Contrib. Plasma Phys. 26 (1986) 1, 1–12

Sudden Jumps, Hysteresis, and Negative Resistance in an Argon Plasma Discharge

I. Discharges with No Magnetic Field

R. A. Bosch and R. L. Merlino

Department of Physics and Astronomy
The University of Iowa
Iowa City, Iowa 52242

Abstract

Jumps and hysteresis in discharge current are produced in a low-pressure (10^{-5}–10^{-4} Torr), magnetic field free, thermionic argon plasma discharge by varying discharge voltage, neutral pressure, or filament temperature. This behavior can be attributed to a region of negative differential resistance in the voltage versus current characteristic of the discharge. A model is presented in order to understand the physical processes leading to this negative resistance.

1. Introduction

The occurrence of sudden jumps and hysteresis in discharge current when the discharge voltage is varied is a well-known phenomenon [1, 2]. This behavior is attributed to a circuit-related instability resulting from the onset of a negative differential resistance in the current-voltage characteristic of a plasma discharge [1, 3–4]. Similar observations of negative resistance related phenomena are found in glow discharges, carbon arcs, thermionic converters, fluorescent lamps and four-layer diodes [1, 2, 5–11].

We undertook a study of this behavior in a hot filament produced argon discharge plasma in a magnetic field free configuration. The results of our investigations of discharges in cusp or magnetic mirror configurations are the subject of a companion paper. Such hot filament discharge devices are important in applications for basic plasma physics studies [12, 13], as ion sources in neutral beam injectors [14–16], and for reactive plasma etching [17]. Although there has been some theoretical work on negative resistance characteristics in a collision dominated plasma column [18] and on the circuit-related instability of transient plasma discharges, e.g., tokamaks and z-pinches [19], little work has been done to uncover the basic physics leading to the negative resistance behavior. Knorr [20] has recently presented a phenomenological treatment of this phenomenon based on catastrophe theory.

In our experiments we found that sudden jumps and hysteresis would occur not only as the discharge voltage was varied; it was also observed as either the neutral pressure, or filament temperature was varied. The phenomenon was also dependent on filament geometry, i.e., number of filaments and their relative spacing. This indicates that the effects are not simply related to an atomic phenomenon such as ionization thresholds but are related to various plasma physics phenomena. The upward jumps in discharge current were associated with the onset of intense plasma noise (electrostatic and electromagnetic) accompanied by an increase in the electron temperature, while the downward jumps were associated with a decrease in the plasma noise level and a decrease in the electron temperature.

Our approach in attempting to understand these effects is to separately examine two aspects of the problem, viz., the processes affecting electron emission from the filaments and the mechanisms which affect plasma production. We will refer to these as filament and plasma production properties. Clearly the full picture requires a self-consistent model of these processes since the discharge current of primary electrons is dependent on plasma properties, while the rate of plasma production depends on the discharge current. A model is presented which closely conforms to the experimental observations and indicates the conditions under which a jump may be expected to occur when the neutral pressure is varied. We then show that this model indicates the existence of a region of negative differential resistance in the discharge voltage versus current characteristic.

By considering the general functional dependences of the various parameters, we are able to show the relation of jumps and hysteresis to plasma turbulence and filament geometry.

The organization of the paper is as follows: in Section II, we give a complete description of the experimental apparatus followed, in Section III, by a presentation of our observations of jumps and hysteresis and their interpretation. A discussion of the results and our physical model is given in Section IV. In Section V we summarize our results and conclusions.

2. Apparatus

The experimental apparatus used is shown in Fig. 1. A cylindrical stainless steel chamber, 90 cm in length and 60 cm in diameter, is evacuated to a base pressure of 5×10^{-7} Torr. A set of coils for the production of magnetic fields are also shown but were not used for this investigation.

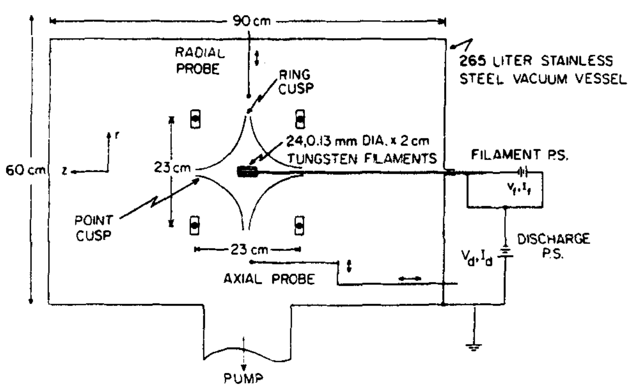

Fig. 1. Schematic cross section of experimental device. Water-cooled coils can produce either a spindle cusp magnetic field, or a mirror field with mirror ratio of 1.2, symmetric about the z-axis. For this paper no magnetic field was used

In the center of the chamber, thoriated tungsten filaments of 0.13 mm diameter are mounted between two 16 mm diameter copper disks. Current follows a copper rod to one of the disks, then flows back coaxially through the filaments, the other disk, and 6.35 mm (1/4 inch) copper tubing. From 1 to 24 filaments can be mounted in this way.

To produce a plasma, argon gas is admitted to the chamber with neutral pressures from 3×10^{-5} to 10^{-3} Torr. A current of 2 to 3 A is passed through each filament for heating, while the filaments are biased from −30 to −200 V with respect to the chamber. A discharge current of primary electrons flows from the filaments, ionizing the argon gas to produce a plasma with densities up to $10^9\,cm^{-3}$. Electron temperatures range from 0.2 to 5 eV, while ion temperatures are several tenths of a volt. The plasma potential depends upon the wall conditions, ranging from +3 V immediately after cleaning to −20 V. None of the effects described in this paper appear to depend on the actual plasma potential.

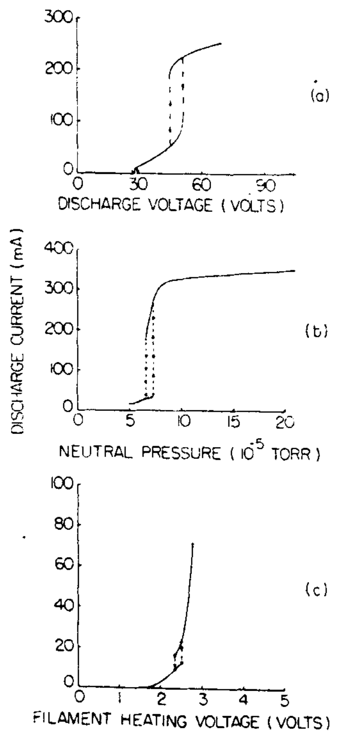

Fig. 2. Hysteresis in discharge current due to variations in three parameters: (a) discharge voltage, (b) neutral pressure, and (c) filament temperature (heating voltage)

3. Experimental Observations and Interpretation

While producing plasma with a discharge current of primary electrons, sudden increases in discharge current and plasma density were observed as the discharge voltage was gradually increased. When the discharge voltage was then reduced, hysteresis occurred with a sudden decrease in discharge current in plasma density occurring at a lower value of the discharge voltage [Fig. 2(a)].

Similar jumps and hysteresis were observed when neutral gas pressure [Fig. 2(b)], or filament heater current [Fig. 2(c)] were varied.

The jumps and hysteresis depended on the filament geometry. The largest jumps were observed when the thermionic cathode consisted of several closely spaced parallel filament wires. For a discharge voltage of 60 volts, and three-wire cathodes, the magnitude of jumps and hysteresis due to pressure variation decreased as the space between wires was varied upward between 2 mm and 1 cm; no jumps occurred with a filament spacing of 2 cm. Smaller jumps were observed for cathodes consisting of two filaments. No jumps or hysteresis were observed when the cathode consisted of a single filament wire or a 0.5 cm × 3 cm × 0.0013 cm strip of tantalum foil.

By placing a 500 Ω series resistor in the discharge circuit, sudden jumps due to variations in discharge voltage could be eliminated, allowing observation of the region of the current versus voltage characteristic with a negative differential resistance. This indicates that in the region of hysteresis, for a single value of discharge voltage, there are three equilibrium states of the plasma with different values of discharge current and plasma density. This result is shown in Fig. 3.

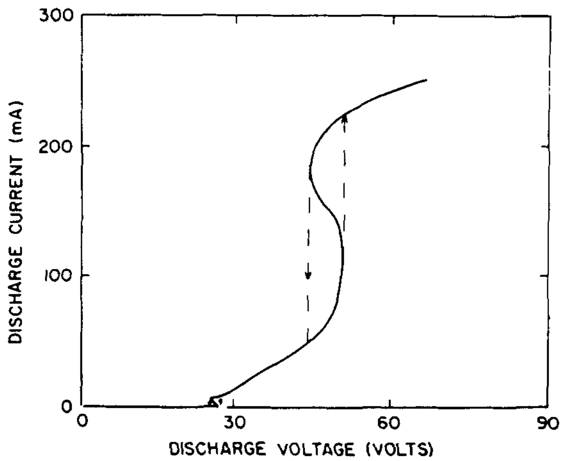

Fig. 3. Typical voltage versus current characteristics of the argon plasma discharge at $P = 7 \times 10^{-5}$ Torr, obtained by placing a 500 Ω series resistor in the discharge circuit. The dashed lines indicate jumps occurring without the resistor

Jumps and hysteresis due to variations in discharge voltage are described by the S-shaped curve that results from the existence of a region of negative differential resistance. When the voltage is increased from zero, it reaches a critical voltage at which $dV/dI = 0$. As voltage is further increased, discharge current must jump upwards to reach an equilibrium state. The jumps and hysteresis can be viewed as an example of a cusp catastrophe [20]. At sufficiently high pressures, the region of negative differential resistance is no longer present in the V vs. I curves. This behavior is shown in Fig. 4. The equilibrium surface describing I as a function of P and V develops a fold at low pressures, resulting in a cusp catastrophe [20].

The experimentally controlled independent variables of the plasma discharge are the discharge voltage (V), neutral pressure (P) and filament temperature (T_f). The plasma potential was observed to be nearly constant throughout the chamber, and it varies little as the independent variables are changed or when jumps occur.

The discharge current (I) and plasma density (n) are functionally dependent upon the independent variables:

$$I = I(V, P, T_f) \tag{1}$$

$$n = n(V, P, T_f). \tag{2}$$

Since jumps and hysteresis occur in these variables, these functions are multi-valued in some regions.

If we consider a change of variables so that V, T_f and n are considered independent, then equation (1) takes on the form:

$$I = I(V, n, T_f). \tag{3}$$

This equation describes the dependence of the discharge current on plasma density and the filament properties T_f and V, so its graph (I vs. n) will be called a filament characteristic curve.

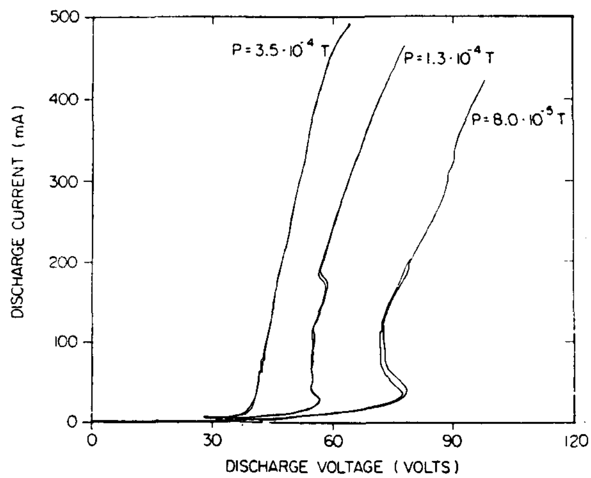

Fig. 4. Discharge current-voltage (I—V) characteristic for various pressures obtained by placing a 500 Ω series resistor in the discharge circuit. These curves show the development of the region of negative differential resistance

Similarly, considering V, P and I as independent variables transforms (2) into:

$$n = n(V, P, I). \tag{4}$$

This equation describes the dependence of plasma density upon parameters affecting the ionization rate: neutral pressure (P), primary electron current (I), and primary electron energy (eV), so its graph (I vs. n) will be called a plasma production curve.

Although the functions in equations (1) and (2) are multivalued, the functions in (3) and (4) may be well-behaved, and such was the case in our observations. For given values of V, T_f and P, the values of I and n can be determined by simultaneous solution of relations (3) and (4), so no information is lost by dealing with equations (3) and (4) instead of equations (1) and (2). An advantage of using equations (3) and (4) is the separation of the dynamics of the filament properties and those of ionization.

For a plasma described by equations (3) and (4), the differential resistance $\mathrm{d}V/\mathrm{d}I = \partial V/\partial I \mid P, T_f$ may be calculated. For constant T_f the filament characteristic equation is:

$$I = I(V, n) \tag{5}$$

and the plasma production curve for constant P is:

$$n = n(I, V). \tag{6}$$

Both of these equations must be satisfied simultaneously, so we can substitute (6) into (5) to get:

$$I = I[V, n(I, V)]. \tag{7}$$

This equation relates the discharge current to discharge voltage for fixed values of P and T_f.

Differentiating equation (7) with respect to I and rearranging terms gives:

$$\frac{dV}{dI}\left\{\left.\frac{\partial I}{\partial V}\right|_n + \left.\frac{\partial I}{\partial n}\right|_V \left.\frac{\partial n}{\partial V}\right|_I\right\} = 1 - \left.\frac{\partial I}{\partial n}\right|_V \left.\frac{\partial n}{\partial I}\right|_V$$

$$= 1 - \frac{\text{slope of filament characteristic curve}}{\text{slope of plasma production curve}}, \tag{8}$$

where dI/dn is the slope.

Experimentally, we find that the terms inside the brackets in equation (8) are positive. Therefore, the differential resistance of the plasma discharge is positive when the slope of the filament characteristic curve is smaller than the slope of the plasma production curve at their intersection.

Experimentally, plots of I vs. n were obtained by using the electron saturation current collected by a planar disc Langmuir probe of 3 mm diameter. Since equation (3) contains no dependence on P, its plot (a filament characteristic curve for specific values of V and T_f) was obtained by varying P. For a filament consisting of three wires spaced 2 mm apart, a filament characteristic curve is shown in Fig. 5, indicating significant jumps and hysteresis. Although a portion of the curve is experimentally inaccessible, the curve can be continued smoothly through the jump region.

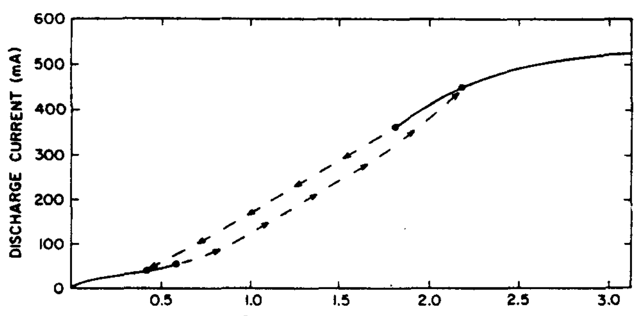

Fig. 5. Graph of discharge current versus electron saturation current as neutral pressure is varied, showing jumps and hysteresis. This graph can be interpreted as a filament characteristic, showing dependence of discharge current on the plasma density

Equation (4) has no dependence on the filament temperature, so plasma production curves for several pressures were obtained by varying the filament heater current. These curves are shown in Fig. 6. Again, the curves can be continued smoothly through the jump region.

Both types of curves bend upward in the jump region. Associated with the upward bends is the onset of broadband plasma noise. As the discharge current increased in this region, a coherent oscillation of electron probe current was observed with frequency around 40 MHz, about half of the calculated value of plasma frequency. Further increase in discharge current results in a broadband noise spectrum extending from the plasma frequency to much lower frequencies, with $\Delta n/n$ of a few percent. When broadband noise was present in the probe current, electromagnetic noise was also detected on an FM radio receiver.

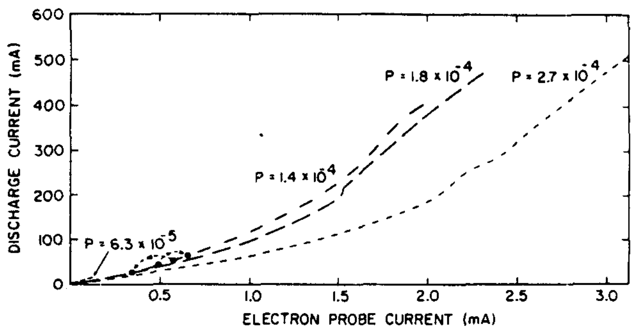

Fig. 6. Graphs of discharge current versus electron saturation current as filament temperature is varied, for four different neutral pressures measured in Torr. The jumps and hysteresis appearing on the curve for $P = 1.4 \times 10^{-4}$ Torr correspond to those of Fig. 2(c). These curves can be interpreted as plasma production curves, showing dependence of plasma density upon discharge current

4. Discussion

4.1. Onset of Jumps and Hysteresis

For given values of V, T_f, and P, the state of the plasma is determined by simultaneous solution of (3) and (4), corresponding to the intersection of the filament characteristic and plasma production curves. In the region of hysteresis, these curves intersect twice, indicating two possible states. However, a portion of each of these two curves is experimentally inacessible in the jump region. If the accessible portions of these curves are smoothly connected, then a third intersection will occur in the inaccessible regions of these curves. Since the slope of the filament characteristic curve is larger than that of the plasma production curve at this point, this intersection corresponds to a state with a negative value of differential resistance, inaccessible due to the resultant circuit-related instability in the discharge current. Such states can be observed by placing a resistor in series with the discharge power supply, as shown in Fig. 3.

The onset of jumps can be described using the schematic illustration shown in Fig. 7. The solid line is a typical filament characteristic curve; the dotted lines are typical plasma production curves.

For fixed values of filament temperature and discharge voltage, the system always lies along the solid curve. Suppose the pressure is P_1, then the system is also described by the dotted curve P_1, so the state of the system is at point A, the intersection of the two curves. As pressure is increased to the value P_2, the pressure curve describing the system moves rightward to the curve labelled P_2, and the system is in state B. As pressure is further increased, the system will move to point C, where the solid and dotted curves are tangent. As pressure is further increased, there are no possible states in the region of C, so the system jumps to state E, the only intersection of the two curves describing the system. If pressure is now reduced, the system moves to point D. Further reduction in pressure results in a jump to point B. The region of the solid curve between points C and D is inaccessible by varying the pressure. For pressures between P_2 and P_3, the filament characteristic curve is intersected three times by the plasma production curve, but one of these states lies on the region of the solid curve between points C and D and is therefore inaccessible.

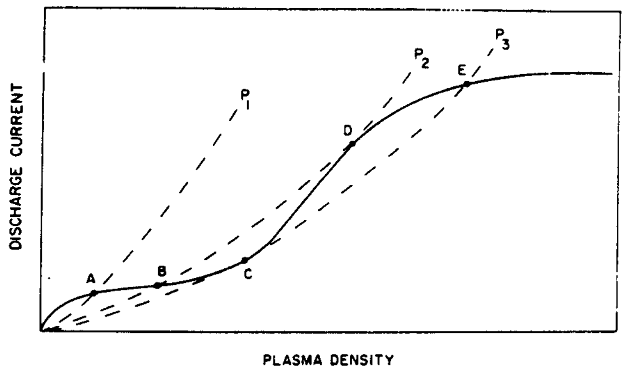

Fig. 7. Schematic illustration of a filament characteristic (solid line) and plasma production curves (dashed lines) used to describe jumps and hysteresis

This interpretation agrees well with our observations: jumps occur when the two types of curves are tangent, and the states before and after a jump lie on the same plasma production curve. The region of the filament characteristic curve between points C and D is inaccessible, also. This can be observed in Fig. 8, which shows three of the curves from Figs. 5 and 6 on the same graph, with labelling of jump points corresponding to Fig. 7. This analysis reduces the explanation of jumps and hysteresis to an explanation of the functional dependence of the plasma production and filament characteristic curves.

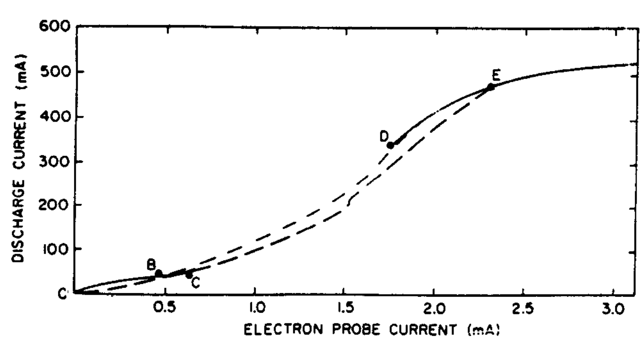

Fig. 8. Experimental filament characteristic curve (solid line) and plasma production curves (dashed lines) previously shown in Figs. 5 and 6. The labelled points B, C, D, and E correspond to those in Fig. 7. The results demonstrate that the two types of curves are tangent at points C and D where jumps occur

4.2. Plasma Production and Filament Characteristic Curves

The plasma production curves have a nearly linear dependence between plasma density and the number of primary electrons (discharge current) for low plasma densities. But at higher densities, the curves bend upward indicating less plasma present per primary electron. The filament characteristic curves also bend upwards in this region, which corresponds to the onset of plasma noise.

Langmuir probe characteristics were taken at various points along the curves shown in Fig. 8. The plots of actual electron density have the same qualitative shape as the plots of saturation current versus discharge current, with more pronounced upward bends in the curves in the region of the onset of turbulence. The ratio of discharge current to plasma density increases by a factor of four when the plasma jumps to the higher state.

An analysis of the Langmuir probe traces indicates that the lower density states are characterized by two populations of electrons. About 90% of the electrons have a temperature of 0.2 eV, while the remainder have a temperature of 2 eV. In the region of the plasma production curve where turbulence onsets, the temperature of the cold species increases and a greater proportion of the electrons are in the hotter species. At higher discharge currents, the cold species is no longer observed and all electrons have a temperature of 3.6 eV.

The observed electron heating and plasma noise appear to result from a beam-plasma instability driven by primary electrons. A similar noise spectrum and electron heating due to a beam-plasma instability has been previously reported by WHELAN and STENZEL [21] in a discharge plasma produced using an oxide coated, indirectly heated cathode.

The maximum current which the filaments can emit is determined by their temperature and composition, and is called the Richardson current. As the neutral pressure or discharge voltage are increased to high values, the discharge current is observed to asymptotically reach this maximum value. The estimated variations in the Richardson current due to filament heating by ion bombardment and by increased discharge current amounts to a few percent. The large variations in discharge current observed at constant T_f are thus due to space-charge effects on the flow of electrons from the filaments. Of course, this is not the case for the plasma production curves obtained by varying T_f.

4.3. Physical Picture of Hysteresis Due to Filament and Plasma Properties

To examine the conditions necessary for negative differential resistance, a physical picture may be constructed. For a planar geometry, the Langmuir relation for a space-charge limited cathode sheath [22] is:

$$I = \left(\frac{m_i}{m_e}\right)^{1/2} I_i, \tag{9}$$

where I_i is the ion current flowing from the plasma to the filaments. We assume that this expression will approximately hold for the non-planar filament geometry.

The Bohm sheath criterion states that the ions approaching the cathode sheath of area A_s must have velocity $(kT_e/m_i)^{1/2}$ (for $T_e \gg T_i$) so the ion current is:

$$I_i = neA_s \left(\frac{kT_e}{m_i}\right)^{1/2}. \tag{10}$$

Combining relations (9) and (10) gives an equation describing the filament characteristic curve:

$$I(V, T_f, n) = neA_s \left(\frac{kT_e}{m_e}\right)^{1/2}. \tag{11}$$

This relationship conforms to the observed increase in discharge current due to increases in density and electron temperature. For fixed T_f and V, n can be considered as the only independent variable, so A_s and T_e are functions of n and:

$$\left.\frac{\partial I}{\partial n}\right|_{V,Tf} = I\left[\frac{1}{n} + \frac{1}{A_s}\frac{\mathrm{d}A_s}{\mathrm{d}n} + \frac{1}{2T_e}\frac{\mathrm{d}T_e}{\mathrm{d}n}\right]. \tag{12}$$

The rate of production of plasma due to ionizing collisions with a neutral gas of pressure P may be written in the form:

$$\text{Production rate} = IP\alpha, \tag{13}$$

where the function α, a measure of ionization probability, is defined by this equation. The plasma escaping at the sheaths on the chamber surface must again obey the Bohm sheath criterion giving:

$$\text{Escape rate} = nA\left(\frac{kT_e}{m_i}\right)^{1/2}, \tag{14}$$

where A is the surface area of the chamber. In a steady-state these rates are equal, so the plasma production curve is given by

$$I = \frac{nA(kT_e/m_i)^{1/2}}{P\alpha}. \tag{15}$$

For constant V and P; T_e and α may be considered as functions of n and:

$$1/(\partial n/\partial I)|_{V,P} = I\left[\frac{1}{n} - \frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}n} + \frac{1}{2T_e}\frac{\mathrm{d}T_e}{\mathrm{d}n}\right]. \tag{16}$$

The condition for stability ($\partial V/\partial I > 0$) determined in equation (8) is that the difference in slopes of these curves at their intersection be negative:

$$\frac{1}{A_s}\frac{\mathrm{d}A_s}{\mathrm{d}n} + \frac{1}{\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}n} < 0. \tag{17}$$

In a nonturbulent plasma, the ionization probability should remain constant so $\mathrm{d}\alpha/\mathrm{d}n = 0$. The relation between sheath thickness, plasma density and temperature may be determined by eliminating I in Child's law using equation (11) to obtain a thickness $\approx \lambda_d(\mathrm{e}V/kT_e)^{3/4}$. As n (or T_e) increases, the cathode sheath thickness will decrease. In our cathode geometries, a thinner cathode sheath will have a smaller surface area, so $\mathrm{d}A_s/\mathrm{d}n < 0$. As a result, the stability criterion is satisfied.

However, as plasma turbulence onsets with increasing density, the electron temperature increases. Enhancement of ionization is thought to be present in beam-plasma instabilities [23], due to the resultant super-thermal tail of electrons. This will cause $\mathrm{d}\alpha/\mathrm{d}n > 0$. A negative differential resistance will result when $\mathrm{d}\alpha/\mathrm{d}n$ becomes sufficiently large to violate equation (17). This interpretation is consistent with our observations: upward jumps were always associated with the onset of turbulence while downward jumps resulted in the cessation of turbulence.

The importance of filament geometry is also evident in this physical picture. Filament geometries where $1/A \, dA_s/dn \ll 0$ (such as a single filament or narrow foil) should be more stable, as we have observed. With multiple filaments, we may have $dA_s/dn \approx 0$ as the sheath transforms from a composite sheath to individual sheaths around each wire, so this cathode geometry will lead to jumps if $dA_s/dn \approx 0$ when turbulence causes $d\alpha/dn > 0$. Experimentally, we have observed increased jumps with closely spaced filaments. It is also possible that filament geometry may influence the term $\alpha^{-1} \, d\alpha/dn$ by affecting the magnitude of the beam-plasma turbulence.

5. Summary and Conclusions

The hysteresis and jumps can be explained by examining the curves which describe filament characteristics and plasma production properties. The intersection of these curves describes the state of the plasma. The shapes of these curves in the jump region give rise to multiple intersections which lead to jumps and hysteresis. At the intersection where the slope of the filament characteristic curve is larger than that of the plasma production curve, the differential resistance of the discharge is negative. This makes the discharge circuit unstable, so these states can be observed only if a series resistor is placed in the discharge circuit.

Jumps and hysteresis are associated with closely spaced filaments and the onset of turbulence. A simple model shows how these conditions can cause the slope of the filament characteristic curve to exceed that of the intersecting plasma production curve, resulting in multiple intersections of the curves and the associated jumps, hysteresis, and negative differential resistance.

Acknowledgments

The authors wish to thank S. CARTIER, G. KNORR, and N. D'ANGELO for useful discussions and A. SCHELLER and D. HUDEK for skillful technical assistance. This work was supported by a Graduate College Block Grant from the University of Iowa.

References

[1] COBINE, J. D., "Gaseous Conductors", McGraw-Hill Book Co., New York (1958).

[2] EMÉLEUS, K. G., "The Conduction of Electricity through Gases", Ch. II, John Wiley & Sons, New York (1951).

[3] KRALL, N., and TRIVELPIECE, A., "Principles of Plasma Physics", McGraw-Hill Book Co., New York (1973).

[4] LOEB, L. B., "Electrical Breakdown of Gases with Steady or Direct Current Impulse Potentials", in Encyclopedia of Physics, Vol. XXII, edited by S. FLÜGGE, Springer-Verlag, Berlin (1956).

[5] BROWN, S., "Introduction to Electrical Discharges and Gases", Chs. XIII and XV, John Wiley & Sons, New York (1966).

[6] FINKELNBURG, W., and MAECKER, H., "Elektrische Bögen und thermisches Plasma", in Encyclopedia of Physics, Vol. XXII, edited by S. FLÜGGE, Springer-Verlag, Berlin (1956).

[7] FRANCIS, G., "The Glow Discharge at Low Pressure", in Encyclopedia of Physics, Vol. XXII, edited by S. FLÜGGE, Springer-Verlag, Berlin (1956).

[8] LESK, I. A., HOLONYAK, N., and DAVIDSOHN, U. S., "Tunnel-Diode and Semiconductor Circuits", Ch. 1, see also Ch. 15, edited by J. M. CARROL, McGraw-Hill Book Co., New York (1963).

[9] Papoular, R., "Electrical Phenomena in Gases", Chs. 12 and 13, American Elsevier Publishing Company, Inc., New York (1965).
[10] Uman, M. F., "Introduction to the Physics of Electronics", Prentice-Hall, Inc., Englewood Cliffs, NJ (1974).
[11] Baksht, F. G., Dyuzhev, G. A., Martsinouskiy, A. M., Moyzhes, B. Ya., Dikus, G. Ye., Sonin, E. B., and Yur'yev, V. G., "Thermionic Converters and Low Temperature Plasma", Technical Information Center/U.S. Dept. of Energy, Springfield, VA (1978).
[12] Gekelman, W., and Stenzel, R. L., Rev. Sci. Instrum. 46 (1975) 1386.
[13] Ellis, R. F., and Arion, D. N., J. Appl. Phys. 54 (1983) 4895.
[14] Coleman, J. W., Post, R. S., Torti, R. P., and Tracey, J. E., Rev. Sci. Instrum. 54 (1983) 1100.
[15] Oka, Y., and Kuroda, T., Appl. Phys. Lett. 34 (1979) 134.
[16] Goebel, D. M., Phys. Fluids 25 (1982) 1093.
[17] Minkiewicz, V. J., Chen, M., Coburn, J. W., Chapman, B. N., and Lee, K., Appl. Phys. Lett. 35 (1979) 393.
[18] Ecker, G., Kroll, W., Spatschek, K. H., and Zoller, O., Phys. Fluids 10 (1967) 1073.
[19] Jensen, T. H., Moeller, C. P., and Freeman, R. L., Phys. Fluids 22 (1979) 1221.
[20] Knorr, G., Plasma Phys. Control. Fusion 26 (1984) 949.
[21] Whelan, D. A., and Stenzel, R. L., Phys. Rev. Lett. 50 (1983) 1133.
[22] Langmuir, I., Phys. Rev. 33 (1929) 954.
[23] Smullin, L. D., "A Review of the Beam-Plasma Discharge", in Relation between Laboratory and Space Plasmas, edited by H. Kikuchi, D. Reidel Publishing Co., Boston (1981).

Received February 25, 1985