10^{-6}	1.1×10^{-3}	10^{-14}

In planetary magnetospheres the plasma often corotates with the planet rotating at a rate of Ω:

$$v_p = (E \times B)/B^2 = \Omega \times r \qquad (22)$$

This, of course, is not true in the distant magnetosphere of the Earth where the solar wind induced convection becomes important. In the vicinity of comets the flowing plasma is the solar wind.

There is no simple way of estimating the magnitude of each force and deciding that one or the other can be neglected. Unfortunately, one must evaluate all for each particular case of interest.

3.2. Gravitoelectrodynamics in Planetary Magnetospheres

It is easy to show that for large particles the gravitational force dominates the motion of charged grains in planetary magnetospheres, because the ratio of the gravitational to electromagnetic force at the equator at a radial distance LR_p (R_p is the planetary radius) in a corotating magnetosphere is

$$R = \frac{F_{EM}}{F_G} = \frac{Q}{m} \frac{|(\Omega \times r - v) \times B|}{\Omega_K^2} r = K_p \frac{\phi_s w_\perp}{a^2 L} \qquad (23)$$

where $\Omega^2_K = GM_p/r^3$ is the Kepler rotation rate at a distance r from the planet whose mass is M_p and which is rotating at the rate Ω. The relative velocity between the plasma and the grain perpendicular to B is $w_\perp = |(\Omega \times r - v) \times B/B|$ measured in meters per second. For Jupiter, $K_p = 4 \times 10^{-19}$ m s V^{-1}, and for Saturn it is 5×10^{-20} m s V^{-1}. For a 1-μm particle orbiting Saturn at $L = 2$ where $w_\perp \sim 10^3$ m/s (Keplerian orbit), we have $R = 2.5 \times 10^{-5} \phi_s \ll 1$ for any reasonable plasma conditions. Thus these particles should to first order move on Keplerian orbits. It is this kind of reasoning that may have convinced many researchers that a

nonzero grain charge has no important dynamical consequences. However, many interesting effects rely on small perturbations from Keplerian orbits. These may be due to, for example, the small perturbing force of a nearby satellite. The electrodynamic force, however, may be much larger than this force. Thus when calculating perturbations from Keplerian orbits, electrodynamic forces must be considered.

A complete discussion of gravitoelectrodynamics is beyond the scope of this review. We will concentrate on the simple case of a planet whose magnetic dipole moment and rotation axis are parallel (Saturn) and grains moving in the equatorial plane. Mendis et al. [1982] and Northrop and Hill [1982, 1983] have shown that circular orbits are possible with an angular velocity

$$\omega = \frac{\omega_g}{2} - 1 \pm \left[1 + 4\left(\frac{\Omega}{\omega_g} + \frac{\Omega_K^2}{\omega_g^2} \right) \right]^{1/2} \qquad (24)$$

where

$$\omega_g = -QB/m \qquad (25)$$

is the gyrofrequency of the grain. The plus sign in (24) refers to prograde motion while the minus sign corresponds to retrograde motion for negatively charged grains ($\omega_g > 0$).

If the grain's orbit is slightly perturbed in the equatorial plane, its motion can be described as an elliptical gyration about the guiding center of a grain which moves on a circular orbit with frequency ω. The frequency of the radial oscillation is given by

$$\omega_r^2 = \omega_g^2 + 4\omega_g \omega + \omega^2 \qquad (26)$$

The minor axis of the ellipse is aligned in the radial direction, and the ratio of semimajor to semiminor axes is

$$\varepsilon = -\omega_r/(2\omega + \omega_g) \qquad (27)$$

We see that not all orbits are stable; stable orbits must satisfy the condition $\omega_r^2 > 0$. The range of parameters (grain charge, mass, radial distance) for which stable orbits are possible has been investigated by *Mendis et al.* [1982].

For perturbations out of the plane the oscillation frequency is given by *Schaffer and Burns* [1987]:

$$\omega_z = -\frac{\omega_g}{2} \pm \Omega_K \left[1 + \frac{\omega_g \Omega}{\Omega_k^2} \right]^{1/2} \qquad (28)$$

These two oscillation frequencies (ω_r, ω_z) are changed slightly when the oblateness of the planet is taken into account [*Schaffer and Burns*, 1987]. Both ω_r and ω_z are close to Ω_K (at least for large particles).

Several interesting consequences result from these considerations. The known planetary magnetic fields are not purely dipolar but contain quadrupole and octupole components. If the variation of the magnetic field seen by a grain moving through the nondipolar magnetic field of planet matches these oscillation frequencies or harmonics thereof, the excursion of the grains from circular orbits can grow exponentially in time. Schaffer and Burns show that the magnetic field variation seen by the grain acts as a periodic forcing term which can produce large resonant responses in the grain orbits. The enhanced vertical excursion of grains at a vertical resonance location may lead to a thickening of the ring. The resonances occur at specific radial distances which may correspond to various observed features of the radial optical depth profile of the Jovian ring. However, the match between predictions and observations is only partial, and more work needs to be done to better understand these magnetic resonances. This should be especially interesting for the Uranian rings which exist in a highly nondipolar field.

Outside of synchronous orbit (where $\Omega = \Omega_K$), negatively charged grains in prograde orbits have $\omega > \Omega_K$ and could thus be in a 1:1 resonance with a satellite interior to the grain orbit. Positively charged grains could be in resonance with a satellite outside the grain orbit. *Mendis et al.* [1982] show that a grain with a frequency ω_r will move in an undulating orbit with a wavelength λ given by

$$\lambda = (2\pi r/\omega_r)\,(\omega - \Omega_{Sat}) \qquad (29)$$

where $\Omega_{Sat} = \Omega_K(r_{Sat})$ is the Kepler rotation of the satellite. Because each successive grain of the same size moving past the satellite will be subject to the same perturbation force, it will follow the same path as its predecessor. The grains will move in phase and form a wave pattern with wavelength λ carried around by the satellite. *Mendis et al.* [1982] suggest that the wavelike pattern observed in Saturn's F ring is produced this way. However, it is not clear that the grains' charges are large enough in the F ring to account quantitatively for the observed wavelength. One must also stress that the wavelength depends on the grain's size, and resonances related to the grain charge pick

out a particular grain size for a given surface potential ϕ_s, i.e., for given plasma parameters. If there are grains of different sizes, the waves should be smeared out.

Northrop and Hill [1982, 1983] pointed out that the stability limit for vertical excursion $(\omega_z^2 = 0)$ for grains whose charge to mass ratio is very large $(\omega_g \gg \Omega_K)$ and thus that orbit Saturn at the local corotation rate occurs at a distance of $r_c = 1.625R_e$ from Saturn. This corresponds reasonably well to the inner edge of the B ring. If the grains are launched in the ring plane with the local Kepler velocity, the orbit will not be circular because of the electromagnetic force. For this case the stability limit is found to be exactly at the inner edge of the B ring at $r_c = 1.5245R_s$ [*Northrop and Hill*, 1983]. For $r < r_c$ a grain with a velocity directed away from the ring plane would not return to the ring but would be lost into the atmosphere of Saturn. Beyond r_c the grain is stably trapped and can be reabsorbed by the ring. Thus a continuous production of grains would yield an erosion of the rings and hence a decrease of optical depth inside r_c but not outside of it. The work of Northrop and Hill, however, does not really apply to dust grains, for which, in general, $\omega_g \ll \Omega_K$, but more to water molecule cluster ions. They suggest that such molecular ions could be created by micrometeoroid impacts onto the ring.

3.3. Transport of Charged Grains in Planetary Magnetospheres

Charged grains are subject to azimuthal (about the planet) as well as radial electromagnetic forces in planetary magnetospheres, so their angular momentum is not constant. This will produce a radial transport and distribute grains throughout the magnetosphere. In general, one would have to follow the motion of individual grains numerically by solving the equation of motion including variations of B, E, and Q along the grain's orbit. This has been done for grains in the Earth's magnetosphere by *Horanyi and Mendis* [1987]. They start with dust grains of various sizes at geosynchronous orbit uniformly distributed in longitude and move the grains subject to gravity, corotation, and solar wind induced convection electric fields and radiation pressure. The plasma distribution in their model, although simplified, includes the plasmasphere which contains dense cold plasma and the plasma sheet with hot tenuous plasma. Figure 6 shows the distribution of grains ($a = 0.1$ μm) 3 days after the injection. A very rapid dispersal of grains has apparently taken place.

It is, of course, rather difficult to understand the details of this figure. But several features can be accounted for qualitatively. Radiation pressure causes the grain orbits to become elliptical and accounts for the shift of the orbits toward 0600 LT. The eccentricity of the orbits increases with time, and once they become large enough, the grains can intersect the Earth and be lost. The authors also
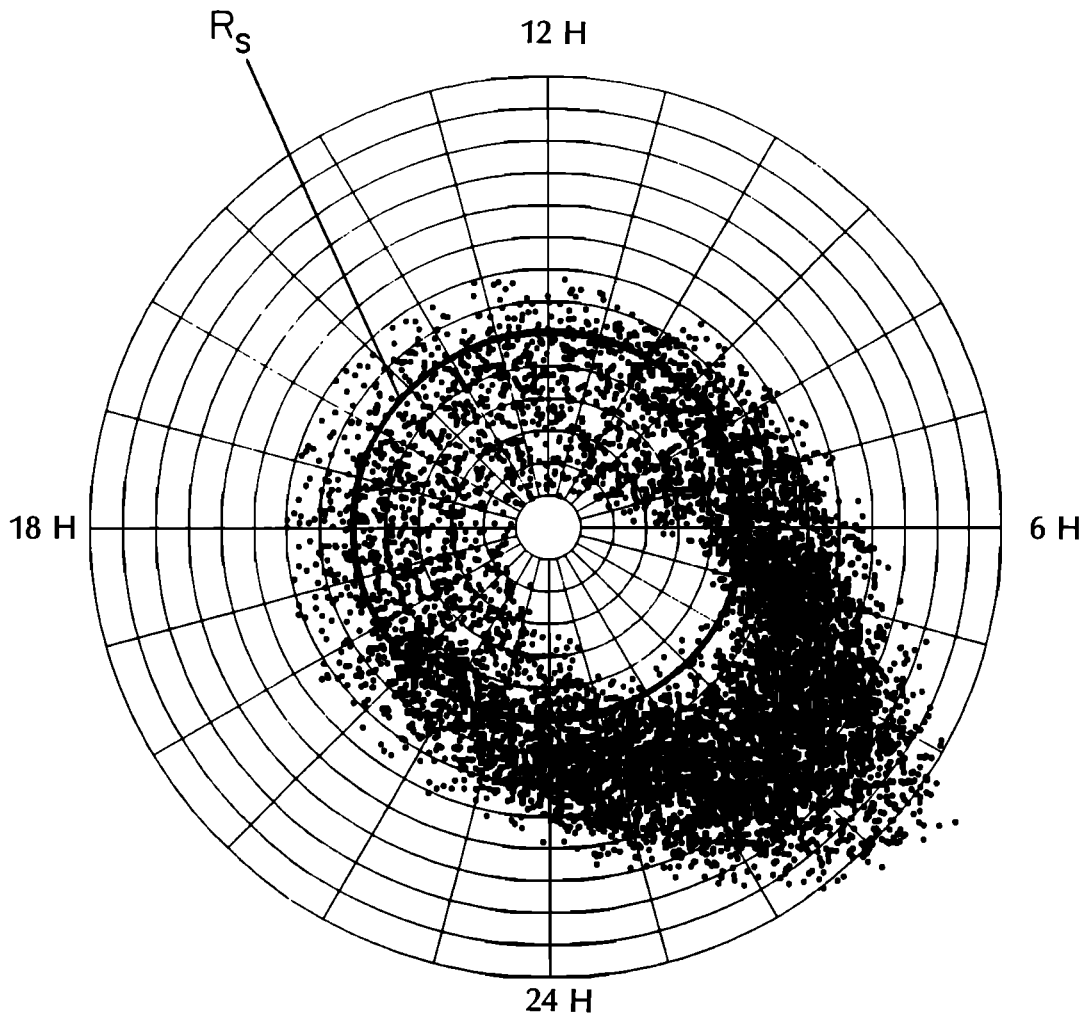
Figure 6. The distribution of 0.1-μm olivine type grains in the Earth's magnetosphere 3 days after injection at synchronous orbit. Each circle represents a dust grain [from *Horanyi and Mendis*, 1986].

assume that once the grains exit the magnetosphere into the solar wind, they are carried away and lost from the system. Plasma drag also leads to an orbital decay inside synchronous orbit and an outward radial drift outside of synchronous orbit. This is strictly true only for a corotating plasma, which is not the case outside the Earth's plasmapause.

The grain charge fluctuates stochastically in a homogeneous plasma because the electron and ion currents fluctuate. These fluctuations of the grain charge can lead to a radial diffusion of grains. Charge fluctuations are, of course, more important for smaller grains than for bigger grains which are gravitationally dominated. Another subtle but quite important effect is due to the temporal variation of Q as the grains move through radial gradients in the plasma parameters, e.g., temperature and density variations which exist, for example, at the plasmapause or the inner edge of the plasma sheet. Density gradients are important only when photoelectrons are included. Without photoelectrons, (8) applies, and the grain charge is independent of the plasma density. It is not obvious that radial gradients of plasma parameters should cause a change of the grain's angular momentum which, after all, requires an azimuthal force. If the grain acquired its equi-

librium charge instantaneously, the average electromagnetic force integrated over one gyro-orbit about the guiding center would only have a radial component and hence not exert a torque. However, the grain will not change its charge instantaneously, and hence there will be a phase lag in the charge with respect to the orbital position. The average electromagnetic force then has an azimuthal (longitudinal) component and does exert a torque on the grain which leads to radial transport. This "gyrophase drift" can be very large for small grains in Jupiter's magnetosphere, especially near plasma temperature gradients at the inner edge of the Io torus [*Northrop et al.*, 1989]. A similar effect also occurs even in a homogeneous plasma because the charge changes as the grain gyrates about its guiding center, since the charge depends on the velocity relative to the plasma which varies along the grain's gyro-orbit. Again there is a phase delay and hence a net azimuthal force.

3.4. Transport of Grains Near Comets

The motion of grains near comets is influenced by radiation pressure, plasma drag, and the electromagnetic force. Gravity plays no important role. Large grains are not affected by the electromagnetic force. But the

distribution of grains of submicron size is very much influenced by it as shown by *Horanyi and Mendis* [1985, 1986]. Because the average direction of the solar wind electric field is perpendicular to the ecliptic, the distribution of small grains in a plane normal to the ecliptic is significantly different from that of bigger grains as shown in Figure 7. It is somewhat surprising that the actual charge of the grains is quite modest. While the surface potential (which is the same for all grains) is about -9 V between the cometary ionopause and the outer shock, it is only +1.5 V in the solar wind.
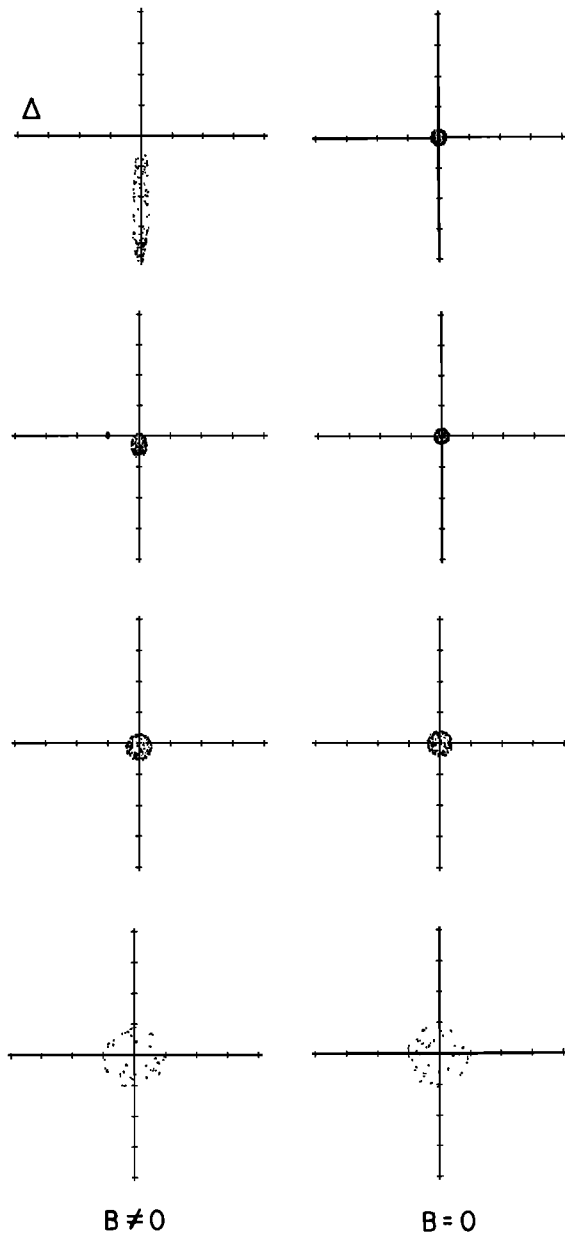
When the direction of the solar wind magnetic field changes as a sector boundary moves across the comet, the electric field direction changes, and the deflection of the grains out of the ecliptic reverses. Figure 8 shows the position of 0.1-μm grains in a plane normal to the ecliptic 10^4 km behind the comet for 8 days after a sector boundary sweeps over the comet [*Horanyi and Mendis*, 1987]. Such excursions of cometary dust tails are frequently observed. However, one must note that the shaping of the dust tail by electric forces depends on the grain's size. If the size distribution of grains has a nonzero spread, one would expect such wavy tails to be smeared out. Thus it is not entirely clear whether this effect can account for the observations. However, it is clear that the distribution of dust grains in a cometary environment can only be understood when the electric forces are taken into account.

Figure 7. The distribution of dust grains 10^4 km behind the nucleus of a comet in a plane perpendicular to the ecliptic for different size grains [from *Horanyi and Mendis*, 1986]. The sizes are, from top to bottom, 0.03 μm, 0.1 μm, 0.3 μm, and 1 μm. The solar wind electric field causes the downward displacement of the small grains. The case $B = 0$ does not include the electromagnetic force. For $B \neq 0$, there is an electric field given by $E = B \times V$ where V is the solar wind velocity.
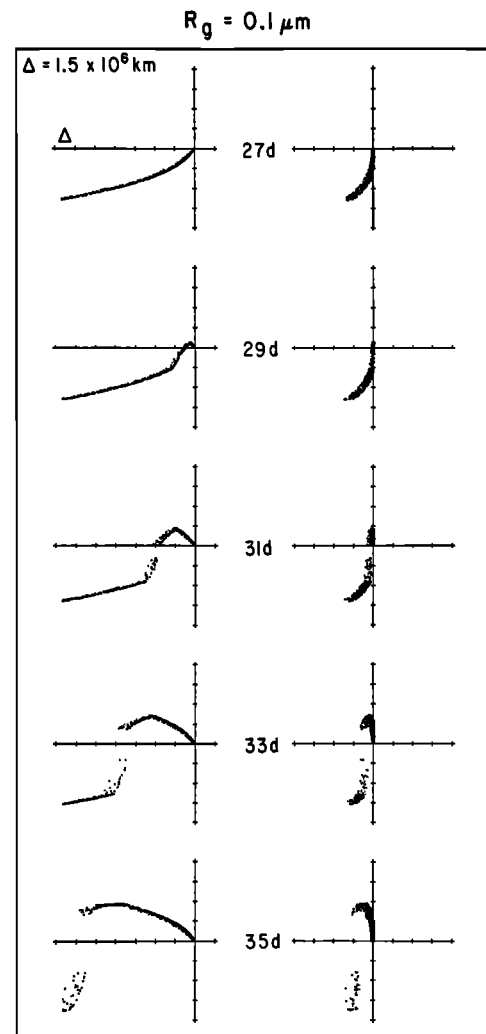

Figure 8. The evolution of a dust tail near a comet with time after a sector boundary has passed across the comet [from *Horanyi and Mendis*, 1987]. Here the grain radius is 0.1 μm. The left-hand panels show projections of the dust tail onto a plane perpendicular to the orbital plane and containing the Sun-comet axis. The Sun is to the right. The comet moves perpendicular to the paper (into the paper). The right-hand panels show the trail in a plane perpendicular to the Sun-comet axis. The comet moves to the right.

4. DUSTY PLASMAS IN PLANETARY RINGS

Voyager observations [*Smith et al.*, 1981, 1982] of the rings of Saturn have revealed that they are not smooth disks but show large radial variations of optical depth at scale lengths ranging from less than a few tens of kilometers to several thousand kilometers. These are not related to any known resonances with Saturn's moons and associated density waves, and there is no universally accepted explanation of the structure. Voyager has also revealed the existence of radially elongated thin (in azimuth) structures (spokes) shown in Figure 9 which appear dark in backward scattered but bright in forward scattered light. This suggests that they contain small micron size or submicron size grains. In this section we will discuss the only detailed spoke formation mechanism published so far [*Goertz and Morfill*, 1983; *Morfill and Goertz*, 1983; *Goertz*, 1984]. We will also show that the radial displacement of the dust grains contained in the spokes causes a significant transport of angular momentum which can lead to the exponential growth of perturbations in the ring surface mass density.

4.1. The Formation of Spokes

The spoke model is based on the assumption that the spokes contain electrostatically levitated small dust particles (radius less than 1 μm) and that the thin radial elongation is due to rapid radial motion of dense plasma clouds whose radii are of the order of several thousand kilometers. The small dust particles coexist with large ring particles (centimeter to meter size). They either reside on the large particles, move between the ring particles, or are lifted off the ring particles above the ring plane. Because of the presence of such plasma clouds the ring charges to a surface potential Φ_R of the order of -6 V (see Figure 10). The negative potential repels the electrons and reduces their density from the value n very far away from the ring to $n_e < n_i = n$, so that at the surface of the ring the current density carried by the plasma electrons ($n_e c_e$) is equal to that of the ions ($n_i c_i$) and the net current is zero. (Without a dense plasma the ring may be charged positively on the sunlit side.) The potential varies from this value at the surface of the ring ($z = 0$) to zero in the plasma (as $z \gg \lambda_D$) through a Debye sheath. The direction perpendicular to the ring plane is the z-direction. The electric field at the
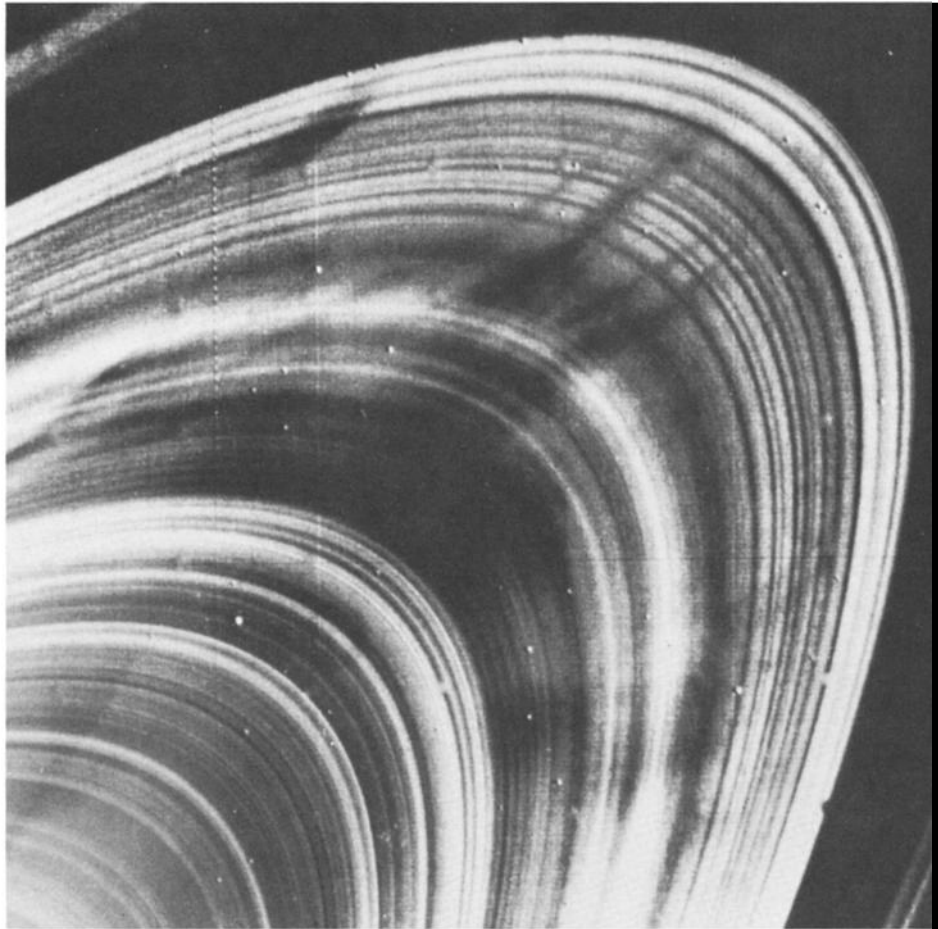

Figure 9. A picture of the radial spokes in Saturn's B ring. The azimuthal width of a spoke is typically a few thousand kilometers, which in the electrostatic levitation model corresponds to the size of the meteorite impact produced plasma cloud. The radial extent is typically several 10^4 km, which corresponds to the distance traveled by the plasma cloud from its origin at the point of a meteorite impact. The length is finite because the plasma cloud has a finite lifetime.
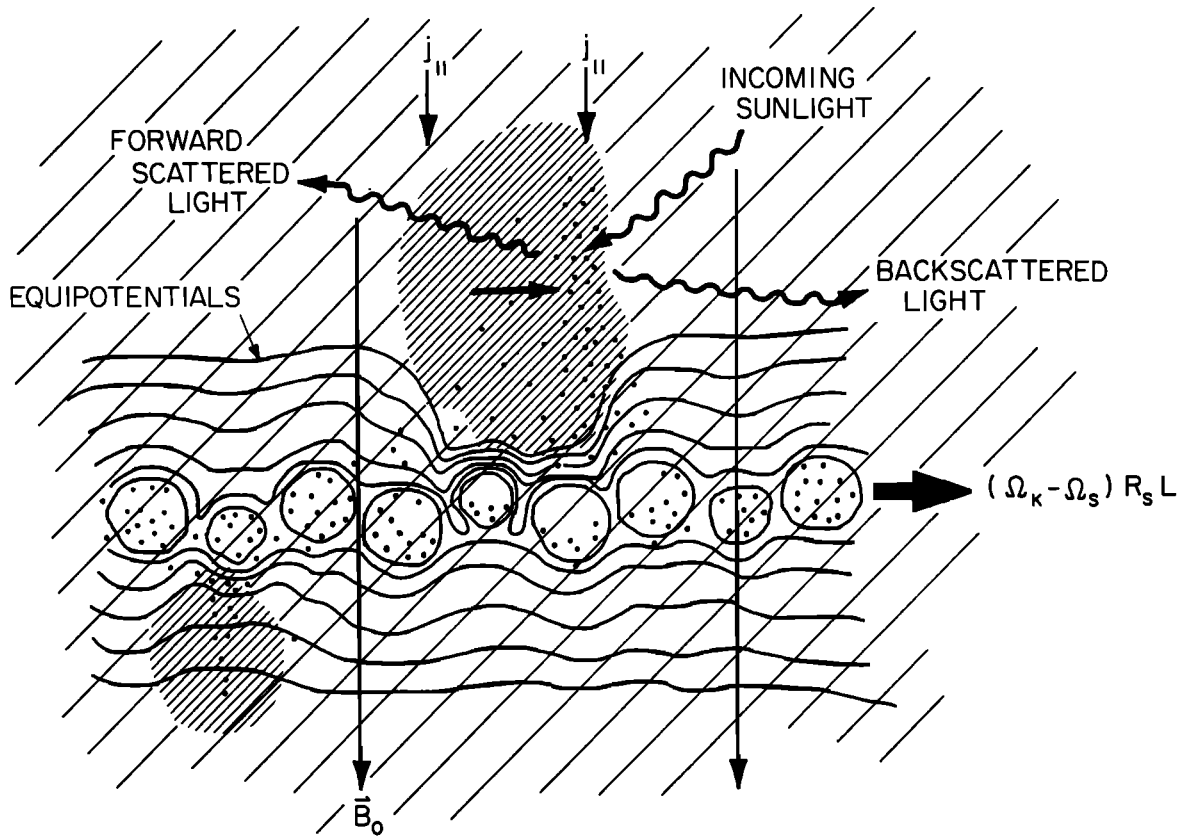
Figure 10. A schematic illustration of the electrostatic levitation model for the formation of spokes in Saturn's rings. Underneath the dense plasma cloud the normal electric field is increased, and more dust particles are elevated. The dust particles move in the azimuthal direction (toward the right) relative to the plasma, causing an azimuthal electric field and field-aligned currents at the edge of the plasma cloud. The azimuthal electric field causes the cloud to move radially, i.e., out of the plane of the paper.

surface $(z = 0)$ is $E_z = \Phi_R /\lambda_D$ and may be strong enough to lift negatively charged dust particles of mass m if the normal electric field E_z satisfies the inequality

$$E_z > \frac{m}{Q} g_\perp = a^2 \left(\frac{\rho}{3\varepsilon_0}\right) g_\perp/\Phi_R \qquad (30)$$

where the vertical component of the acceleration due to gravity, including the self-gravity of the ring, is $g_\perp$. The bulk mass density of a grain is ρ ($\sim 10^3$ kg m^{-3}). While a dust particle resides on a big ring particle, or moves on a Kepler orbit between the big ring particles, its average charge is very small because the surface charge density $(\sigma_q = E_z/4\pi\varepsilon_0)$ times the area of a dust particle is small $(\sigma_q \pi a^2 <<e)$. In other words, it is very unlikely for a particular submicron size particle to have even one excess electron. This has been called into question by P. Goldreich (private communication, 1982), who claims that the charge on one dust particle fluctuates greatly between about $\pm 4\pi\varepsilon_0 \Phi_R a$ with only the average being small. This can be a large number $(\sim\pm 200e)$. However, if a dust particle, by chance, collected two or more electrons, it would very rapidly discharge these electrons through

conduction. Consider two neighboring dust particles residing on a big ring particle, one with an excess of negative charge and the other with an excess of positive charge. The electric field produced by these charges in the big ring particle would be very large between them, and the conduction current through the big ring particle would rapidly neutralize the excess charges. The discharge time of a spherical dust grain embedded in a material of resistivity η is $\eta\varepsilon_0$. The charging time due to absorption of electrons or ions for dust particles in the ring plane is $t_c = 1/(4\pi a^2 F_i)$, where $F_i = F_e = nc_i$ is the thermal ion flux to the ring $(c_i$ is the ion thermal velocity) which in equilibrium at the ring surface is equal to the electron flux. For a particle of $a = 0.5$ μm, a plasma density $n = 10^{-2}$ cm^{-3}, and a plasma temperature of $kT_e = kT_i = 2$ eV, we obtain $t_c \simeq 3000$ s for a hydrogen plasma. Thus the resistivity must be larger than 3×10^{14} Ω m for this charging time to be smaller than the discharge time by currents through the resistive medium of the big ring particles. It seems unlikely that the resistivity of the ring (or the plasma in the ring plane) is that large. We thus believe that the surface charge density of the ring is uniform and does not fluctuate from one dust particle to the next as suggested by

Goldreich. In that case the probability of a dust particle having one excess electron is given by

$$\Psi_e = \sigma_q (4\pi a^2/e) = (\Phi_R/\lambda_D) (a^2/e)\varepsilon_0 \qquad (31)$$

This is much smaller than 1 for particles satisfying (30). But once a dust grain acquires one excess electron, the normal electrostatic force away from the ring plane is larger than the gravitational force directed toward the ring plane, and the dust grain will be lifted out of the ring plane provided its radius is less than about 10^{-2} cm (see, for example, Goertz [1984]). However, this does not guarantee that this dust grain will escape the Debye sheath because while being in the Debye sheath where the ion density is larger than the electron density, its charge fluctuates randomly, and it is likely to acquire an ion and fall back onto the ring particle. The time to escape the Debye sheath is

$$t_D \simeq \lambda_D (m/e\,\Phi_R)^{1/2} \qquad (32)$$

Thus the net probability for a grain to be lifted above the ring plane and escape from the Debye sheath can be written as

$$\Psi = \Psi_e e^{-t_D/t_c} \qquad (33)$$

This probability has a peak at a grain radius given by Goertz [1984]:

$$a_0^{-7/2} = (4\pi/7)n\,c_i\,\lambda_D\,(\pi\rho/3\Phi_R)^{1/2} \qquad (34)$$

where ρ is the mass density of the grain. This has recently been verified by a Monte-Carlo simulation of the charging-discharging process in the ring's Debye sheath (L. H. Shan and C. K. Goertz, The electrostatic levitation of dust particles in Saturn's ring, unpublished manuscript, 1989) (hereinafter Shan and Goertz, 1989). The flux of escaping dust particles is given by Goertz [1984] as

$$F = \int \frac{\sigma_D^N(a)}{t_D(a)} \Psi(a)\,da \qquad (35)$$

where $\sigma_D^N(a)$ is the dust column number density in the ring plane. If the dust particles stay above the ring plane for a time t (roughly one half Kepler period or 5 hours near synchronous orbit), the optical depth of the elevated dust is (for continuous dust elevation)

$$\tau_D = t \int \frac{\sigma_D^N(a)}{t_D(a)} P(a)\pi a^2\,da \qquad (36)$$

The optical depth of dust in the ring plane is τ_0

$$\tau_0 = \int \sigma_D^N N(a)\pi a^2\,da \qquad (37)$$

Most of the dust particles presumably reside on the big ring particles and form a thick regolith layer with $\tau_0 > 1$. If we approximate the function $\Psi(a)$ by a δ function $\Psi(a) = \Psi(a_0)\delta(a - a_0)$, we find

$$\tau_D \approx \tau_0 \Psi(a_0)t/t_D \qquad (38)$$

Morfill and Goertz [1983] have shown that if the plasma near the ring is only of ionospheric origin, its density is small ($n = 10^{-2}$ cm^{-3}). In that case, $a_0 \approx 10^{-4}$ cm, $P(a_0) = 3 \times 10^{-6}$, $t_D = 100$ s, and we find that $\tau_D \sim \tau_0 \times 5 \times 10^{-4}$. Such a small optical depth dust halo is difficult to detect.

However if n_0 were increased to 300 cm^{-3}, one obtains $a_0 = 5 \times 10^{-5}$ cm, $\Psi(a_0) = 10^{-4}$, $t_D = 0.1$s, and thus $\tau_D = \tau_0 \times 10^{-3} \times t$ where t is either the residence time of the dust grains above the ring plane or the duration during which the enhanced plasma density exists (whichever is smaller). Goertz and Morfill [1983] argue that enhanced plasma densities can be produced by meteorite impacts on the ring. Such an enhanced density is, of course, a transient phenomenon, and the time t needs to be calculated by considering the fate of the meteorite impact produced plasma cloud.

After a dust grain escapes from the Debye sheath into the charge neutral plasma above (or below) the ring plane, its charge will increase with time because it is now in a charge neutral region where the electron flux is much larger than the ion flux because $c_e \gg c_i$. The time scale for acquiring negative charge is now $t_c(m_e/m_i)^{1/2}$ which for $a = 5 \times 10^{-4}$ cm, $kT_e = 2$ e V, and $n = 300$ cm^{-3} is only a few milliseconds. Thus the dust grains will all obtain their equilibrium charge Q discussed in section 2. However, because Q/m is still small, the grains will move on nearly Keplerian orbits as shown in section 3. The plasma cloud, on the other hand, corotates with Saturn. Thus there is an azimuthal current which causes an azimuthal charge separation field as indicated in Figure 10. The magnitude of this azimuthal field, E_ϕ, has been calculated by Goertz and Morfill [1983] by balancing the current carried by the relative motion between the negative dust particles and the plasma with the field-aligned current which eventually closes by a Pedersen current in Saturn's ionosphere:

$$E_\phi = \frac{4\tau_D}{\Sigma_p \varepsilon_0 a_0^2} Q(\Omega_K - \Omega_s)R_s L \qquad (39)$$

where Σ_p is the ionosphere Pedersen conductance (~0.1 mho), Ω_s is Saturn's rotation rate, and R_s is Saturn's radius. The plasma cloud will $E_\phi \times B$ drift in the radial direction with a speed $V_R = E_\phi/B$ where $B(B = B_0 L^{-3})$ is the magnetic field of Saturn at the distance LR_s. The time for the plasma cloud of radius R_c to pass across a certain point is

$$t = R_c/V_R \qquad (40)$$

Since τ_D is proportional to t, we can combine (38) through (40) to yield a radial velocity

$$V_R^2 = \left(\frac{9\tau_0\,\Psi(a_0)}{\varepsilon_0\,t_D\,a_0^2}\right)\frac{Q}{B}\,\frac{(\Omega_K-\Omega_s)}{\Sigma_p}R_c r \qquad (41)$$

Goertz and Morfill [1983] have shown that this velocity ranges from zero exactly at synchronous orbit ($L_s = 1.866$) to several tens of kilometers per second inside $L = 1.8$ and outside $L = 1.9$. The plasma cloud moves away from synchronous orbit if $Q < 0$ (as it should be). As it does, it leaves behind a radial trail of elevated dust particles, the spoke.

At synchronous orbit where $\Omega_K = \Omega_s$ the radial velocity is zero, and a cloud at synchronous orbit would not move. However, only clouds whose radius is at least 1000 km will leave trails that are resolved by Voyager. Thus any meteorite impact within ±1000 km of synchronous orbit would produce clouds expanding both inward and outward from synchronous orbit. At a distance of 500 km from synchronous orbit the radial velocity is already 4 km s^{-1}, which is sufficient to produce very long trails in short times.

This model also yields the right order of magnitude for the optical depth of a spoke 1000 km wide (equal to $2R_c$). At $L \approx 1.9$ the radial velocity V_R is about 10 km s^{-1}, and hence it takes about 100 s to pass a certain point. In this time the number of dust particles elevated will produce an optical depth of $\tau_D = \tau_0 \times 10^{-1}$. If the dust grains completely cover the big ring particles ($\tau_0 = 1$), the predicted optical depth is 0.1, in good agreement with observations. The most probable size of the dust grains to be elevated is 0.5 μm. This size is also indicated by the enhanced forward scattering of light by the spokes. *Morfill et al.* [1983] have estimated that the rate of meteorite impacts on the B ring is sufficient to account for the simultaneous presence of the approximately 10 spokes that are observed at any instant of time.

4.2. Electromagnetically Induced Mass Transport in the Rings

Small dust particles elevated above the ring plane at a radial distance r are subject to electromagnetic forces which tend to force them into corotation with the planetary magnetic field. This can be seen from (24) in the limit that $\omega_g \gg \Omega_K$. Dust particles residing on the big ring particles or moving on Kepler orbits between the big ring particles in the ring plane have a very small charge to mass ratio and are not significantly affected by electromagnetic forces. Inside synchronous orbit the elevated dust particles lose angular momentum to Saturn, and outside of it they gain angular momentum. For $Q < 0$ the dust particles elevated at a radial distance r move toward synchronous orbit by an amount Δr and will settle back onto the ring at a radial

distance $r + \Delta r$. Since the specific angular momentum (angular momentum per mass) of the dust settling down at $r + \Delta r$ is different from the Keplerian specific angular momentum of the absorbing ring particles, these big ring particles will experience a torque when the dust is absorbed and hence change their angular momentum. The magnitude of this torque is proportional to the flux of dust onto the ring material and the radial hopping distance Δr. The big ring particles absorbing the dust will move onto a new circular Keplerian orbit corresponding to their new angular momentum. Averaged over many episodes of dust absorption, this yields a radial velocity of the ring material W which was given by *Goertz et al.* [1986] and *Goertz and Morfill* [1988]:

$$W(r+\Delta r) = 2rF(r)[(L_D/L_K)-1]/\sigma_R(r+\Delta r) \qquad (42)$$

where $F(r)$ is the flux of dust elevated at r. The ring surface (or column) mass density is σ_R. L_D is the dust specific angular momentum at $r+\Delta r$, and L_K is the Kepler angular momentum at $r+\Delta r$. The authors show that

$$2r[(L_D/L_K)-1] = -\Delta r(1-\eta) \qquad (43)$$

where

$$\eta = w_g/\Omega_K \qquad (44)$$

The 1 on the right-hand side in (43) is due to the radial variation of the specific Keplerian angular momentum. The factor η expresses the change of the specific angular momentum due to the azimuthal electromagnetic force. We now make the assumption that the flux of dust elevated is proportional to the ring surface (or column) mass density σ_R. This is quite reasonable, since one would expect dust to be produced by grinding collisions between the big particles. Thus

$$F(r) = b\,\sigma_R(r) \qquad (45)$$

The continuity equation for the ring surface mass density can be written as

$$\frac{\partial}{\partial t}\sigma_R(r) + \frac{1}{r}\frac{\partial}{\partial r}[rW(r)\sigma_R(r)] = \frac{\partial^2}{\partial r^2}[D\,\sigma_R(r)]$$

$$+[F(r-\Delta r) - F(r)] \qquad (46)$$

The first term on the right-hand side represents diffusion due to particle collisions, and the second term represents the gardening effect of the dust being elevated and absorbed at different radial distances. A similar equation can be written for the surface number density of the ring n_R:

$$\frac{\partial}{\partial t}n_R(r) + \frac{1}{r}\frac{\partial}{\partial r}[rW(r)n_R(r)] = \frac{\partial^2}{\partial r^2}[D\,n_R(r)] \qquad (47)$$

The gardening effect does not change the number density. The surface number density is related to the surface mass density by

$$n_R = (3/4\pi)(\sigma_R/R_b^3 \rho_b) \qquad (48)$$

where R_b is the radius of the big ring particles whose mass density is ρ_b. The optical depth is given by

$$\tau = \pi R_b^2 n_R = 3\sigma_R/4R_b \rho_b \qquad (49)$$

We should note that the radial transport discussed here is similar to the ballistic transport of *Ip* [1983], *Lissauer* [1984] and *Durisen* [1984], who consider only the radial displacement of uncharged debris produced by meteorite impacts. In that case, Δr is small, and, most importantly, it can be either positive or negative depending on the velocity of the debris as it is ejected from the ring. Thus their $F(r)$ must be evaluated by integrating over all ejection velocities, and the resulting angular momentum change is small because debris has almost the same probability of settling down at a larger radial distance than at a smaller one. For charged dust the sign of Δr depends only on the sign of the charge of the dust particles.

Let us now consider radial variations of σ_R and n_R whose scale lengths are much larger than Δr but smaller than r. In that case we write from (42) and (43)

$$W(r - \Delta r) = W(r) = -\Delta r(1 - \varepsilon)F(r)/\sigma(r)$$

$$F(r - \Delta r) - F(r) = -\Delta r \partial F(r)/\partial r \qquad (50)$$

$$\frac{1}{r}\frac{\partial}{\partial r} r \sigma_R W \approx \frac{\partial}{\partial r} \sigma_R W$$

and obtain from (46)

$$\frac{\partial \sigma_R}{\partial t} + \frac{\partial}{\partial r}(\sigma_R W_{\text{eff}}) = \frac{\partial^2}{\partial r^2}(D \sigma_R) \qquad (51)$$

where the effective radial mass transport velocity is given by

$$W_{\text{eff}} = -\frac{r}{2}\left(1 - \frac{\Omega_s}{\Omega_k}\right) t_{DS} \frac{\langle\sigma_D\rangle}{\langle\sigma_R\rangle}\omega_g^2 \qquad (52)$$

This uses the relations [*Goertz et al.*, 1986]

$$\Delta r = -\omega_g r(\Omega_K - \Omega_s)t_{DS}^2/2 \qquad (53)$$

and the fact that the quantity b is related to the average column mass density of the elevated dust $\langle\sigma_D\rangle$ by

$$b = \frac{\langle\sigma_D\rangle}{\langle\sigma_R\rangle}\frac{1}{t_{DS}} \qquad (54)$$

where t_{DS} is the average time a dust particle spends outside the Debye sheath where the electromagnetic force is

significant between elevation and reabsorption (roughly 5 hours). Inside synchronous orbit the ring mass transport is inward, and outside of it transport is outward, independent of the dust particle's sign of charge. The radial displacement Δr of a dust particle does, however, depend on the sign of its charge. This perhaps puzzling fact is easily understood by realizing that the radial current carried by the radial displacement of the dust particles is always away from synchronous orbit no matter what the sign of Q is. The $\mathbf{j} \times \mathbf{B}$ force due to this current will decrease the angular momentum of the ring inside L_s and increase it outside. The change of ring angular momentum is taken up by the planet's ionosphere where Pedersen currents exert the required $\mathbf{j} \times \mathbf{B}$ force.

The effective radial velocity w_{eff} can be quite large considering that $t_{DS} \sim 10^4$ and $\langle\sigma_D\rangle/\langle\sigma_R\rangle> \approx 2.5 \times 10^{-8}$. *Goertz et al.* [1986] show that in the B ring at $L = 1.8$, for example, the effective velocity is $7 \times 10^{-12}(Q/e)^2$ cm s^{-1} which for $Q/e \approx 200$ becomes about 3×10^{-7} cm s^{-1}. The effective diffusive transport velocity for a typical ring viscosity of 20 cm^2 s^{-1} is only 2×10^{-9} cm s^{-1}. Clearly, electromagnetic angular momentum transport is, at least, equal in importance to viscous transport. Since the electromagnetic transport is away from synchronous orbit, one expects a minimum of σ_R there. This is, indeed, observed, as shown in Figure 11.

For small scale length variations of σ_R we can make the ansatz

$$\sigma_R = \sigma_0(r) + \delta\sigma e^{-i(kr-\omega t)} \qquad (55)$$

with $\sigma_0(r)$ being the slowly varying density discussed above. Inserting this into (51), we obtain the dispersion relation (neglecting diffusion)

$$i\omega = b\{e^{ik\Delta r} - 1 + ik\Delta r(1-\varepsilon)e^{ik\Delta r}\} \qquad (56)$$

which indicates growth $(Re(i\omega) > 0)$ for wave numbers in bands centered on $k\Delta r = \pm(l + \frac{1}{2})\pi$ with $l = 0,1,2$, etc. The growth rates are of the order of b, and density perturbations with radial wavelengths $\lambda = \pm 2\Delta r(l + \frac{1}{2})$ should grow to appreciable amplitudes in a relatively short time of order $b^{-1} \approx 4 \times 10^{11}$ s or 10^4 years. Of course, diffusion will prevent very small wavelengths from growing because diffusive damping is proportional to $1/\lambda^2$.

In addition, we must note that Δr is not a fixed number. For example, the charge on a dust particle escaping from the ring's Debye sheath fluctuates, causing a fluctuation of Δr about a mean value. *Goertz and Morfill* [1988] have shown that the growth rate for $l \geq 1$ becomes, in fact, negative if the fluctuation of Δr exceeds 50%. In addition, one should note that Saturn's magnetic field may not be constant over 10^4 years. If the field changes direction, the sign of Δr changes. The duration of constant magnetic field since the last change may not have been long enough
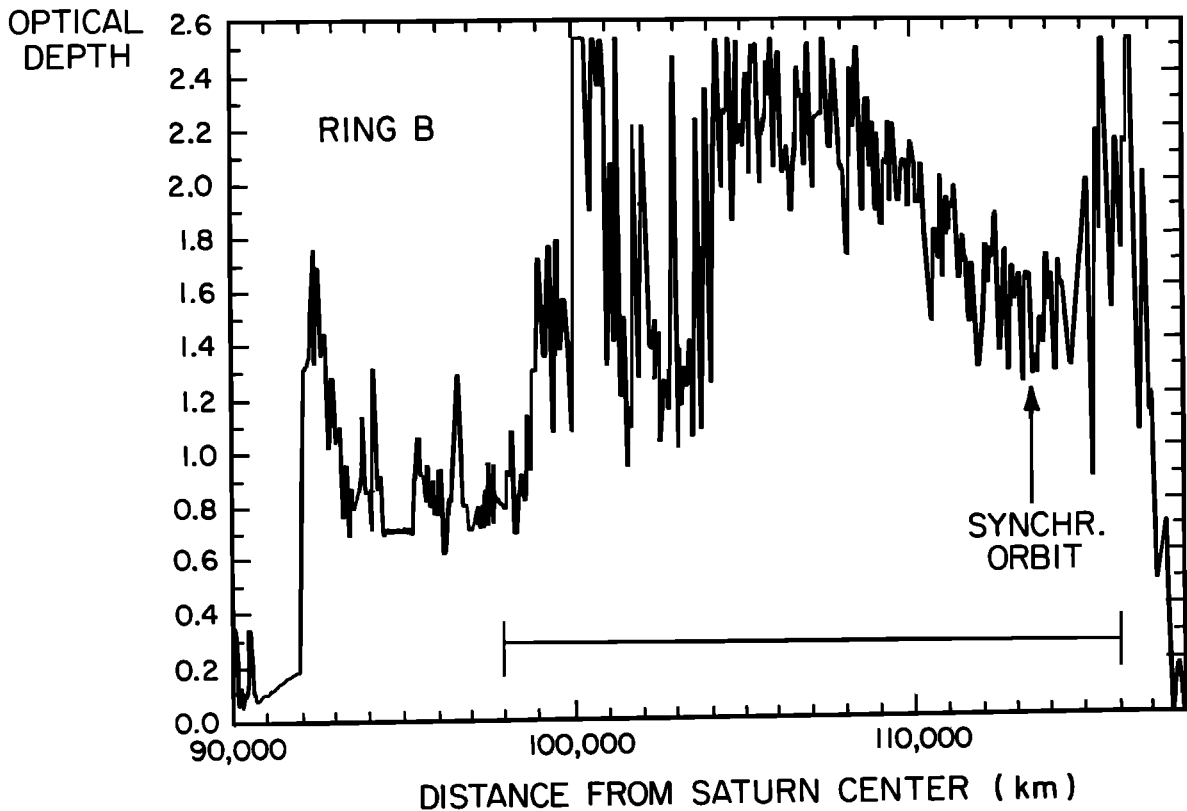
Figure 11. The variation of optical depth in Saturn's rings as a function of radial distance. The minimum occurs near synchro-nous orbit. The line indicates the observed extent of spoke activity. The figure is from *Esposito et al.* [1983].

to allow perturbations to have grown to very large amplitudes. Note that a magnetic field turning would not affect the time-averaged effective radial velocity w_{eff} and hence the evolution of the optical depth minimum near synchronous orbit because it is proportional to B^2. (I am grateful to P. Cloutier for pointing this out to me.)

For $Q/e = 200$ we find that at $L = 1.8$, $\Delta r \sim 50$ km. The unstable wavelength bands are thus centered at $\lambda = (400$ km)$/2l + 1)$. The $l = 0$ and $l = 1$ wavelengths are quite similar to the scale lengths of the optical depth variations observed in that region of Saturn's rings. (See the fine structure shown in Figure 11.)

5. COLLECTIVE EFFECTS IN A DUSTY PLASMA

In the previous sections we have dealt with the effects of charges on the motion of single dust grains. Because the effects scale with the grain size, different size grains move on different orbits, and one expects that the effects are washed out if the size distribution has a finite width, and they may thus be difficult to observe. On the other hand, collective effects due to, for example electric fields, may change the whole distribution. In this section we will discuss several collective phenomena, which rely on perturbation of the dust density.

5.1. Density Waves in Planetary Rings

Resonantly driven density waves in planetary rings (see, for example, *Shu* [1984]) are associated with noticeable perturbations of the dust density. In that case the electro-static force on a grain cannot be calculated by assuming that the grain charge is constant. In this section we require that at all times charge neutrality is satisfied which restricts the perturbation to wavelengths much larger than the plasma Debye length. This, however, does not mean that electrostatic forces can be entirely neglected when calculating the dispersion relation of density waves, because the plasma potential V_p depends on the dust density N. Thus there is an electrostatic force on a grain given by

$$\mathbf{F} = -Q\nabla V_p = -Q(N)\frac{\partial V_p}{\partial N}\nabla N \qquad (57)$$

This force is directed away from density enhancements and thus acts like a pressure force. *Goertz et al.* [1988] have shown that the dispersion relation of density waves is the same as that derived by, for example, *Shu* [1984] except that the effective random thermal velocity of the dust grains, c_d, must be replaced by $(c_d^2 + u^2)^{1/2}$ where the equivalent electrostatic random velocity u is given by

$$u^2 = \langle (Q_0 N_0 \partial V_{p0}/\partial N_0) \rangle/m \qquad (58)$$

where N_0 is the unperturbed dust density and the functions $Q_0(N)$ and $V_{p0}(N)$ are given by Figure 5 in section 2.2. The averaging is over the vertical thickness of the ring. Long wavelengths are not significantly altered by the electrostatic force; only small wavelengths are. The critical wavelength is given by $\lambda_c = u^2/G\sigma_{R0}$ where G is the gravitational constant and σ_{R0} is the unperturbed surface mass density. For $\lambda \ll \lambda_c$ the radial phase and group velocities of the density waves are not measures of the random thermal velocity c_d but of $u^2 + c_d^2)^{1/2}$. Since u can be as large as several 10^5 m/s for centimeter size grains, this difference can be significant even in the dense rings. For micron size particles in the tenuous rings, u can be as large as 1 m/s, which is probably significantly larger than c_d.

Without electrostatic effects a ring would be unstable against gravitational collapse if $c_d \Omega K/\pi G\sigma_0 < 1$ for waves with $\lambda \approx 2c_d^2/G\sigma_0$. Thus a ring with a small velocity dispersion (e.g., $c_d < 0.1$ cm/s for $\sigma_0 = 60$ g/cm^2) is unstable to gravitational collapse. However, the electrostatic force can prevent such a collapse if the condition $(u^2 + c_d^2)^{1/2} \Omega_K/\pi G \sigma_{R^0} < 1$ is satisfied. In fact, a finite thickness ring with zero random velocity dispersion c_d can be maintained against gravitational collapse purely by electrostatic repulsion [Havnes and Morfill, 1984]. The equilibrium vertical thickness H is given by $H = \alpha u/\mu$ where $-\mu^2 z$ is the vertical component of the planet's gravitational field at a position z above the ring plane ($\mu \approx \Omega_K$ for a spherical planet) and α is a factor of order 1 which depends on the maximum dust density of the ring in the equatorial plane [see Goertz et al., 1988].

5.2. Waves in a Dusty Plasma

The distribution of grains in a planetary magnetosphere is given by

$$f(a, v) = Ng(a)h[v(a)] \tag{59}$$

where $g(a)$ is the size distribution and $h[v(a)]$ is the velocity distribution. The velocity depends on the size (section 3). The functions $g(a)$ and $h(v)$ are normalized so that

$$\int\int g\left(\frac{a}{a_0}\right) h[v(a)]\, da\, dv = 1 \tag{60}$$

The dust density is N, and a_0 normalizes the size. Because of the large mass of the dust grains one would not expect electric fields to affect the grain velocity very much. Small perturbations of the grain velocity will cause only small density perturbations. Even though these are small, they can have a large effect on the electric fields in the dusty plasma because each grain carries a large charge. In other words, even though Q/m is much smaller for dust particles than for the plasma ions or electrons, the plasma frequency

of the dust may be nonnegligible because it scales as Q^2/m. As an example, we discuss electrostatic waves in a dusty plasma.

We consider a system which consists of electrons of density n_e, ions of density n_i, and dust of density N. The unperturbed dusty plasma is charge neutral. To simplify matters, we will assume that the velocity distribution of the electrons and ions is given by a water bag model, i.e.,

$$f_e(v_e) = \frac{n_e}{12\pi c_e^3} \qquad -\frac{c_e}{2} < v_e < \frac{c_e}{2}$$

$$f_e(v_e) = 0 \qquad |v_e| \geq \frac{c_e}{2} \tag{61}$$

$$f_i(v_i) = \frac{n_i}{12\pi c_i^3} \qquad -\frac{c_i}{2} < v_i < \frac{c_i}{2}$$

$$f_i(v_i) = 0 \qquad |v_i| \geq \frac{c_i}{2} \tag{62}$$

The grains are assumed to be cold and to move relative to the plasma with a velocity $w(a)$, i.e.,

$$h[v(a)] = \delta[v - w(a)] \tag{63}$$

Electrostatic waves with frequency ω and wave vector $\mathbf{k}$ obey the following dispersion relation

$$1 = \frac{\omega_{pe}^2}{(\omega^2 - k^2 c_e^2)} + \frac{\omega_{pi}^2}{(\omega^2 - k^2 c_i^2)} + \omega_{pd}^2 \int \frac{[g(x)/x]dx}{[\omega - \mathbf{k} \cdot \mathbf{w}(x)]^2} \tag{64}$$

where $x = a/a_0$ and the dust plasma frequency is

$$\omega_{pd}^2 = \frac{N}{\varepsilon_0} \frac{Q^2(a_0)}{m(a_0)} = \frac{12\pi\varepsilon_0 U^2 N}{\rho a_0} \tag{65}$$

The mass density of the dust is $\rho \approx 10^3$ kg m^{-3}. The surface potential of all grains is U. In planetary magnetospheres the relative velocity $\mathbf{w}$ is given by

$$\mathbf{w} = (\omega - \Omega) \times \mathbf{r} \approx (\Omega_K - \Omega) \times \mathbf{r}(1 + \omega_g/2\Omega_K) \tag{66}$$

where we have used (29) for ω. Except for very small particles, $\omega_g \ll \Omega_K$, and we may neglect the last term of (66). In that case the dispersion relation is given by

$$1 = \frac{\omega_{pe}^2}{(\omega^2 - k^2 c_e^2)} + \frac{\omega_{pi}^2}{(\omega^2 - k^2 c_i^2)} + \frac{\kappa^2 \omega_{pd}^2}{(\omega - \mathbf{k} \cdot \mathbf{w})^2} \tag{67}$$

where

$$\mathbf{w} = (\Omega_K - \Omega) \times \mathbf{r} \tag{68}$$

and

$$\kappa = \int \frac{g(x)}{x}\, dx / \int g(x)\, dx \tag{69}$$

Equation (67) has the structure of the well-known two-stream instability dispersion relation. *Bliokh and Yarashenko* [1985] have suggested that waves destabilized by the dust beams may explain the spokes. However, it is obvious that the finite electron and/or ion thermal velocities will stabilize these waves. Since in all magnetospheres, $w \ll c_e$, plasma waves are stable. The ion acoustic wave instability requires $T_e > T_i$, in addition to $w > c_s/(1+k^2\lambda_D^2)^{1/2}$ which could be satisfied in planetary magnetospheres. The growth rate would be of the order of $\gamma \sim \omega_{ps}^{1/3}\omega_{Pd}^{2/3}$. We note here that the effective dust plasma frequency $\kappa^{1/2}\omega_{Pd}$ is surprisingly large in planetary rings. This can be seen by expressing N in terms of the optical depth τ and vertical thickness H of the ring:

$$N = \tau/4\pi a_0^2 H\beta \tag{70}$$

where

$$\beta = \int g(x)x^2 \, dx/\int g(x) \, dx \tag{71}$$

Thus

$$\kappa\omega_{Pd}^2 = 6.6 \times 10^5 \, (\kappa/\beta)\tau(kT)^2/H\, a_{0\mu}^3 \tag{72}$$

where we have assumed that $eU = 2.5kT$; $a_{0\mu}$ is a_0 expressed in micrometers, and kT is expressed in electron volts. For example, in Saturn's G ring the mass distribution varies as m^{-3} between 0.3 and 3 μm [*Grün et al.*, 1984]. Thus $\kappa/\beta = 27$ for $a_{0\mu} = 1$, and the effective dust plasma frequency for $kT = 10^2$ eV is 0.4 s^{-1}. In a cold ion plasma, ion acoustic waves may grow rapidly. But their growth would saturate by ion heating. It seems difficult to believe that plasma waves whose amplitude is at most of the order of kT/e or $m_i w^2/2$ should cause noticeable perturbations of the dust flow velocity and hence dust density. Thus we do not believe that plasma waves can cause the strong optical depth modulation associated with the spokes. Nevertheless, the idea of dust-driven plasma waves provides an intriguing mechanism for heating ions at the expense of the gravitational energy of orbiting dust grains.

The effect of charged dust on the propagation of low-frequency electromagnetic waves has been investigated by *Pilipp et al.* [1987] who assumed a δ function for $g(a)$. As expected, the dispersion relation of such waves is significantly altered only near the dust gyrofrequency ω_g at which frequency the waves are in resonance with the grains and hence can be damped by them. Their treatment also includes the collisional coupling between the plasma, dust, and neutral gas which is especially relevant for the situation of interstellar molecular clouds which they have treated.

Finally, we note that in this section we have implicitly assumed that U, and hence Q, are independent of the dust density, which we know not to be true. This does not really affect the result of Pilipp et al., who only treat

incompressible modes. For the ion acoustic waves the effect is negligible as long as $w \gg u$. The dependence of Q on N would add an equivalent thermal velocity term, $-k^2u^2$, in the denominator of the last term on the right-hand side of (67).

5.3. Strongly Coupled Dusty Plasmas

Because the charge on a grain can be very large and the distance between grains small in a dense planetary ring (or dust cloud), the electrostatic interaction energy between grains can become comparable to their random motion thermal energy. If that is the case, the grains cannot be treated as a gas anymore—in fact, the grains could form a regular structure or lattices. This has recently been suggested by *Ikezi* [1986]. The charged grains are embedded in a charge-neutralizing background of plasma, and this ensemble can be treated as a strongly coupled one-component plasma (OCP) which has been subject to intense study (see, for example, *Rogers and deWitt*, 1987]). It was shown by several authors [e.g., *Slattery et al.*, 1980] that regular, latticelike structures can be formed if the coupling constant Γ exceeds a critical value of 170. The coupling constant is written as

$$\Gamma = Q^2/4\pi\varepsilon_0 d\,\varepsilon_{th} \tag{73}$$

where d is the average distance between grains and $\varepsilon_{th} = mc_d^2/2$ is the thermal energy of the random grain motion. Since $d = (3/4\pi N)^{1/3}$ and Q is a function of N (see section 3), the coupling constant increases with N until it reaches a maximum at a certain critical value N_c. For $N > N_c$ the charge Q decreases rapidly, and the coupling constant becomes small. Using the results of section 3 and writing $eU/kT = \bar{U}$, we have for grains whose mass density is $\rho = 10^3$ kg m^{-3}

$$\Gamma = 4.3 \times 10^{-14} \frac{(kT)^{5/3} n_0^{1/3}}{a^{4/3} c_d^2} P^{1/3} \bar{U}^2 \tag{74}$$

where plasma thermal energy kT is expressed in electron volts and the plasma density n_p in m^{-3}. The quantity P is defined in (19). We can approximate $\bar{U}$ by

$$\bar{U} = -2e^{-P/P_0} = -2e^{-\gamma} \tag{75}$$

where $P_0 \approx 10^{-9}$. We see that Γ has a maximum for $y = 1/6$. This corresponds to

$$N_c = (n_0/aT)(P_0/6) \tag{76}$$

The random thermal velocity of the grains is difficult to estimate. *Borderies et al.* [1984] calculate the equilibrium random velocity of grains in a planetary ring by balancing the rate at which energy is fed into random motions by the viscous stress with the rate of energy dissipation by inelas-

tic collisions. They find that $c_d \sim 10^{-3}$ m/s for optically thick rings. In planetary magnetospheres, typical values for kT and n_p are about 10^2 eV and 10^7 m^{-3}, respectively. Thus

$$\Gamma = 8 \times 10^{-5} (y^{1/3} e^{-2y} / a^{4/3}) A \qquad (77)$$

where the scaling factor A is

$$A = (kT/100)^{5/3} (n_p/10^7)^{1/3} /(c_d/10^{-3})^2 \qquad (78)$$

and a is expressed in meters. One can see that if the grain size is sufficiently small, the coupling constant may become larger than the critical value. Unfortunately, not enough is known about the plasma parameters in the vicinity of the planetary rings to determine the coupling constant there. In addition, charged grains may not make inelastic collisions (because they do not actually get in contact), and the velocity c_d may be very different from the estimates of Borderies et al.

The narrow rings of Uranus and the incomplete rings of Neptune are puzzling because a narrow ring should quickly spread if inelastic collisions occur, and an incomplete ring should become azimuthally uniform because of the Kepler velocity shear. If the electrostatic interaction energy were large enough, these rings may actually be examples of strongly coupled plasmas. Even though Γ may not exceed 170 in these rings, the electrostatic interaction energy which is a function of Γ may be important. It is negative and has a minimum for $N = N_c$. Thus the total energy of a narrow ring (gravitational plus electrostatic) does not decrease continuously with ring width (inversely proportional to N) but may have a minimum. This occurs only for narrow (radial width less than a few tens of kilometers) and tenuous rings. Very massive rings and those containing only large particles have no energy minimum (Shan and Goertz, 1989). If an energy minimum occurs at a certain width, a ring with this radial width would be stable because in order for the ring to spread in radial distance, energy would have to be absorbed by the ring.

6. SUMMARY

Dusty plasmas contain charged grains whose motion is influenced not ony by gravity and radiation pressure but also by plasma drag and electromagnetic forces. Even though the electromagnetic forces are usually small, they cause several interesting effects such as resonant orbit perturbations, modification of density wave dispersion characteristics in planetary rings, and angular momentum transport in planetary rings. The important charge to mass ratio is a complicated function of plasma parameters and the number density of dust particles. Only a relatively small number of cases has been investigated in detail. In many instances the surface potential of a grain is approximately equal to $-2.5kT_e/e$. However, the magnitude of the charge on a grain is not necessarily equal to $4\pi\epsilon_0 a |\phi_s|$. When the dust density number becomes large, the charge is smaller than this value. When secondaries are important, the surface potential can have three equilibrium values, and grains of different signs of charge can coexist in a plasma. This may have important consequences for the rate at which grains collide and coagulate to form bigger particles. Such coagulation must have occurred during the early stages of the solar system evolution from its solar nebula stage. In addition, the grain charge fluctuates randomly in a plasma and systematically as a grain gyrates about the magnetic field or moves through gradients of plasma density and/or temperature. These fluctuations can cause the angular momentum of a grain in a planetary magnetosphere to change and can lead to radial transport. It is evident that many more cases are relevant and need to be studied in detail.

The transport of angular momentum by charged dust can have significant effects on the evolution of planetary rings. It seems that the dust particles contained in the spokes of Saturn can cause mass transport away from synchronous orbit and may be responsible for the optical depth minimum observed there. In addition, an instability can grow in the ring which modulates the ring surface mass density and optical depth. The growth rate is positive for certain wavelengths which are quite similar to the observed size of the radial optical depth variations in the Saturnian B ring. Several questions need to be answered before these models can be accepted. For example, the distribution of particle size in the spokes produced by the electrostatic levitation mechanism is crucial for determining the growth rate. If the distribution is too broad, the instability does not grow. Simple arguments suggest that the width is not very broad, and recent Monte-Carlo type simulations have confirmed this. However, no systematic search of parameter space (plasma density, temperature, duration of enhanced plasma density) has been performed, and the sensitivity of the model to these parameters is not clear. Observationally, one needs to obtain a spectrum of the density perturbations in the ring. So far, only spectra averaged over the whole ring have been published. These do not show the structures predicted by the model. However, the most unstable wavelength changes as a function of radial distance, and the spectra should be taken for data limited to small radial distance ranges. The spectra should show a systematic variation with radial distance. Near synchronous orbit the spectral power should peak at smaller wavelengths than further away from synchronous orbit.

The characteristics of plasma waves in a dusty plasma have barely been studied at all. It is surprising that the effective plasma frequency of small charged grains can be quite large. The relative drift between grains and the plasma may excite two-stream or ion acoustic instabilities.

The saturation mechanism for such waves is probably ion heating, but that remains to be seen. In a magnetized dusty plasma the gyrofrequency may be a continuous distribution (because Q/m is a continuous variable). The dispersion characteristics of electromagnetic waves should thus be very interesting.

Finally, the electrostatic interaction energy of grains in a plasma may become larger than their random motion thermal energy. In that case, dust grains may form ordered structures, so-called Coulomb lattices. This would be an exciting phenomenon, and the study of the properties of such a configuration would be challenging, indeed.

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C. K. Goertz, Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52240.