Kinetic Turbulence

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Abstract The weak collisionality typical of turbulence in many diffuse astrophysical plasmas invalidates an MHD description of the turbulent dynamics, motivating the development of a more comprehensive theory of kinetic turbulence. In particular, a kinetic approach is essential for the investigation of the physical mechanisms responsible for the dissipation of astrophysical turbulence and the resulting heating of the plasma. This chapter reviews the limitations of MHD turbulence theory and explains how kinetic considerations may be incorporated to obtain a kinetic theory for astrophysical plasma turbulence. Key questions about the nature of kinetic turbulence that drive current research efforts are identified. A comprehensive model of the kinetic turbulent cascade is presented, with a detailed discussion of each component of the model and a review of supporting and conflicting theoretical, numerical, and observational evidence.

1 Introduction

The study of turbulence in astrophysical plasmas has almost exclusively employed a magnetohydrodynamic (MHD) description of the turbulent dynamics, treating the magnetized plasma as a single fluid, an approximation valid for large-scale, low-frequency dynamics in the strongly collisional limit. Yet, the plasmas in a wide variety of turbulent astrophysical environments often violate one or more of the conditions required by the MHD approximation, particularly on the small scales at which dissipation mechanisms act to damp the turbulent fluctuations, ultimately leading to heating of the plasma. The study of the turbulent dynamics at small scales and of the physical mechanisms responsible for the dissipation of the turbulence generally requires a kinetic treatment. Thus, it is necessary to leave behind the comfortable

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surroundings of the theory of MHD turbulence and enter the uncharted territory of the evolving theory of *kinetic turbulence*.

1.1 Quantitative Characterization of Plasma Turbulence

Turbulent systems are typically described theoretically by a spectral decomposition of the broadband spatial fluctuations into a sum of plane wave modes, each characterized by its three dimensional wavevector, phase, and amplitude. An energy spectrum of the turbulent fluctuations therefore provides a useful quantitative description of the turbulent system. In a magnetized plasma, the three-dimensional wavevector space can be reduced to two dimensions by assuming axial symmetry about the direction of the equilibrium magnetic field, requiring only the specification of the turbulent power with respect to the cylindrical components of the wavevector, $k_\perp$ and $k_\parallel$. The nature of the dynamics in the different ranges of the kinetic turbulent cascade can be quantitatively characterized by two properties: (1) the *one-dimensional magnetic energy spectrum* in perpendicular wavenumber, $E_B(k_\perp)$; and (2) the *wavevector anisotropy*, or the distribution of turbulent power in wavevector space. Here $E_B(k_\perp)$ is defined such that the total magnetic energy is given by $E_B = \int dk_\perp E_B(k_\perp)$. For Alfvénic turbulence that is driven isotropically at the outer-scale wavenumber k_0, the conjecture of critical balance implies that the turbulent power fills a region of the cylindrical wavevector space satisfying $k_\parallel \lesssim k_0^{1-q} k_\perp^q$. Specification of the scaling of the boundary of this region, $k_\parallel \propto k_\perp^q$, is sufficient to completely characterize the anisotropic distribution of turbulent power.

1.2 Limits of MHD Treatment of Astrophysical Turbulence

The limitations of an MHD treatment of astrophysical turbulence can be illuminated by considering the domain of applicability of MHD turbulence theory within the broader context of plasma turbulence. Beginning with the general theory of the turbulent cascade of kinetic energy in hydrodynamic systems, we consider the modifications required to describe the turbulent energy cascade in the magnetized plasma systems relevant to astrophysical environments.

1.2.1 From Fluid to Kinetic Models of the Turbulent Cascade

The limitations of MHD turbulence theory can be illustrated most clearly by a qualitative comparison of the features of nonlinear cascade of energy in hydrodynamic turbulence, MHD turbulence, and kinetic turbulence.

In hydrodynamic systems, turbulent motions are driven at some large scale L, denoted the driving or energy injection scale. Nonlinear interactions serve to transfer the turbulent kinetic energy to motions at ever smaller scales, until reaching a small scale l_ν at which dissipation via viscous damping is sufficient to terminate the turbulent cascade. For typical hydrodynamic systems, a large dynamic range exists between the driving and dissipation scales, $L/l_\nu \gg 1$. In that case, one may define an *inertial range* of scales l within which the effects of the driving and dissipation are negligible, $L \gg l \gg l_\nu$. Within the inertial range, there exists no particular characteristic length scale, so the dynamics of the turbulence in the inertial range is found to be self-similar, and a simple application of dimensional analysis is sufficient to describe accurately the steady-state hydrodynamic turbulent cascade of energy (Kolmogorov, 1941). A qualitative diagram of the kinetic energy wavenumber spectrum for the hydrodynamic turbulence cascade is shown in Figure 1(a).

In magnetohydrodynamic systems, the turbulence theory must be modified in three important ways. First, the dynamics of two turbulent fields, the velocity and the magnetic field, must be described, so the cascade of both kinetic and magnetic energy is mediated by nonlinear turbulent interactions. Second, fluctuations of the two turbulent fields are dissipated by distinct mechanisms, viscosity for the velocity and resistivity for the magnetic field. The characteristic length scales of viscous dissipation l_ν and resistive dissipation l_η need not be equal, and their ratio is characterized by the magnetic Prandtl number $\mathrm{Pr}_m \equiv l_\nu/l_\eta$. Third, the magnetic field in the plasma establishes a preferred direction, leading to distinct dynamics in the direction parallel to the magnetic field and in the plane perpendicular to the magnetic field. In addition, the magnetic tension provided by the magnetic field supports a type of linear wave, the Alfvén wave, which has no counterpart in the hydrodynamic case, transforming the nature of the turbulent motions from hydrodynamic vortices to magnetohydrodynamic waves. This third complication is the most significant change from hydrodynamic turbulence, and leads to the inherent anisotropy of MHD turbulence, where turbulent energy is transferred more rapidly to small perpendicular scales $l_\perp$ than to small parallel scales $l_\parallel$. Nonetheless, despite these significant differences, the overall qualitative picture of the turbulent energy cascade in MHD turbulence bears a striking resemblance to the hydrodynamic case.

Consider, in particular, the simplified case of MHD turbulence in a $\mathrm{Pr}_m = 1$ plasma, so there exists a single dissipation scale $l_d = l_\nu = l_\eta$. One may define an MHD inertial range, $L \gg l_\perp \gg l_d$, directly analogous to the hydrodynamic case. Due the anisotropy of the turbulent energy transfer, the turbulent dynamics are optimally described with respect to the perpendicular scale $l_\perp$. The evolution of the parallel scale is determined in terms of the perpendicular scale by the condition of *critical balance* (Goldreich and Sridhar, 1995), so that $l_\parallel \propto l_\perp^q$. The exponent q describes the *scale-dependent anisotropy* of the MHD turbulent cascade, where $q = 2/3$ in the Goldreich-Sridhar model (Goldreich and Sridhar, 1995), and $q = 1/2$ in the Boldyrev model (Boldyrev, 2006). Similar to the hydrodynamic case, in the MHD inertial range, there exists no characteristic length scale, so the dynamics of MHD turbulence is found to be self-similar as well. Therefore, for the $\mathrm{Pr}_m = 1$ case, the MHD turbulence theory appears nearly the same as the hydrodynamic turbulence theory, with a few minor changes: (1) the turbulent cascade is described by the perpendicular scale $l_\perp$ rather than an isotropic scale l; (2) there exists a scale-

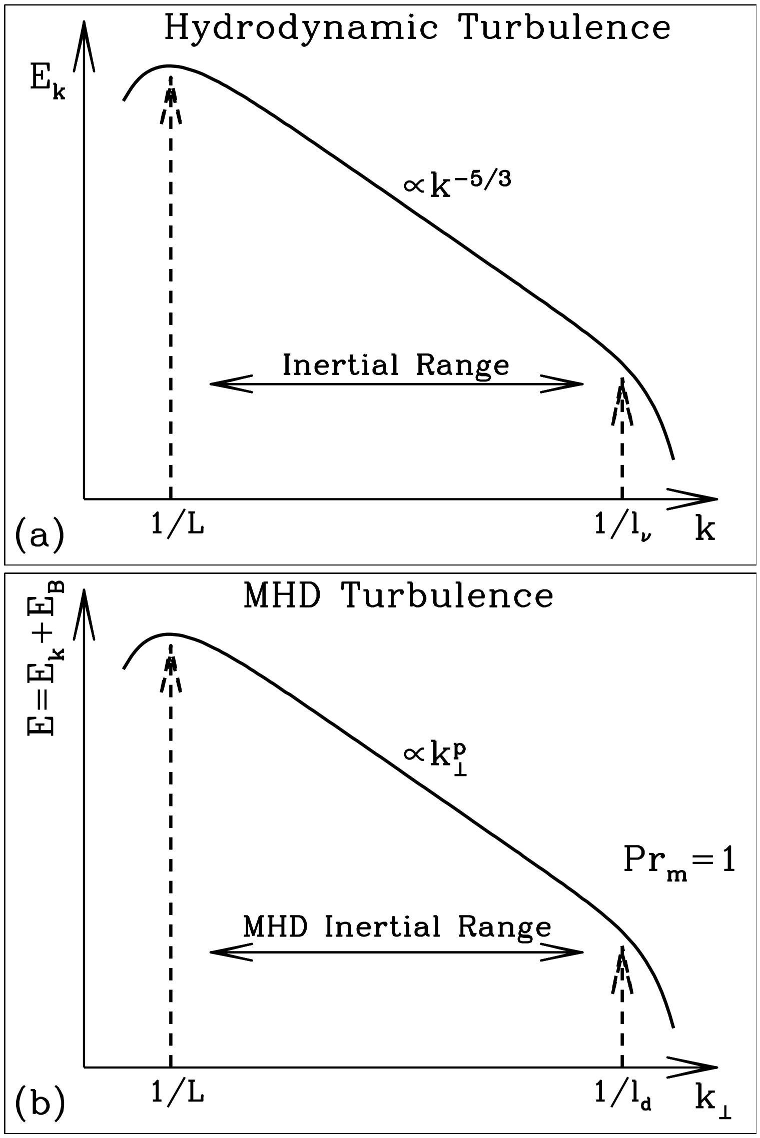

Fig. 1 (a) Wavenumber spectrum for kinetic energy in hydrodynamic turbulence, from the driving scale, L, through the inertial range, to the viscous dissipation scale, l_ν. (b) Perpendicular wavenumber spectrum for total energy $E = E_k + E_B$ in MHD turbulence with $\mathrm{Pr}_m = 1$, from the driving scale, L, through the inertial range, to the viscous and resistive dissipation scale, $l_d = l_\nu = l_\eta$.

dependent anisotropy due to the parallel scaling $l_\parallel \propto l_\perp^q$; and (3) the exponent p in the self-similar power law solution for the one-dimensional energy spectrum $E \propto k_\perp^p$ may differ quantitatively from the hydrodynamic solution. But the general qualitative picture—a self-similar MHD turbulent cascade of energy from the driving scale L, through an inertial range, to the dissipative scale l_d—remains essentially the same as the hydrodynamic cascade, as is evident by comparing the diagram of the wavenumber spectrum for total energy $E = E_k + E_B$ in the MHD turbulent cascade in Figure 1(b) to the hydrodynamic case in Figure 1(a).

In kinetic plasma systems, this simple qualitative model of the turbulent cascade changes dramatically due to the existence of three characteristic length scales and new physics associated with each of these scales. The three characteristic length scales that come into play in typical conditions for turbulent astrophysical plasmas are the ion mean free path λ_i, the ion Larmor radius ρ_i, and the electron Larmor radius ρ_e. The MHD approximation requires the following four conditions:

1. Nonrelativistic conditions, $v_{ts}/c \ll 1$
2. Strongly collisional conditions, $\lambda_i/l \ll 1$
3. Large-scale motions, $\rho_i/l \ll 1$
4. Low-frequency dynamics, $\omega/\Omega_i \ll 1$

Here $v_{ts} = \sqrt{2T_s/m_s}$ is the thermal velocity[1] of species s, ω is the typical frequency of the turbulent fluctuations, and Ω_i is the ion cyclotron frequency. It is clear that, in the MHD approximation, all three of the characteristic scales above are assumed to be infinitesimal compared to the typical scale of the turbulent motions, l. However, in astrophysical plasmas of interest, the turbulent dynamics frequently violate conditions (2) and (3) above[2]. Therefore, it is important to examine more closely how these characteristic scales enter into the dynamics of the turbulent cascade in astrophysical plasmas, leading to a violation of the MHD approximation and requiring the transition to a kinetic description of the turbulent dynamics.

1.2.2 Violation of the MHD Approximation

Spacecraft measurements of turbulence in the solar wind provide invaluable guidance for the construction of a theoretical model that describes the energy spectrum of the kinetic turbulent cascade. Recent measurements of solar wind turbulence with unprecedented temporal resolution enable us to probe the turbulent dynamics down to the scale of the electron Larmor radius (Sahraoui et al., 2009; Kiyani et al., 2009; Alexandrova et al., 2009; Chen et al., 2010; Sahraoui et al., 2010; Alexandrova et al., 2012). Therefore, we now have a fairly complete observational picture of the kinetic turbulent cascade in the solar wind over a dynamic range of 10^6 from the the large energy injection scale at $L \sim 10^6$ km down to the scale of the electron

[1] Here T_s is expressed in units of energy, absorbing the Boltzmann constant.

[2] Note that condition (4) is not generally independent of condition (3). For MHD Alfvén waves, the condition $\omega \ll \Omega_i$ may be alternatively written $\rho_i/l_\parallel \ll \sqrt{\beta_i}$, where the ion plasma beta is $\beta_i = 8\pi n_i T_i/B^2$. Thus, if $\sqrt{\beta_i} \sim \mathscr{O}(1)$, then condition (4) is roughly equivalent to condition (3).

Larmor radius at $\rho_e \sim 1$ km. From the large body of turbulence measurements in the solar wind (Sahraoui et al., 2009; Kiyani et al., 2009; Alexandrova et al., 2009; Chen et al., 2010; Sahraoui et al., 2010; Alexandrova et al., 2012), we can construct a general diagram for the perpendicular wavenumber spectrum of the magnetic energy in turbulent astrophysical plasmas, shown in Figure 2(a). It is important to emphasize here that, although the general form of the magnetic energy spectrum is well established from observations, the interpretation of this spectrum in terms of the characteristic plasma scales requires significant input from plasma kinetic theory, and many of the features of Figure 2(a) remain topics of active research.

In Figure 2(a), the plasma turbulence is driven at some large scale $L \gg \rho_i$. It is generally assumed, in the absence of arguments to the contrary, that the turbulence is driven isotropically with respect to the magnetic field, so that the perpendicular and parallel components of the driving wavevector $\mathbf{k}_0$ are equal, $k_{\|0} \sim k_{\perp 0} \sim k_0 \sim 1/L$. If the plasma conditions at the driving scale satisfy the MHD approximation, then the large scale end of the turbulent cascade is described by MHD turbulence theory. Although the turbulent fluctuations in an MHD plasma may, in general, be composed of a mixture fast, Alfvén, and slow waves, observational and numerical evidence suggests that Alfvén waves dominate the turbulent dynamics in typical astrophysical plasmas (this point is discussed further below). For the Alfvénic turbulent cascade, the one-dimensional magnetic energy spectrum as a function of perpendicular wavenumber $k_\perp$ scales as $E_B \propto k_\perp^p$, where the spectral index is $p = -5/3$ in the Goldreich-Sridhar model (Goldreich and Sridhar, 1995) or $p = -3/2$ in the Boldyrev model (Boldyrev, 2006). The wavevector anisotropy of the anisotropic Alfvénic cascade scales as $k_\| \propto k_\perp^q$, where the values for q are given in §1.2.1; this anisotropic cascade of energy through wavevector space is depicted in Figure 2(b).

As the MHD turbulent cascade transfers energy to smaller scales (higher wavenumber), it eventually reaches the one of the characteristic length scales λ_i, ρ_i, or ρ_e, at which point the MHD approximation is violated. Here we focus on exploring how these length scales enter into the model for kinetic turbulence and what effect they have on the turbulent dynamics. For typical conditions in astrophysical plasmas, the characteristic length scales are ordered by $\lambda_i > \rho_i > \rho_e$, so the ion mean free path λ_i is usually reached first.

The ion mean free path λ_i characterizes the collisionality for the motion of plasma particles parallel to the magnetic field[3], so it must be compared to the parallel wavenumber $k_\|$. For $k_\| \lambda_i \ll 1$, the plasma is strongly collisional; for $k_\| \lambda_i \gg 1$, the plasma is weakly collisional. Fluid approximations, such as hydrodynamics or MHD, break down for plasma conditions $k_\| \lambda_i \gtrsim 1$, so kinetic theory is formally required to describe the plasma dynamics in moderately to weakly collisional regimes.

[3] The Lorentz force limits the perpendicular motion of plasma particles to the particle Larmor radius. Since typical astrophysical conditions yield $\rho_i \ll \lambda_i$, the plasma is essentially always collisionless in the perpendicular direction. Note, however, that because plasma particles cannot move beyond the Larmor radius in the perpendicular direction from the magnetic field, this embodies the large-scale *perpendicular* motions, $l_\perp \gg \rho_i$, with a fluid-like behavior, even under weakly collisional conditions.

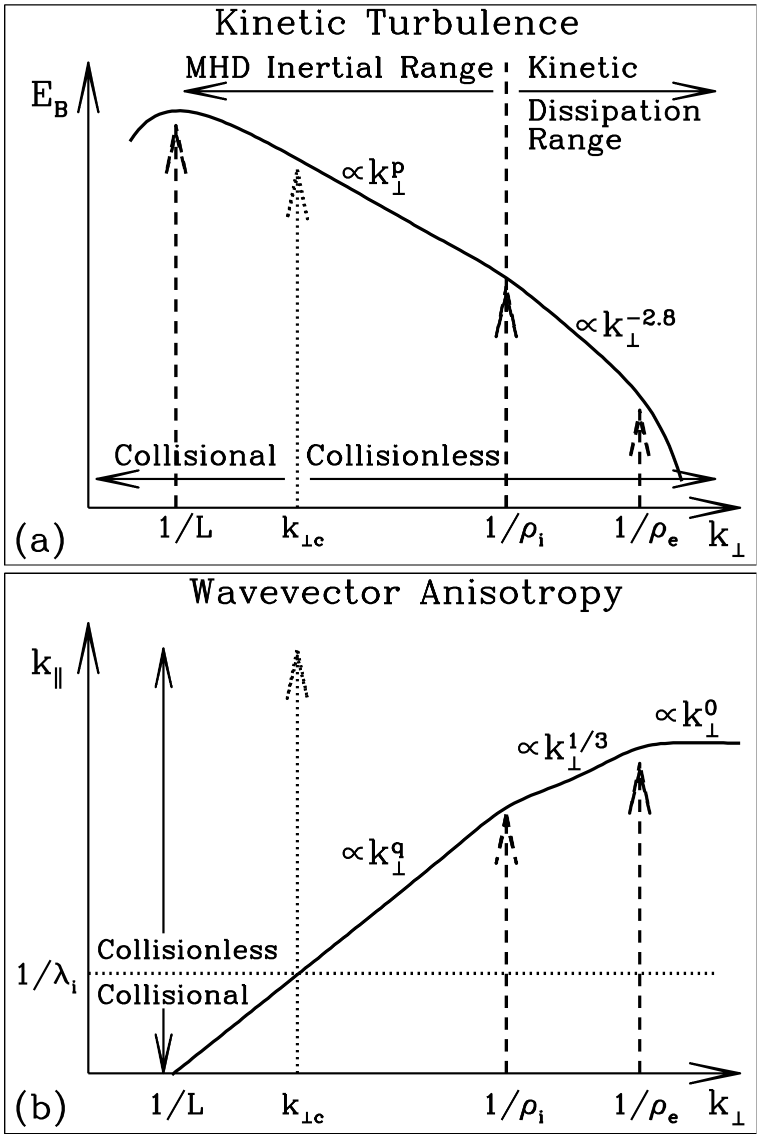

Fig. 2 (a) Perpendicular wavenumber spectrum for magnetic energy in kinetic turbulence, from the driving scale, L, through the MHD inertial range to the ion Larmor radius ρ_i, where the turbulent cascade enters the kinetic dissipation range, and down to the electron Larmor radius ρ_e. The transition from collisional to collisionless dynamics occurs at $k_{\perp c}$. (b) Wavevector anisotropy in kinetic turbulence, scaling as $k_\perp^q$ in the MHD inertial range, $k_\perp^{1/3}$ in the kinetic dissipation range, and $k_\perp^0$ (no parallel cascade) beyond electron scales. The transition from collisional to collisionless dynamics occurs at $k_\parallel \rho_i \sim 1$.

As depicted in Figure 2(b), at some point in the MHD inertial range, the parallel scales may reach the scale of the ion mean free path, $k_\parallel \lambda_i \sim 1$, marking the transition from collisional dynamics that is well described by MHD at $k_\parallel \lambda_i \ll 1$ to collisionless dynamics that requires a kinetic description at $k_\parallel \lambda_i \gg 1$. The condition of critical balance determines the relation between the parallel and perpendicular wavenumbers of strong MHD turbulence (Goldreich and Sridhar, 1995), so we may define the perpendicular wavenumber $k_{\perp c}$ that corresponds to the transition of collisionality at $k_\parallel \lambda_i \sim 1$. This transition in the perpendicular wavenumber spectrum typically occurs at perpendicular scales larger than the ion Larmor radius, $k_{\perp c}\rho_i < 1$, as shown in Figure 2(a). For perpendicular wavenumbers $k_\perp \ll k_{\perp c}$, the strongly collisional dynamics is well described by MHD, and for $k_\perp \gg k_{\perp c}$ the weakly collisional dynamics require a kinetic description, as depicted in Figure 2(a).

For the weakly collisional range $k_\perp \gg k_{\perp c}$, it has been shown rigorously from kinetic theory that the Alfvénic turbulent fluctuations remain essentially fluid in nature (Schekochihin et al., 2009). The Alfvénic turbulent cascade continues to be accurately described by the equations of reduced MHD (Strauss, 1976) and remains undamped down to the perpendicular scale of the ion Larmor radius, $k_\perp \rho_i \sim 1$ (Schekochihin et al., 2009). Therefore, although the MHD approximation is formally violated at scales $k_\perp \gtrsim k_{\perp c}$, the MHD description of the anisotropic Alfvénic cascade remains applicable, regardless of whether the dynamics is collisional or collisionless, for all scales larger than the ion Larmor radius, $k_\perp \rho_i \ll 1$. Therefore, we denote the range of scales $L \gg l_\perp \gg \rho_i$ in the *kinetic* turbulent cascade as the *MHD inertial range*.

MHD Inertial Range: The range of perpendicular scales from the large scale of energy injection to the scale of the ion Larmor radius, $L \gg l_\perp \gg \rho_i$, including both collisional and collisionless regimes.

On the other hand, compressible turbulent fluctuations associated with the MHD fast and slow waves in the MHD inertial range require a kinetic description at all moderately to weakly collisional scales, $k_\parallel \lambda_i \gtrsim 1$ or $k_\perp \gtrsim k_{\perp c}$. These modes are damped both collisionally by ion viscosity at $k_\parallel \lambda_i \sim 1$ (Braginskii, 1965) and collisionlessly by ion Landau damping at $k_\parallel \lambda_i \gg 1$ (Barnes, 1966). Therefore, it is expected that the damped compressible modes will play at most a subdominant role relative to the undamped Alfvénic fluctuations in turbulent astrophysical plasmas. In the weakly collisional solar wind, for example, compressible fluctuations generally contribute less than 10% of the turbulent magnetic energy (Tu and Marsch, 1995; Bruno and Carbone, 2005) in the MHD inertial range. Therefore, we turn our attention back to the dynamics of the dominant Alfvénic turbulent fluctuations.

When the Alfvénic turbulent cascade reaches the perpendicular scale of the ion Larmor radius, $k_\perp \rho_i \sim 1$, the MHD description of the Alfvénic fluctuations breaks down completely for two reasons. First, finite Larmor radius effects lead to a decoupling of the ions from the turbulent electromagnetic fluctuations at perpendicular wavenumbers $k_\perp \rho_i \gtrsim 1$. The result is that the non-dispersive Alfvén wave in the limit

$k_\perp \rho_i \ll 1$ transitions to the dispersive kinetic Alfvén wave in the limit $k_\perp \rho_i \gg 1$. The dispersive nature of the Alfvénic fluctuations accelerates the rate of the turbulent nonlinear energy transfer, leading to a steepening of the magnetic energy spectrum, with a break in the spectrum at the $k_\perp \rho_i \sim 1$, as shown in Figure 2(a). Second, collisionless damping of the electromagnetic fluctuations occurs due to the Landau resonance with the ions, with a peak in the ion damping rate around $k_\perp \rho_i \sim 1$. In addition, electron Landau damping can also contribute significantly for all scales $k_\perp \rho_i \gtrsim 1$. The combined effect of the ion and electron collisionless damping can lead to a further steepening of the spectrum for scales $k_\perp \rho_i \gtrsim 1$ (Howes et al., 2011; Howes et al., 2011). Finally, the cascade reaches the perpendicular scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$, where collisionless damping becomes sufficiently strong to terminate the turbulent cascade, leading to an exponential drop off of the magnetic energy spectrum (Terry and et al., 2012; Alexandrova et al., 2012; TenBarge et al., 2013). MHD turbulence theory cannot describe the dispersive wave behavior or the dissipation that occurs via kinetic mechanisms at scales $k_\perp \rho_i \gtrsim 1$. Therefore, we denote the range of scales $l_\perp \lesssim \rho_i$ in the kinetic turbulent cascade as the *kinetic dissipation range*.

Kinetic Dissipation Range: The range of perpendicular scales at or below the scale of the ion Larmor radius, $l_\perp \lesssim \rho_i$, where wave dispersion and collisionless dissipation play important roles.

1.3 Importance of Kinetic Turbulence

Of fundamental importance in the study of astrophysical turbulence is to determine the pathway by which the energy of turbulent motions is ultimately converted to plasma heat. Astrophysical turbulence is generally driven by violent events or instabilities at large scales, but fluctuations are dissipated strongly only at scales of order or smaller than the ion Larmor radius. A kinetic turbulent cascade arises to transfer energy via nonlinear couplings from the large energy injection scales, through the MHD inertial range, down to the scale of the ion Larmor radius. The turbulent fluctuations begin to be damped when the cascade reaches the scale of the ion Larmor radius, marking the entry into the kinetic dissipation range. Since the dynamics within the kinetic dissipation range is typically weakly collisional, the dissipation of the turbulent electromagnetic fluctuations must be accomplished via collisionless mechanisms governed by plasma kinetic theory. The energy thus removed from the turbulent fluctuations ultimately leads to thermal heating of the protons, electrons, and minority ions in the plasma. The observational signature of astrophysical objects depends strongly on the nature of the plasma heating, so to interpret observational data requires a detailed characterization of the small-scale, kinetic plasma turbulence.

For example, as matter in an accretion disk spirals slowly into a black hole, it converts a tremendous amount of gravitational potential energy into heat. Several physical mechanisms contribute to this process. First, the magnetorotational instability (Balbus and Hawley, 1991; Balbus and Hawley, 1998) taps free energy from the differential rotation of the accretion disk to drive turbulence on the scale height of the disk, $L \sim H$. The turbulence effectively transports angular momentum outward in the disk, enabling accretion disk plasma to fall down the gravitational potential and mediating the conversion of gravitational potential energy into kinetic and magnetic energy of the MHD turbulent fluctuations. The high temperatures characteristic of the plasma in a black hole accretion disk lead to a collisional mean free path $\lambda_i \sim H$, so the turbulent dynamics is weakly collisional. A kinetic turbulent cascade is responsible for the transfer of turbulent energy through the MHD inertial range down to the scale of the ion Larmor radius, where the turbulent electromagnetic fluctuations are damped via collisionless mechanisms in the kinetic dissipation range. An entropy cascade ultimately mediates the final conversion of this turbulent free energy into plasma heat. Therefore, the radiation that is emitted from the hot, magnetized plasma is a strong function of the black hole properties and of the character of the small-scale plasma fluctuations, where the plasma heating occurs. To interpret observational data from the Chandra X-ray Observatory, for example, one must unravel the details of the kinetic turbulent cascade.

Developing a mature model of the kinetic turbulent cascade is critical to understanding the turbulent dynamics of the kinetic dissipation range, the physical mechanisms responsible for the damping of the turbulent fluctuations, and the resulting heating of the plasma species. The ultimate goal is to develop a predictive capability to estimate accurately the heating of the protons, electrons, and minority ions in the plasma based on the plasma parameters and the characteristics of the turbulent driving.

2 Key Questions about Kinetic Turbulence

The unprecedented availability of high temporal resolution solar wind turbulence measurements from current spacecraft missions has enabled the observational characterization of the kinetic turbulent cascade from the large scales of energy injection down to the scale of the electron Larmor radius. This has spurred the heliospheric physics community to engage actively the topic of the turbulence in the dissipation range of the solar wind, and has engendered considerable controversy about a number of significant issues related to the fundamental character of kinetic turbulence. In particular, the nature of both the turbulent fluctuations in this regime and the physical mechanisms responsible for their dissipation remains highly contested within the scientific community. Four key questions relevant to the study of the dissipation range of solar wind turbulence are identified and are discussed at length in a forthcoming review (Howes, 2015):

1. What are the limits of validity of using a fluid description of the turbulence in the dissipation range, and which aspects of the turbulence require a kinetic description?
2. Are the linear plasma wave properties relevant to the turbulent fluctuations of the dissipation range?
3. What are the characteristic dynamics of the dissipation range fluctuations?
4. What physical mechanisms are responsible for the dissipation of the turbulent fluctuations and the ultimate conversion of their energy to plasma heat?

Although these significant questions about the nature of kinetic turbulence remain controversial, a promising model of the kinetic turbulent cascade (Howes, 2008; Howes et al., 2008a; Schekochihin et al., 2009; Howes et al., 2011) has been developed that appears to be broadly consistent with most observations of solar wind turbulence. This model involves an anisotropic cascade of Alfvénic fluctuations beginning as a cascade of Alfvén waves in the MHD inertial range and transitioning to a cascade of kinetic Alfvén waves subject to collisionless damping in the kinetic dissipation range. Yet the cascade of energy from large to small scales described by this kinetic turbulence model may not explain all of the fluctuations observed in the solar wind. For example, fluctuations can be generated by the action of kinetic temperature anisotropy instabilities (Bale et al., 2009) that are driven by the spherical expansion of the solar wind, an effect beyond the scope of this model. Plausible arguments exist that suggest some of these additional effects may coexist peacefully with the kinetic turbulence, proceeding without being significantly affected by or significantly affecting the kinetic turbulent cascade. The remainder of this chapter aims to describe in detail the model of the kinetic turbulent cascade and to discuss the supporting and conflicting theoretical, observational, and numerical evidence.

3 A Model of the Kinetic Turbulent Cascade

A basic theoretical model of the kinetic turbulent cascade in astrophysical plasmas has been developed with the aim to describe completely the flow of energy from the large driving scales of the turbulence to its ultimate fate as thermal heat of the plasma (Howes, 2008; Howes et al., 2008a; Schekochihin et al., 2009; Howes et al., 2011). We present here a brief outline of this model, before delving into a detailed description of each component of the model and a discussion of supporting and conflicting evidence.

Violent events or instabilities first drive turbulent fluctuations of the magnetic field and plasma at some large scale, generating a mixture of finite amplitude fast, Alfvén, and slow waves. If the fluctuations are driven isotropically with velocities approximately equal to the Alfvén velocity in the plasma, a cascade of strong compressible MHD turbulence will mediate the transfer of the turbulent kinetic and magnetic energy to smaller scales. The fast waves cascade to smaller scales isotropically, while the critically balanced Alfvén wave cascade produces an anisotropic distribution of Alfvén and slow wave fluctuations in this collisional part of the MHD inertial range. The parallel scales of the turbulent fluctuations eventually reach the ion collisional mean free path, marking the transition from strongly to weakly collisional dynamics. The compressible fast and slow wave fluctuations suffer collisional damping at the moderately collisional scale of the transition, and collisionless damping at the smaller, weakly collisional scales. The incompressible Alfvénic fluctuations remain undamped through this transition, so the damped fast and slow waves are expected to contribute subdominantly to the turbulence compared to the Alfvén waves. The Alfvén waves continue their cascade undamped through the collisionless remainder of the MHD inertial range until their perpendicular scales reach the ion Larmor radius, marking the transition to the kinetic dissipation range.

The anisotropic Alfvénic fluctuations at this transition transfer energy into a cascade of kinetic Alfvén waves at perpendicular scales below the ion Larmor radius. In addition, collisionless wave-particle interactions via the Landau resonance with the ions lead to a peak in the ion kinetic damping at the ion Larmor radius, dissipating some fraction of the turbulent electromagnetic fluctuation energy. The undamped remainder of the turbulent energy continues as a cascade of kinetic Alfvén waves to smaller perpendicular scales, forming the kinetic dissipation range at all scales below the ion Larmor radius. Throughout this range, electron Landau damping may cause significant collisionless damping of the turbulent fluctuations, with the strength of the damping increasing as the perpendicular scale decreases. At the perpendicular scale of the electron Larmor radius, the electron Landau damping becomes sufficiently strong to terminate the cascade, leading to an exponential decay of the turbulent energy spectrum at the electron scale.

Thermodynamically, the transfer of free energy from the kinetic and magnetic energy of the turbulent electromagnetic fluctuations to free energy in velocity space structure of the particle distribution functions is not equivalent to irreversible thermal heating of the plasma. Irreversible plasma heating, and the associated increase of entropy, ultimately requires collisions. This is accomplished in a weakly collisional plasma by the ion and electron entropy cascades, dual cascades in physical and velocity space that drive fluctuations to small enough velocity-space scales that arbitrarily weak collisions are sufficient to achieve irreversibility. This final process marks the thermodynamic end of the kinetic turbulent cascade, completing the conversion of large-scale turbulent fluctuation energy to thermal heat of the plasma.

This model of the kinetic turbulent cascade implies certain answers to the questions posed in §2, so we elucidate those answers here:

1. A fluid description is applicable for all turbulent fluctuations at scales larger than the collisional transition, and for the Alfvénic dynamics at all scales larger than the ion Larmor radius. The dynamics and kinetic damping of the compressible fluctuations at all moderately to weakly collisional scales, and of the Alfvénic fluctuations at the scales of the ion Larmor radius and below, require a kinetic description.
2. The properties of the turbulent fluctuations at scales sufficiently below the driving scale are related to the characteristics of the linear kinetic plasma waves.
3. The dissipation range fluctuations are kinetic Alfvén waves.

4. Ion and electron Landau damping are the physical mechanisms by which the turbulent electromagnetic fluctuations are damped, and the ion and electron entropy cascades mediate the irreversible transition of free energy in the particle distribution functions to thermal heat.

In the following sections, we describe in detail all of the facets of this model of the kinetic turbulent cascade, providing supporting theoretical, observational, and numerical evidence and reviewing findings in conflict with this model. A general diagram of the magnetic energy spectrum and the distribution of turbulent power in wavevector space is shown in Figure 3.

3.1 MHD Inertial Range: From Driving Scales to the Collisional Transition

The turbulence in astrophysical environments is typically driven by some external mechanism, often a violent event or large-scale instability, that generates plasma motions at some large scale, $L \gg \rho_i$. This energy injection scale, often denoted the outer scale of the turbulence, is an important characteristic of any turbulent astrophysical system, and is conveniently parameterized by the wavenumber, $k_0 \sim 1/L$. For the investigation of kinetic turbulence, a convenient dimensionless measure of the driving scale is the *driving wavenumber*, $k_0\rho_i$, where $k_0\rho_i \ll 1$ indicates that the turbulence is driven at large scale compared to the ion Larmor radius. It is generally assumed, in the absence of arguments to the contrary, that the turbulence is driven isotropically with respect to the magnetic field, so that the perpendicular and parallel components of the driving wavevector are equal, $k_{\parallel 0} \sim k_{\perp 0} \sim k_0$.

If the MHD approximation is satisfied for the turbulent dynamics of the plasma at the driving scale, then the large scale section of the MHD inertial range is described by MHD turbulence theory (Sridhar and Goldreich, 1994; Goldreich and Sridhar, 1995; Galtier et al., 2000; Lithwick and Goldreich, 2001; Boldyrev, 2006). If the amplitude of the driven turbulent velocities are comparable to the Alfvén velocity in the magnetized plasma, then a cascade of strong MHD turbulence arises to transfer energy nonlinearly to higher wavenumbers; for smaller amplitudes, weak MHD turbulence will be generated (Sridhar and Goldreich, 1994; Goldreich and Sridhar, 1995). Since most turbulent astrophysical environments are believed to be driven strongly, and weak turbulence eventually transitions to strong turbulence as the cascade progresses (Sridhar and Goldreich, 1994), we focus here on the case of strong MHD turbulence. In general, the finite-amplitude turbulent fluctuations may be considered to be a mixture of the three propagating MHD wave modes, the incompressible Alfvén waves and compressible fast and slow waves, as well as the non-propagating entropy mode. The nature of the turbulent cascades of these various characteristic fluctuations have been elucidated by numerical simulations of MHD turbulence: the fast waves cascade isotropically in wavevector space, while the Alfvén waves, slow waves and entropy mode fluctuations cascade anisotropically according to the condition of critical balance (Maron and Goldreich, 2001; Cho and Lazarian, 2003).

The fast wave cascade produces an isotropic one-dimensional magnetic energy spectrum $E_B(k) \propto k^{-3/2}$, as observed in simulations (Cho and Lazarian, 2003). Two competing models exist that describe the nature of strong MHD turbulence for Alfvén waves, the Goldreich-Sridhar model (Goldreich and Sridhar, 1995) and the Boldyrev model (Boldyrev, 2006). The magnetic energy spectrum of the Alfvénic turbulent cascade is predicted to scale as $E_B \propto k_\perp^p$, where the spectral index is $p = -5/3$ in the Goldreich-Sridhar model (Goldreich and Sridhar, 1995) and $p = -3/2$ in the Boldyrev model (Boldyrev, 2006). The anisotropy of the Alfvénic cascade, for isotropic driving at wavenumber k_0, is given by $k_\parallel = k_0^{1-q} k_\perp^q$, where $q = 2/3$ in the Goldreich-Sridhar model (Goldreich and Sridhar, 1995), and $q = 1/2$ in the Boldyrev model (Boldyrev, 2006). The slow waves and entropy modes are passively cascaded by the Alfvén waves, and therefore adopt the same spectrum and anisotropic distribution of power as the Alfvén waves (Maron and Goldreich, 2001; Lithwick and Goldreich, 2001). For anisotropic turbulent fluctuations with $k_\perp \gg k_\parallel$, the frequencies of the fast wave fluctuations, which scale as $\omega \propto k$, are generally much higher than the frequencies of the Alfvén and slow wave fluctuations, which scale as $\omega \propto k_\parallel$, so the dynamics of the fast wave cascade are expected to decouple from the dynamics of the Alfvén and slow wave cascades (Lithwick and Goldreich, 2001; Howes et al., 2012).

The turbulent cascade transfers energy nonlinearly to higher wavenumber fluctuations, as dictated by the MHD turbulence theory, until the parallel wavenumber reaches the transition from collisional to collisionless dynamics, $k_\parallel \lambda_i \sim 1$. The perpendicular wavenumber, $k_{\perp c}$, that corresponds to $k_\parallel \lambda_i \sim 1$, differs for the anisotropic Alfvén wave cascade and the isotropic fast wave cascade. For the anisotropic Alfvénic cascade, the perpendicular wavenumber of this transition is given by $k_{\perp c} \sim k_0 (k_0 \lambda_i)^{-1/q}$, whereas, for the isotropic fast wave cascade, it is given by $k_{\perp c} \sim k_0 (k_0 \lambda_i)^{-1}$, or more simply $k_{\perp c} \lambda_i \sim 1$. Since $q < 1$, this means that the fast wave cascade reaches the collisional transition first, at a smaller wavenumber than the Alfvén wave cascade.

For many astrophysical plasmas, the transition for both fast and Alfvén waves occurs within the MHD inertial range, $k_{\perp c} \rho_i < 1$. The compressible fast waves, slow waves, and entropy modes undergo strong collisional damping by ion viscosity (Braginskii, 1965) in the moderately collisional conditions at $k_\perp \sim k_{\perp c}$. Any energy in the compressible turbulent fluctuations that passes through this transition is expected to be transferred nonlinearly to the kinetic counterparts of the MHD fast and slow waves (Klein et al., 2012) in the weakly collisional conditions at wavenumbers $k_\perp \gg k_{\perp c}$ (Schekochihin et al., 2009). The Alfvén waves are incompressible, involving no motions parallel to the magnetic field, so they are essentially unaffected by the transition in collisionality, and the Alfvén wave cascade continues unabated to higher wavenumbers, $k_\perp \gg k_{\perp c}$.

3.2 MHD Inertial Range: From the Collisional Transition to the Ion Larmor Radius

Critical balance predicts a scale-dependent wavevector anisotropy given by $k_\perp/k_\parallel = (k_\perp/k_0)^{1-q}$, where $q < 1$ for either the Goldreich-Sridhar or Boldyrev models. Therefore, at perpendicular wavenumbers within the MHD inertial range sufficiently higher than the driving wavenumber, $k_\perp \gg k_0$, the Alfvénic fluctuations become anisotropic in the sense that $k_\perp \gg k_\parallel$. In the limit of the MHD inertial range $k_\perp \rho_i \ll 1$, the kinetic dynamics of these anisotropic Alfvénic fluctuations is described rigorously by the equations of reduced MHD (Strauss, 1976), and the Alfvén wave cascade remains undamped down to the perpendicular scale of the ion Larmor radius, $k_\perp \rho_i \sim 1$ (Schekochihin et al., 2009). It has also been shown that the slow wave and Alfvén wave cascades do not exchange energy in the MHD inertial range (Schekochihin et al., 2009), and the fast waves likewise are not expected to exchange energy with the Alfvén waves due to the mismatch in frequency, as discussed in §3.1. Therefore, the dynamics of the Alfvénic cascade throughout the MHD inertial range is correctly described by the MHD turbulence theory, even at the weakly collisional scales, $k_\perp \gg k_{\perp c}$.

The magnetic energy spectrum in the solar wind seems to bear this out. Spacecraft measurements in the super-Alfvénic solar wind are generally interpreted by assuming the Taylor hypothesis (Taylor, 1938), that frequency of measured temporal fluctuations is directly related to the wavenumber of spatial variations that are swept past the spacecraft. At the frequencies $f \lesssim 0.4$ Hz, corresponding to spatial scales larger than the ion Larmor radius, the magnetic energy spectrum in the solar wind has a spectral index of approximately $-5/3$ (Goldstein et al., 1995), apparently consistent with the prediction of the Goldreich-Sridhar theory for strong MHD turbulence. It is worth noting, however, that the velocity spectrum was found have a spectral index closer to $-3/2$ (Podesta et al., 2007), in conflict with the Goldreich-Sridhar model. Recent work on the evolution of the residual energy, $E_r = E_k - E_B$, in MHD turbulence, however, suggests that these spectral indices may indeed be consistent with the Boldyrev theory, and that the difference in the spectral indices of the kinetic and magnetic energy spectra is an inherent property of the MHD turbulent cascade (Boldyrev et al., 2011, 2012).

The cascade of compressible turbulent fluctuations that passes through the collisional transition will suffer moderate to strong collisionless damping by the Landau resonance with the ions (Barnes, 1966) at all higher wavenumbers, $k_\perp \gg k_{\perp c}$. The damping of the compressible fluctuations in the moderate to weakly collisional regimes at $k_\perp \gtrsim k_{\perp c}$ leads to the theoretical prediction that compressible fluctuations will play a subdominant role relative to the undamped Alfvénic fluctuations in turbulent astrophysical plasmas. Studies of interstellar scintillation (Armstrong et al., 1981, 1995) show evidence for a power-law spectrum of density fluctuations over 12 orders of magnitude in the interstellar medium, suggesting that compressible fluctuations are not entirely damped. But it is not possible from remote astrophysical observations to deduce the relative contributions of compressible and incompressible components of the turbulence.

In situ spacecraft measurements of turbulent fluctuations in the solar wind, however, allow a direct determination. The entire turbulent cascade in the solar wind, including the driving scales, is weakly collisional, $\lambda_i/L \gg 1$, so spacecraft measurements constrain role of compressible fluctuations in collisionless conditions, $k_\perp \gg k_{\perp c}$. Measurements show that the turbulent fluctuations in the MHD inertial range appear to be dominantly incompressible (Tu and Marsch, 1995; Bruno and Carbone, 2005), where the incompressible motions have been shown to be Alfvénic in nature (Belcher and Davis, 1971). The compressible fluctuations generally contribute less than 10% of the turbulent magnetic energy (Tu and Marsch, 1995; Bruno and Carbone, 2005). These compressible fluctuations have typically been interpreted as a possible mixture of fast MHD waves and pressure balanced structures (PBSs) (Tu and Marsch, 1995; Bruno and Carbone, 2005), where the latter are equivalent to non-propagating slow mode fluctuations with $k_\parallel = 0$ (Tu and Marsch, 1994; Kellogg and Horbury, 2005). Note, however, that a recent study using a novel method of synthetic spacecraft data (Klein et al., 2012) suggests that these compressible fluctuations are not associated with the kinetic counterpart of the fast MHD wave, but rather consist of an anisotropic distribution of kinetic slow wave fluctuations (Howes et al., 2012). Clearly, more investigation of the kinetic physics of compressible turbulent fluctuations in astrophysical environments, including their damping via collisional and collisionless mechanisms and the resulting plasma heating, is needed. Nonetheless, since only a small fraction of the turbulent energy appears to be associated with the compressible fluctuations, we focus our attention henceforth on the dominant Alfvénic turbulent fluctuations, as they reach the perpendicular scale of the ion Larmor radius, $k_\perp \rho_i \sim 1$.

3.3 Transition at the Ion Larmor Radius

Spacecraft measurements of turbulence in the solar wind demonstrate that the $-5/3$ scaling of the magnetic energy spectrum in the MHD inertial range breaks at a frequency around $f \sim 0.4$ Hz, leading to a steeper spectrum at higher frequencies in the kinetic dissipation range. Numerous observational studies have attempted to correlate the position of the break with a characteristic plasma time or length scale, such as the ion cyclotron frequency, the ion Larmor radius, or the ion inertial length (Goldstein et al., 1994; Leamon et al., 1998, 1999; Leamon et al., 2000; Smith et al., 2001; Perri et al., 2010; Smith et al., 2012; Bourouaine et al., 2012), but contradictory results have been found. Establishing a convincing correlation has likely been elusive because three competing effects may contribute to the dynamics at this transition between the MHD inertial range and kinetic dissipation range: (1) the transition from non-dispersive to dispersive linear wave physics as the ions decouple from the turbulent electromagnetic fluctuations; (2) a peak in the ion kinetic damping; and (3) the possible role of kinetic instabilities, such as temperature anisotropy instabilities (Bale et al., 2009), in generating electromagnetic fluctuations at this scale.

Based on theoretical considerations of the kinetic plasma physics, the kinetic turbulence model presented here predicts that the transition between the relatively well understood MHD inertial range and the significantly more controversial kinetic dissipation range occurs at the perpendicular scale of the ion Larmor radius, $k_\perp \rho_i \sim 1$ (Howes, 2008; Howes et al., 2008a; Schekochihin et al., 2009; Howes et al., 2011). The boundary conditions (in wavevector space) for the nonlinear transfer of energy into the kinetic dissipation range are given by the nature of the turbulent fluctuations at the end of the MHD inertial range. At this transition at $k_\perp \rho_i \sim 1$, the wavevector anisotropy of Alfvénic turbulent fluctuations is given by $k_\perp / k_\parallel \sim (k_0 \rho_i)^{q-1}$, so for a sufficiently large MHD inertial range, $k_0 \rho_i \ll 1$, this implies $k_\perp \gg k_\parallel$ since $q < 1$ (Goldreich and Sridhar, 1995; Boldyrev, 2006). This significant wavevector anisotropy at the transition is supported by multi-spacecraft measurements of turbulence in the near-earth Solar wind (Sahraoui et al., 2010). It follows that, beyond this transition, the characteristic wavevector of the fluctuations satisfies $k_\perp \rho_i \gtrsim 1$ and $k_\parallel \rho_i \ll 1$; the Alfvénic solution of linear kinetic theory with such a wavevector is the kinetic Alfvén wave (Hasegawa and Sato, 1989; Stix, 1992). Therefore, the Alfvén waves of the MHD inertial range are predicted to transfer their energy, via nonlinear interactions at the transition $k_\perp \rho_i \sim 1$, to kinetic Alfvén waves (Leamon et al., 1998; Gruzinov, 1998; Leamon et al., 1999; Quataert and Gruzinov, 1999; Howes et al., 2008a; Schekochihin et al., 2009). Nonlinear gyrokinetic simulations of this transition appear to support this hypothesis (Howes et al., 2008b), reproducing the qualitative changes in the electric and magnetic field energy spectra measured in the solar wind at the scale of the spectral break (Bale et al., 2005).

Another important effect that occurs at the transition at $k_\perp \rho_i \sim 1$ is a peak in the collisionless damping rate of the electromagnetic fluctuations due to the Landau resonance with the ions (Leamon et al., 1998, 1999; Leamon et al., 2000; Howes et al., 2008a; Howes, 2008; Schekochihin et al., 2009; Howes et al., 2011). This ion kinetic damping becomes increasingly strong as the ion plasma beta increases, and is generally non-negligible for plasmas with beta of order unity or larger, $\beta_i \gtrsim 1$, leading to a significant fraction of the dissipated turbulent energy heating the ions (Howes, 2010, 2011), in approximate agreement with empirical estimates of the plasma heating in the solar wind (Cranmer et al., 2009; Breech et al., 2009). Some measurements of the magnetic energy spectrum in the dissipation range of the solar wind show a significant steepening to a slope of approximately -4 at the ion scales, flattening to -2.8 spectrum further into the dissipation range (Sahraoui et al., 2010), evidence suggesting significant ion kinetic damping.

In a steady-state kinetic turbulent cascade, the turbulent energy reaching the transition at $k_\perp \rho_i \sim 1$ that is not damped at that scale will carry on, launching a turbulent cascade of kinetic Alfvén waves in the kinetic dissipation range at $k_\perp \rho_i \gtrsim 1$. Although the Alfvén and slow wave cascades do not exchange energy in the MHD inertial range, they may exchange energy at this transition (Schekochihin et al., 2009), so it is possible that the kinetic Alfvén wave cascade can gain energy that is transferred nonlinearly from compressible fluctuations in the MHD inertial range.

3.4 Kinetic Dissipation Range: Between the Ion and Electron Larmor Radius

Although direct spacecraft measurements of the kinetic dissipation range of turbulence in the near-Earth solar wind have been possible for more than a decade, the nature of the turbulent fluctuations in this regime remains a controversial topic. Characterizing these fluctuations is one of the key goals in heliospheric physics today, especially because the relevant physical dissipation mechanisms that ultimately lead to heating of the plasma depend strongly on the nature of the turbulent fluctuations themselves.

Many early investigations of the dissipation range in solar wind turbulence implicitly assumed that the turbulent fluctuations in the dissipation range are related to the linear wave modes in the plasma. Two main hypotheses have been proposed, that the turbulence is composed of either kinetic Alfvén waves (Leamon et al., 1998; Gruzinov, 1998; Leamon et al., 1999; Quataert and Gruzinov, 1999; Howes et al., 2008a; Schekochihin et al., 2009) or whistler waves (Stawicki et al., 2001; Gary et al., 2010; Narita and Gary, 2010). Although these two possibilities generally remain the leading candidates, several other possibilities have been suggested: ion Bernstein waves (Sahraoui et al., 2012), ion cyclotron waves (Jian et al., 2009), non-propagating pressure balanced structures (PBSs), or inherently nonlinear structures, particularly highly intermittent coherent structures and current sheets (Servidio et al., 2011).

Direct spacecraft measurements of turbulence in the solar wind at the frequencies $f \gtrsim 1$ Hz, corresponding to the kinetic dissipation range, provide important constraints on the nature of the turbulent fluctuations. A number of recent studies employing high temporal resolution spacecraft measurements have found a nearly power-law scaling of the magnetic energy spectrum between the ion and electron scales with a spectral index of approximately -2.8 (Sahraoui et al., 2009; Kiyani et al., 2009; Alexandrova et al., 2009; Chen et al., 2010; Sahraoui et al., 2010). These observations of the turbulence over the dissipation range scales raise two important questions that any model for kinetic turbulence must answer: (1) What causes the magnetic energy spectrum to steepen in the dissipation range; and (2) Does significant dissipation of the turbulent fluctuations occur between the ion and electron scales?

In the model of the kinetic turbulent cascade, the boundary conditions in wavevector space determined by the anisotropic Alfvénic cascade through the MHD inertial range suggest that the turbulent energy is transferred nonlinearly to a cascade of kinetic Alfvén waves in the kinetic dissipation range, as discussed in §3.3. Here we describe the properties of the kinetic Alfvén wave cascade at perpendicular scales below the ion Larmor radius, $k_\perp \rho_i \gtrsim 1$.

Although MHD Alfvén waves are non-dispersive, kinetic Alfvén waves become dispersive due to the averaging of the ion response over the finite ion Larmor radius, a physical effect that increasingly decouples the ions from the electromagnetic fluctuations with wavevectors satisfying $k_\perp \rho_i \gtrsim 1$ (Hollweg, 1999; Schekochihin et al., 2009). A useful formula combining the linear frequency in the Alfvén and kinetic Alfvén wave regimes Howes et al. (2006); Schekochihin et al. (2009) is given by

$$\omega = k_\parallel v_A \sqrt{1+\frac{(k_\perp \rho_i)^2}{\beta_i + 2/(1+T_e/T_i)}} \tag{1}$$

In addition, the kinetic Alfvén wave is significantly compressible, generating a non-zero parallel magnetic field fluctuation, $\delta B_\parallel$, particularly in the limit of low to moderate plasma beta, $\beta_i \lesssim 1$ (Hollweg, 1999; TenBarge et al., 2012).

The model for kinetic turbulence predicts the quantitative scaling of the magnetic energy spectrum and the wavevector anisotropy for the kinetic Alfvén wave cascade. The Kolmogorov hypothesis—that the energy transfer rate is constant due to local (in wavenumber space) nonlinear interactions—can be used to predict the magnetic energy spectrum for the kinetic Alfvén wave cascade in the absence of significant dissipation. For $k_\perp \rho_i \gg 1$, the linear wave frequency increases due to dispersion, yielding a scaling $\omega \propto k_\parallel k_\perp$. This leads to more rapid nonlinear energy transfer, steepening the magnetic energy spectrum to a predicted scaling $E_B \propto k_\perp^{-7/3}$ when dissipation is neglected (Biskmap et al., 1999; Cho and Lazarian, 2004; Krishan and Mahajan, 2004; Shaikh and Zank, 2005; Galtier, 2006; Howes et al., 2008a; Schekochihin et al., 2009). Extending the concept of critical balance—that the linear wave frequency and nonlinear energy transfer frequency remain in balance—to the kinetic Alfvén wave regime leads to a predicted wavevector anisotropy given by $k_\parallel \propto k_\perp^{1/3}$ (Cho and Lazarian, 2004; Howes et al., 2008a; Schekochihin et al., 2009).

In addition to the effects of wave dispersion, collisionless damping via wave-particle interactions can also play an important role in kinetic turbulence for all scales $k_\perp \rho_i \gtrsim 1$. In addition to the peak in ion Landau damping at $k_\perp \rho_i \sim 1$ discussed in §3.3, electron Landau damping may also play a significant role for all scales $k_\perp \rho_i \gtrsim 1$, becoming increasingly strong as the perpendicular wavenumber increases (Howes et al., 2008a). Although early models of the turbulent energy cascade in the kinetic dissipation range suggested that such strong collisionless Landau damping would lead to an exponential cutoff of the spectrum before reaching the perpendicular scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$ (Howes et al., 2008a; Podesta et al., 2010), subsequent solar wind observations called this prediction into question (Sahraoui et al., 2009; Kiyani et al., 2009; Alexandrova et al., 2009; Chen et al., 2010; Sahraoui et al., 2010) and recent kinetic numerical simulations have demonstrated that this idea is incorrect (Howes et al., 2011).

In addition to collisionless damping via the Landau resonance, if the kinetic Alfvén wave frequency reaches the ion cyclotron frequency, $\omega \to \Omega_i$, collisionless damping may occur via the cyclotron resonance with the ions. However, the very large MHD inertial range typical of astrophysical plasma turbulence leads to highly anisotropic fluctuations at small scales, $k_\parallel \ll k_\perp$, so the kinetic Alfvén wave frequency typically remains very small compared to the ion cyclotron frequency, $\omega \ll \Omega_i$. Therefore, ion cyclotron damping is not predicted to play a strong role in the dissipation of astrophysical turbulence (Howes et al., 2008a), with a few exceptions, such as the inner heliosphere (Howes, 2011).

There is significant evidence accumulating in support of a kinetic Alfvén wave cascade at the perpendicular scales between the electron and ion Larmor radius, but there also remains observational evidence that appears to be unexplained by this model. The scaling predictions for the kinetic Alfvén wave cascade in the absence of dissipation have been corroborated by simulations using electron MHD, a fluid limit which describes the dynamics of kinetic Alfvén waves in the limit $k_\parallel \ll k_\perp$, but does not resolve the physics of collisionless dissipation. Specifically, these simulations reproduce the predicted magnetic energy scaling, $E_B \propto k_\perp^{-7/3}$ (Biskmap et al., 1999; Cho and Lazarian, 2004, 2009; Shaikh and Zank, 2009), and wavevector anisotropy, $k_\parallel \propto k_\perp^{1/3}$ (Cho and Lazarian, 2004, 2009). The magnetic energy spectrum from these fluid simulations, however, is not consistent with the observed spectral index of approximately -2.8 (Sahraoui et al., 2009; Kiyani et al., 2009; Alexandrova et al., 2009; Chen et al., 2010; Sahraoui et al., 2010).

It has been recently suggested that the combined effects of collisionless dissipation and nonlocal energy transfer can lead to a further steepening of the magnetic energy spectrum beyond $k_\perp^{-7/3}$ for scales $k_\perp \rho_i \gtrsim 1$ (Howes et al., 2011). For a hydrogenic plasma of protons and electrons, the dynamic range between the ion and electron Larmor radius for unity temperature ratio is $\rho_i/\rho_e \simeq 43$, so there is little room for an asymptotic range of perpendicular scales satisfying the requirements $1/\rho_i \ll k_\perp \ll 1/\rho_e$. Therefore, it should come as no surprise that a self-similar spectrum with a spectral index of $-7/3$ is not observed—throughout the range of perpendicular scales between the ion and electron Larmor radius, the transition at $k_\perp \rho_i \sim 1$ and strong kinetic dissipation at $k_\perp \rho_e \sim 1$ may significantly affect the turbulent dynamics.

Nonlinear gyrokinetic simulations of turbulence in the kinetic dissipation range seem to bear this out. A simulation over the entire range of scales from the ion to the electron Larmor radius, which resolves the physics of collisionless ion and electron damping, produces a nearly power-law magnetic energy spectrum with a spectral index of -2.8, in remarkable quantitative agreement with the solar wind measurements (Howes et al., 2011). Additional gyrokinetic simulations support the predicted scaling of the wavevector anisotropy, $k_\parallel \propto k_\perp^{1/3}$ (TenBarge and Howes, 2012; TenBarge et al., 2013).

Direct spacecraft measurements of dissipation range turbulence in the solar wind have yielded other lines of evidence in support of or in conflict with the model of a kinetic Alfvén wave cascade. A *k*-filtering analysis of multi-spacecraft data from the Cluster mission establishes that the plasma-frame fluctuation frequencies are consistent with linear dispersion relation of the kinetic Alfvén wave and inconsistent with that of the whistler wave (Sahraoui et al., 2010). A study combining measurements of the ratio of electric to magnetic field fluctuation amplitudes and of the magnetic compressibility have shown that the small-scale fluctuations agree well with predictions for kinetic Alfvén waves and are inconsistent with that for whistler waves (Salem et al., 2012). An examination the compressibility of turbulent fluctuations in the weakly collisional plasma in the MHD inertial range finds evidence of negligible energy in the fast wave mode, suggesting that all large-scale turbulent energy is transferred, via the anisotropic Alfvénic cascade, to kinetic Alfvén waves, with little energy coupling to whistler waves (Howes et al., 2012; Klein et al., 2012). Investigations of the magnetic helicity of turbulent fluctuations as a function of the angle of the wavevector with respect to the local magnetic field direction finds a broad region of positive helicity at oblique angles (He et al., 2011; Podesta and Gary, 2011), as expected for kinetic Alfvén waves (Howes and Quataert, 2010), but a small region corresponding to nearly parallel wavevectors that is consistent with either ion cyclotron waves or whistler waves (He et al., 2011; Podesta and Gary, 2011).

The presence of either ion cyclotron or whistler waves with nearly parallel wavevectors is not explained by the model for kinetic Alfvén wave turbulence, but these fluctuations may be driven by the action of kinetic temperature anisotropy instabilities in the spherically expanding solar wind (Bale et al., 2009). These instabilities typically generate relatively isotropic fluctuations (with respect to the local mean magnetic field direction), having wavevector components $k_\perp \rho_i \sim k_\parallel \rho_i \sim 1$. Since the anisotropic Alfvénic cascade produces fluctuations with $k_\parallel \ll k_\perp$, and since Alfvénic frequencies in equation (1) scale linearly with the parallel component, $\omega \propto k_\parallel$, these anisotropy-driven fluctuations are expected to have a much higher frequency and to occupy a different regime of wavevector space than turbulent fluctuations of the turbulent cascade. Therefore, it is reasonable to expect that any kinetic-instability-driven fluctuations may persist without significantly affecting, or being significantly affected by, the turbulent fluctuations of the anisotropic Alfvénic cascade.

In conclusion, although the nature of the kinetic turbulence at the perpendicular scales between the electron and ion Larmor radius has not been established conclusively, there appears to be significant evidence for an anisotropic cascade of kinetic Alfvén waves in the kinetic dissipation range. Collisionless dissipation via the Landau resonance with the electrons appears to be play a non-negligible role in steepening the magnetic energy spectrum beyond the dissipationless prediction. But this damping is not strong enough to halt the cascade, so the kinetic turbulence continues down to the perpendicular scale of the electron Larmor radius, at which point strong kinetic dissipation can effectively terminate the turbulent cascade.

3.5 *Kinetic Dissipation Range: Termination at Electron Larmor Radius*

Ultimately, the kinetic turbulent cascade reaches the perpendicular scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$. At this scale, the linear collisionless damping rate due to electron Landau damping reaches a value $\gamma/\omega \sim 1$, sufficiently strong that the turbulent magnetic energy cascade is terminated. A simplified analytical treatment of the turbulent cascade undergoing this dissipation suggests the spectrum will develop an exponential fall-off $E_B \propto k_\perp^{-2.8} \exp(-k_\perp \rho_e)$ setting in at the perpendicular scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$ (Terry and et al., 2012). As the amplitudes of the turbulent fluctuations are diminished by damping, the strong kinetic Alfvén wave turbulence eventually drops below critical balance and becomes weak dissipating kinetic Alfvén wave turbulence (Howes et al., 2011). It has been conjectured that the transition back to weak turbulence leads to an inhibition of the parallel cascade, so the parallel number of the fluctuations remains constant (Howes et al., 2011), as shown in in Figure 2(b).

A recent study of a sample of 100 solar wind magnetic energy spectra at the electron scales shows that all of these spectra may be fit by an empirical form $E_B \propto k_\perp^\alpha \exp(-k_\perp \rho_e)$, where $-2.5 \geq \alpha \geq -2.8$ (Alexandrova et al., 2012). A nonlinear gyrokinetic simulation of the turbulence over the range $0.12 \leq k_\perp \rho_e \leq 2.5$ yields an energy spectrum demonstrating an exponential fall-off that is quantitatively fit by $E_B \propto k_\perp^{-2.8} \exp(-k_\perp \rho_e)$, further supporting the model of the kinetic turbulent cascade (TenBarge et al., 2013). A refined model of the turbulent cascade (Howes et al., 2011)—incorporating the weakening of the nonlinear turbulent energy transfer due to dissipation, the effect of nonlocal energy transfer, and the linear collisionless damping via the Landau resonance—fits the shape of the spectrum well. This provides compelling evidence that collisionless damping is the dominant mechanism for the dissipation of the kinetic turbulent cascade, marking the end of the kinetic dissipation range (TenBarge et al., 2013).

A number of recent works have suggested instead that dissipation in current sheets is the dominant dissipation mechanism for plasma turbulence, based on fluid simulations using MHD (Dmitruk et al., 2004; Servidio et al., 2009, 2010, 2011) and Hall MHD (Dmitruk and Matthaeus, 2006), hybrid simulations with kinetic ions and fluid electrons (Parashar et al., 2009, 2010; Markovskii and Vasquez, 2011; Servidio et al., 2012), and observational studies of large-scale discontinuities in the solar wind (Osman et al., 2011, 2012). However, all the numerical work upon which this conclusion has been based employ a fluid description of the electrons which does not resolve the dominant collisionless dissipation mechanism of Landau damping. In a collisionless plasma, current sheets supporting small-scale reconnection with a guiding magnetic field are expected to form at the perpendicular scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$ (Birn and Priest, 2006). A recent gyrokinetic simulation over the range of electron scales $0.12 \leq k_\perp \rho_e \leq 2.5$, which resolves both collisionless electron Landau damping and the formation of current sheets at $k_\perp \rho_e \sim 1$, finds dissipation via current sheets to be sub-dominant compared to linear collisionless damping (TenBarge and Howes, 2013).

These results establish fairly secure observational and numerical grounds that the electromagnetic fluctuations of the kinetic turbulent cascade are dissipated at the perpendicular scale of the electron Larmor radius, $k_\perp \rho_e \sim 1$. Further work is ongoing to identify the dominant physical mechanisms for the dissipation of these turbulent electromagnetic fluctuations. Although this dissipation terminates the kinetic dissipation range of electromagnetic fluctuations at electron scales, there remains the final matter of identifying the physical mechanism mediating the conversion of this turbulent fluctuation energy irreversibly to thermodynamic heat in a weakly collisional astrophysical plasma.

3.6 Irreversible Heating Via the Ion and Electron Entropy Cascades

At the perpendicular scales of the ion and electron Larmor radius, collisionless wave-particle interactions via the Landau resonance damp the turbulent electromagnetic fluctuations. In the absence of collisions, this process conserves a generalized energy, generating nonthermal structure in velocity space of the corresponding plasma particle distribution functions (Howes, 2008; Schekochihin et al., 2009; Plunk et al., 2010). Boltzmann's H-theorem dictates that, in a kinetic plasma, collisions are required to increase the entropy and therefore achieve irreversible thermodynamic heating (Howes et al., 2006). In the weakly collisional plasmas of astrophysical environments, an *entropy cascade*—a nonlinear phase mixing process (Dorland and Hammett, 1993) that drives a dual cascade in physical and velocity space—mediates the transfer, at sub-Larmor radius scales, of the nonthermal free energy in the particle distribution functions to sufficiently small scales in velocity space that arbitrarily weak collisions can manifest irreversibility, increasing the entropy and thermodynamically heating the plasma (Schekochihin et al., 2009; Plunk et al., 2010). This inherently kinetic physical mechanism represents the final element of the kinetic turbulent cascade, governing the final transition of the turbulent energy to its ultimate fate as plasma heat.

The ion entropy cascade in two-dimensional plasma systems (in the plane perpendicular to the local mean magnetic field) has been thoroughly examined theoretically (Schekochihin et al., 2009; Plunk et al., 2010) and verified in gyrokinetic numerical simulations (Tatsuno et al., 2009; Plunk and Tatsuno, 2011). In the inherently three-dimensional system of Alfvénic plasma turbulence (Howes et al., 2011), the effects of the ion entropy cascade in generating structure in the perpendicular component of velocity space (Howes, 2008) and in manifesting ion heating at physical scales well below the peak in the collisionless ion damping (Howes et al., 2011) have been identified numerically. Yet, a thorough analysis of the ion and electron entropy cascades in kinetic turbulence remains to be undertaken.

4 Conclusion

Turbulence is found ubiquitously throughout the universe, playing a governing role in the conversion of the energy of large-scale motions to astrophysical plasma heat. Extending our understanding of astrophysical turbulence from the limited theory of MHD turbulence to the more comprehensive theory of kinetic turbulence opens up the possibility of ultimately achieving a predictive capability to determine the plasma heating due to the dissipation of turbulence. This chapter has outlined a theoretical model of the kinetic turbulent cascade describing the flow of energy from the large driving scales, through the MHD inertial range, to the transition at the ion Larmor radius, and into the kinetic dissipation range, where the energy is ultimately converted to plasma heat. Although significant progress has already been made, much research remains to be done to refine the kinetic turbulence model and test its predictions using numerical simulations and observational data.

Since kinetic turbulence includes a number of the physical processes that are inherently kinetic—such as collisionless wave-particle interactions and the entropy cascade—kinetic numerical simulations will play an essential role in testing the predictions of this model. The higher dimensionality of kinetic systems—in general, requiring three dimensions in physical space and three dimensions in velocity space—demands a huge investment of computational resources to perform numerical simulations. It is tempting to reduce the dimensionality in physical space to two-dimensions to lower the computational costs, but doing so fundamentally limits the applicability of the results to turbulent astrophysical plasmas. The reason is because the anisotropic Alfvénic turbulence dominating the kinetic turbulent cascade is inherently three-dimensional in physical space: the dominant nonlinearity responsible for the turbulent cascade requires both dimensions perpendicular to the magnetic field, and Alfvénic fluctuations require variation in the parallel dimension (Howes et al., 2011). Therefore, kinetic simulation results can only be directly compared to astrophysical systems if physical space is modeled in three dimensions.

Observational tests of the kinetic turbulence model should exploit intuition from kinetic plasma theory to unravel the dependence of the turbulent properties on the plasma parameters. The suitably normalized MHD linear dispersion relation depends only on two dimensionless parameters, $\hat{\omega}_{\rm MHD} = \omega/(kv_A) = \hat{\omega}_{\rm MHD}(\beta,\theta)$: the plasma beta β, and the angle between the wavevector and the magnetic field θ (Klein et al., 2012). In contrast, the linear physics of Vlasov-Maxwell kinetic theory depends on five dimensionless parameters, $\tilde{\omega}_{\rm VM} = \omega/\Omega_i = \tilde{\omega}_{\rm VM}(k_\parallel\rho_i, k_\perp\rho_i, \beta_i, T_i/T_e, v_{ti}/c)$: the normalized parallel wavenumber $k_\parallel\rho_i$, the normalized perpendicular wavenumber $k_\perp\rho_i$, the ion plasma beta β_i, the ion-to-electron temperature ratio T_i/T_e, and the ratio of the ion thermal velocity to the speed of light v_{ti}/c (Stix, 1992; Howes et al., 2006).[4] The dynamical behavior of the kinetic plasma—for example, the frequencies, collisionless damping rates, and eigenfunctions of the fluctuations—varies as these dimensionless parameters are changed. Therefore, since the kinetic plasma physics depends on these parameters, observational investigations of kinetic turbulence should strive to analyze measurements in terms of the ion plasma beta β_i, the ion-to-electron temperature ratio T_i/T_e, and length scales normalized to a characteristic plasma kinetic length scale, such as the ion Larmor radius or ion inertial length.

Finally, we conclude this chapter on kinetic turbulence with a schematic diagram that depicts the salient features of the kinetic turbulent cascade, as shown in Figure 3 for the case of turbulence in the near-Earth solar wind (the relevant panel is noted in parentheses in the description below). The turbulence is driven isotropically at a large scale, corresponding to a normalized wavenumber $k_0\rho_i \sim 10^{-4}$ (a,b).

[4] In the limit that the turbulent astrophysical fluctuations satisfy the gyrokinetic approximation, $k_\parallel \ll k_\perp$ and $\omega \ll \Omega_i$ (Frieman and Chen, 1982; Howes et al., 2006; Schekochihin et al., 2009), the linear physics depends on only three dimensionless parameters, $\overline{\omega}_{\rm GK} = \omega/(k_\parallel v_A) = \overline{\omega}_{\rm GK}(k_\perp\rho_i, \beta_i, T_i/T_e)$ (Howes et al., 2006).

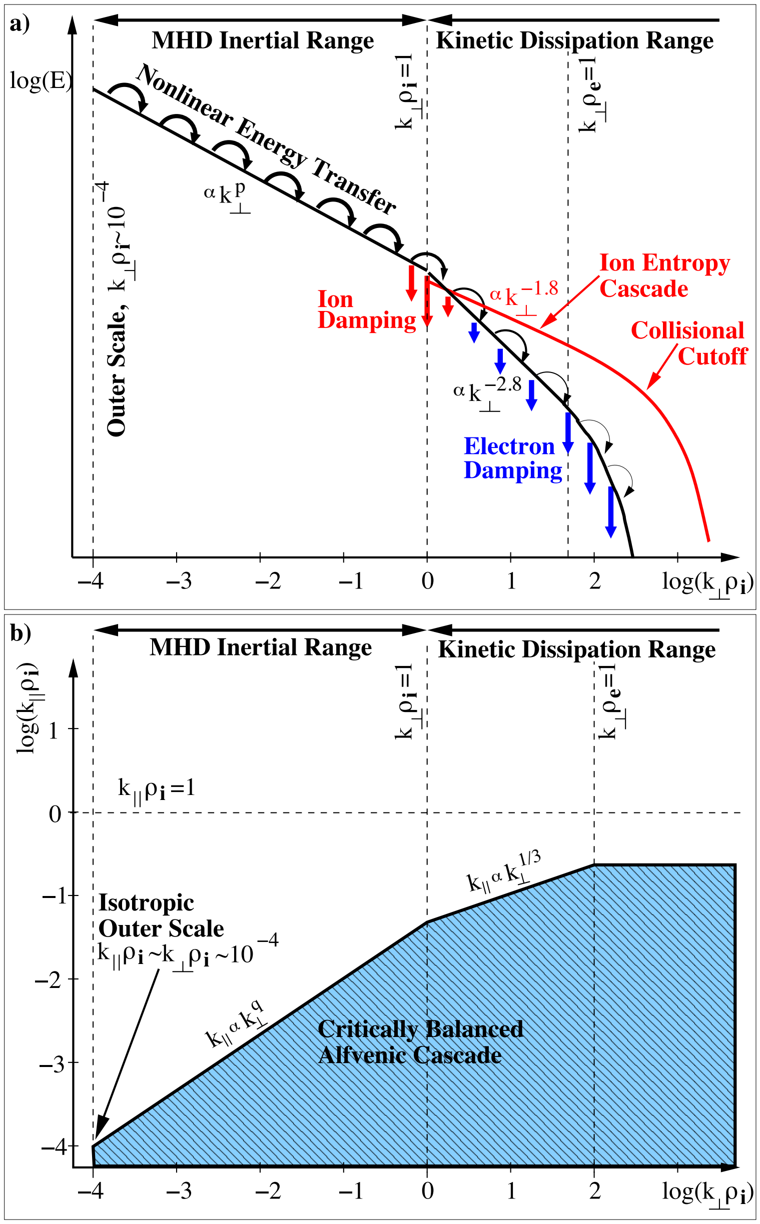

Fig. 3 (a) Diagram of the magnetic energy spectrum and ion entropy cascade in kinetic turbulence. (b) Anisotropic distribution of power in $(k_\perp, k_\parallel)$ wavevector space in kinetic turbulence.

Nonlinear interactions serve to transfer the energy from this low driving wavenumber to higher wavenumber through the MHD inertial range, generating a magnetic energy spectrum scaling as $E_B \propto k_\perp^p$, where $p = -5/3$ or $-3/2$ (a). This cascade is anisotropic in wavevector space, such that turbulent fluctuations fill the shaded region below $k_\parallel \propto k_\perp^q$, where $q = 2/3$ or $1/2$ (b). The Alfvénic turbulence transitions from the MHD inertial range to the kinetic dissipation range at a perpendicular wavenumber $k_\perp \rho_i \sim 1$, a scale at which collisionless ion Landau damping peaks (a). The energy transferred via wave-particle interactions to the ion distribution function feeds the ion entropy cascade at wavenumbers $k_\perp \rho_i \gtrsim 1$, a dual cascade in physical and velocity space that mediates the transfer of nonthermal structure to sufficiently small scales in velocity space that weak collisions can thermalize the energy (a). The remaining turbulent energy that is not collisionlessly damped at $k_\perp \rho_i \sim 1$ is transferred nonlinearly to a kinetic Alfvén wave cascade in the kinetic dissipation range, $k_\perp \rho_i \gtrsim 1$, leading to a magnetic energy spectrum $E_B \propto k_\perp^{-2.8}$ (a) and a wavevector anisotropy $k_\parallel \propto k_\perp^{1/3}$ (b). In addition, electron Landau damping becomes stronger as the wavenumber increases over the entire range $k_\perp \rho_i \gtrsim 1$ (a). Finally, electron Landau damping becomes sufficiently strong to terminate the kinetic turbulent cascade, leading to an exponential fall-off of the magnetic energy spectrum at $k_\perp \rho_e \sim 1$ (a), inhibiting the transfer of energy to higher parallel wavenumber (b), and possibly launching an electron entropy cascade (not shown).

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