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DUST-ACOUSTIC WAVES IN DUSTY PLASMAS

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Abstract—New acoustic waves originating from a balance of dust particle inertia and plasma pressure are investigated. It is shown that these waves can propagate linearly as a normal mode in a dusty plasma, and non-linearly as supersonic solitons of either positive or negative electrostatic potential.

There has been much interest in plasmas containing dust particles because of the importance of such plasmas in the study of the space environment, such as asteroid zones, planetary rings, cometary tails, as well as the lower ionosphere of the Earth (Horanyi) and Mendis, 1985, 1986a, 1986b; Whipple *et al.*, 1985; Whipple, 1986; de Angeles *et al.*, 1988). Dusty plasmas usually contain a small amount of dust grains of micrometre or submicrometre size which are negatively charged because of field emission, ultra-violet ray irradiation, and plasma currents, etc. (Feuerbacher *et al.*, 1973; Fechting *et al.*, 1979; Whipple *et al.*, 1985; Havnes *et al.*, 1987).

Most theoretical works on dusty plasmas are concerned with the dynamics of the dust particles, such as their creation, trajectory, impact and fragmentation characteristics, etc. rather than their collective interaction with the plasma. Collective effects in microplasmas have been studied by Verheest (1967), and James and Vermuelen (1968), using many-fluid models. They consider cold particle fluids, with the collective effect arising from a distribution of the fluid velocities. In this manner, they obtain dispersion relations for collective oscillations in such plasmas. On the other hand, recently de Angelis *et al.* (1988) studied the propagation of ion acoustic waves in a dusty plasma, in which a spatial inhomogeneity is created by a distribution of immobile dust particles (Whipple *et al.*, 1985). They applied their results in interpreting the low frequency noise enhancement observed by the *Vega* and *Giotto* space probes in the dusty regions of Halley's comet.

Physically, the low-frequency behaviour of a dusty plasma is very similar to that of a plasma consisting of negative ions (D'Angelo *et al.*, 1966). In fact, for the case in which the wavelength and the inter-particle distance are much larger than the grain size, the dust grains can be treated as negatively charged point masses (like negative ions). Here, however, the charge-to-mass ratio of a dust particle can take on any value. Thus, with minor corrections, many results from the theory of negative ion plasmas can be adapted to dusty plasmas.

In this paper, we study the long-wavelength low-frequency collective oscillations in a dusty plasma. We shall consider modes in which the dust particle dynamics is crucial, rather than modes which are simply affected by the dust. In particular, we study the collective motion of the negatively charged dust particles in a background of hot electrons and ions in thermodynamic equilibrium. We find that a new type of sound wave, namely, the dust-acoustic waves, can appear. These waves are usually of very low frequency, but in some cases the latter can be comparable with that of the ion-acoustic waves. We also investigate the non-linear characteristics of the dust-acoustic waves, and show that they can propagate as solitons with either negative or positive electrostatic potential. The two types of solitons have quite distinct speed–amplitude relations.

In realistic dusty plasmas, the charge, size and mass of the dust grains are usually widely distributed. Furthermore, the charge of the grains can be affected by the presence of the fluctuating electric field, and the grains may break up or coalesce. Since the inclusion of these effects would considerably complicate our investigation, we shall in the following assume that the dust grains are of uniform mass and behave like point charges.

Consider a three-component plasma consisting of electrons and ions having Boltzmann distributions with temperatures T_e and T_i, respectively, as well as negatively charged, heavy dust particles. For one-dimensional propagation of acoustic-like low-frequency waves in such a plasma, the dynamics of the dust particles is governed by the fluid equations,

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$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nv)=0, \tag{1}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}=\frac{Ze}{m}\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=-4\pi\, e(n_i-n_e-Zn), \tag{3}$$

where n, m, $-Ze$, and v are, respectively, the number density, mass, charge, fluid velocity of the dust particles, and ϕ is the self-consistent ambipolar potential. Here, x and t are the space and time variables, and e is the electronic charge. In equation (2) we have assumed, for simplicity, that the dust particles are cold. In equation (3), the ion and electron number densities (n_i and n_e) are given by the Boltzmann distribution,

$$n_i=n_{i0}\exp\left(-\frac{e\phi}{T_i}\right), \tag{4}$$

$$n_e=n_{e0}\exp\left(\frac{e\phi}{T_e}\right), \tag{5}$$

where n_{i0} and n_{e0} are the respective equilibrium number densities. Quasi-neutrality in equilibrium leads to,

$$n_{i0}=n_{e0}+Zn_0, \tag{6}$$

where n_0 is the equilibrium number density of the dust particles. Thus, our problem differs from those of Verheest (1967) and James and Vermuelen (1968) in that here the background plasma is hot and in equilibrium, while the dust fluid is cold.

The dispersion relation of linear low-frequency waves corresponding to equations (1)–(6) can be easily obtained by carrying out the usual normal mode analysis. Accordingly, we assume all the perturbations to be of the form $\exp[i(kx-\omega t)]$ and obtain the linear dispersion relation,

$$\omega^2=\beta^2C_s^2k^2\left[1+\frac{k^2\lambda_{De}^2}{1+\eta\delta}\right]^{-1}, \tag{7}$$

where $\beta^2=Z(\delta-1)/(1+\eta\delta)$, $C_s=(T_e/m)^{1/2}$, $\delta=n_{i0}/n_{e0}$, $\eta=T_e/T_i$, and $\lambda_{De}=(T_e/4\pi n_0e^2)^{1/2}$. In order that $\omega^2>0$, we require $\delta>1$, which is always satisfied in view of the equilibrium condition (6). Clearly, equation (7) shows that the waves are of acoustic nature, whose phase velocity ω/k in the long wavelength limit ($k\lambda_{De}\ll 1$) is βC_s. Furthermore, in this limit, equation (7) becomes,

$$\omega=\beta C_sk\left(1-\frac{1}{2}\frac{k^2\lambda_{De}^2}{1+\eta\delta}\right), \tag{8}$$

which shows that the waves are weakly dispersive for finite wavelengths.

For finite amplitude waves, the linear approximation breaks down, whereas the non-linear and dispersive effects can become equally important. For weak non-linearity and dispersion, equations (1)–(5) can be reduced to a more tractable and simpler equation of the Boussinesq form using a procedure discussed by Rao (1990). Omitting the details, which are fairly straightforward, we obtain,

$$\frac{\partial^2\phi}{\partial t^2}-\beta^2C_s^2\frac{\partial^2\phi}{\partial x^2}-\frac{\beta^2C_s^2}{1+\eta\delta}\lambda_{De}^2\frac{\partial^4\phi}{\partial x^4}$$
$$-\frac{1}{2}\left[\frac{\delta\eta^2-1}{1+\eta\delta}\beta^2C_s^2-3ZC_s^2\right]\frac{\partial^2}{\partial x^2}(\phi^2)$$
$$+\frac{1}{6}\left[\frac{\delta\eta^3+1}{1+\eta\delta}\beta^2C_s^2-15\frac{Z^2C_s^2}{\beta^2}\right]\frac{\partial^2}{\partial x^2}(\phi^3)=0, \tag{9}$$

where ϕ has been normalized with respect to T_e/e. Equation (9) is a generalized Boussinesq equation, having an additional non-linear ($\partial^2\phi^3/\partial x^2$) term when compared with the standard Boussinesq equation (see, for example, Makhankov, 1974). The third term represents the dispersive effects. The last term is in general negligible, except in the case when the coefficient of the $\partial^2\phi^2/\partial x^2$ term turns out to be small. Equation (9) governs the propagation of acoustic waves which are weakly non-linear and dispersive.

For uni-directional, "near-sonic" ($\omega/k=\beta C_s$) propagation, equation (9) can be further reduced. Assuming $\partial/\partial t\sim -\beta C_s\partial/\partial x$, we obtain,

$$\frac{\partial\phi}{\partial t}+\beta C_s\frac{\partial\phi}{\partial x}+\frac{1}{2}\frac{\beta C_s}{1+\eta\delta}\lambda_{De}^2\frac{\partial^3\phi}{\partial x^3}$$
$$+\frac{1}{2}\left[\frac{\delta\eta^2-1}{1+\eta\delta}\beta C_s-3\frac{ZC_s}{\beta}\right]\phi\frac{\partial\phi}{\partial x}$$
$$-\frac{1}{4}\left[\frac{\delta\eta^3+1}{1+\eta\delta}\beta C_s-15\frac{Z^2C_s}{\beta^3}\right]\phi^2\frac{\partial\phi}{\partial x}=0, \tag{10}$$

which is a generalized Korteweg–de Vries (K–dV) equation with an extra non-linear ($\phi^2\partial\phi/\partial x$) term when compared with the usual K–dV equation (Washimi and Taniuti, 1966). Note that equation (10) can also be derived using the standard reductive perturbation analysis with the stretched variables,

$$\zeta=\varepsilon(x-\beta C_st),$$
$$\tau=\varepsilon^3t, \tag{11}$$

where ε is a smallness parameter (Bharuthram and Shukla, 1986).

Equation (9) or (10) can be exactly solved for stationary solutions of the form $\phi(x,t)=\phi(\xi)$, where $\xi = x - Mt$ and M represents the speed of the stationary wave structure normalized by βC_s. In the stationary frame, equation (9) becomes,

$$\frac{d^2\phi}{d\xi^2} = a_1\phi + a_2\phi^2 + a_3\phi^3, \qquad (12)$$

where

$$a_1 = (1+\delta\eta)(M^2-1),$$
$$a_2 = \frac{1}{2}\left[1-\delta\eta^2+3\frac{Z(1+\eta\delta)}{\beta^2}\right],$$
$$a_3 = \frac{1}{6}\left[1+\delta\eta^3-15\frac{Z^2(1+\eta\delta)}{\beta^4}\right]. \qquad (13)$$

In equation (12), we have normalized ξ and M with respect to λ_{De} and βC_s, respectively, and used the boundary conditions appropriate for localized solutions, namely,

$$\phi, \frac{d\phi}{d\xi}, \frac{d^2\phi}{d\xi^2} \to 0 \quad \text{as } \xi \to \pm\infty. \qquad (14)$$

The localized solutions of equation (12) are given by

$$\phi_{\mp}(\xi) = \frac{6a_1 \operatorname{sech}^2[\kappa(\xi-\xi_0)]}{\beta_{\mp}-\beta_{\pm}\tanh^2[\kappa(\xi-\xi_0)]}, \qquad (15)$$

where ξ_0 is a constant of integration, and

$$\beta_{\mp} = -2a_2 \mp (4a_2^2-18a_1a_3)^{1/2} \qquad (16)$$

$$\kappa = \sqrt{a_1}/2. \qquad (17)$$

In order that κ is real (so that the non-linear wave is indeed localized), one requires $M^2 > 1$, that is, only supersonic solutions exist for equation (9). Similar solutions exist also for the generalized K–dV equation (10). In equation (15), both the solutions are allowed, as is discussed below.

The two solutions given by equation (15) represent localized structures with positive and negative potentials. While a_1 and a_2 are always positive, the sign of a_3 depends on the magnitudes of δ and η. However, for the realistic case $\delta \gtrsim 1$ and $\eta \gg 1$, we find,

$$a_1 \approx \eta\delta(M^2-1) > 0,$$
$$a_2 \approx \frac{\delta^2\eta^2}{\delta-1} > 0,$$
$$a_3 \approx -\frac{7}{3}\frac{\delta^3\eta^3}{(\delta-1)^2} < 0. \qquad (18)$$

The coefficients $\beta_{\mp}$ then simplify to

$$\beta_{\mp} \approx -\frac{2\delta^2\eta^2}{\delta-1}\left\{1\pm\left[1+\frac{21}{2}(M^2-1)^{1/2}\right]^{1/2}\right\}. \qquad (19)$$

Thus, $\phi_-(\xi) < 0$ everywhere and has an inverted bell-shaped structure. We note that the potential-dip solutions are unique to the dust-particle acoustic waves. This is in contrast to the usual ion-acoustic waves which propagate non-linearly only as localized potential-humps.

Unlike the previous case, the $\phi_+(\xi)$ solution has the usual bell-shaped structure with $\phi_+(\xi) > 0$ everywhere, similar to that of the usual ion-acoustic wave solitons.

Finally, we shall discuss the existence of finite amplitude dust particle acoustic solitons using the full equations (1)–(5). In the stationary frame $\xi = x - Mt$, they can be integrated once to yield the "energy integral"

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi) = 0, \qquad (20)$$

where the effective potential $V(\phi)$ is given by,

$$V(\phi) = \frac{\delta}{\eta}[1-\exp(-\eta\phi)] + 1 - \exp(\phi)$$
$$+(1-\delta)\frac{M^2\beta^2}{Z}\left[\left(1+\frac{2Z}{M^2\beta^2}\phi\right)^{1/2}-1\right]. \qquad (21)$$

The existence conditions for localized solutions can be easily obtained by analysing $V(\phi)$. The conditions are (Chen, 1974)

$$(1+\eta\delta)(M^2-1) > 0, \qquad (22)$$

$$\delta\exp(-\eta\phi_0) - \exp(\phi_0) - (\delta-1)$$
$$\times\left(1+\frac{2Z}{M^2\beta^2}\phi_0\right)^{-1/2} \gtrless 0 \quad \text{for } \phi_0 \gtrless 0, \qquad (23)$$

where the amplitude ϕ_0 is related to the Mach number M by,

$$M^2 = \frac{Z}{2\eta(1-\delta)\beta^2}\{\delta[1-\exp(-\eta\phi_0)]$$
$$+\eta[1-\exp(\phi_0)]\}^2\{\eta\phi_0(1-\delta)$$
$$+2\delta[1-\exp(-\eta\phi_0)]+2\eta[1-\exp(\phi_0)]\}^{-1}. \qquad (24)$$

Note that in the small, but finite, amplitude limit, equation (22) exactly reduces for $M^2 \sim 1$ to the generalized Boussinesq equation in the stationary frame, namely equation (12).

In this paper, we have shown that in dusty plasmas, dust-acoustic waves can propagate both linearly and non-linearly. The phase velocity of the linear waves is approximately given by $(n_0ZT_e/n_{i0}m)^{1/2}$, where we recall that n_0, Z and m are the number density, charge and mass of the dust particles, T_e is the electron (here set equal to the ion) temperature, and n_{i0} is the ion density. Clearly, depending on the concentration, charge, as well as mass density of the dust grains, the phase velocity and the corresponding wave frequency can assume a wide range of values. Non-linearly, the dust-acoustic waves can propagate as solitons of either negative or positive electrostatic potential, corresponding to a hump or depletion in the electron density, respectively. The very distinctive relationships between the amplitude and speed of these two types of solitons may be useful in the eventual identification of the latter in the laboratory or space.

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