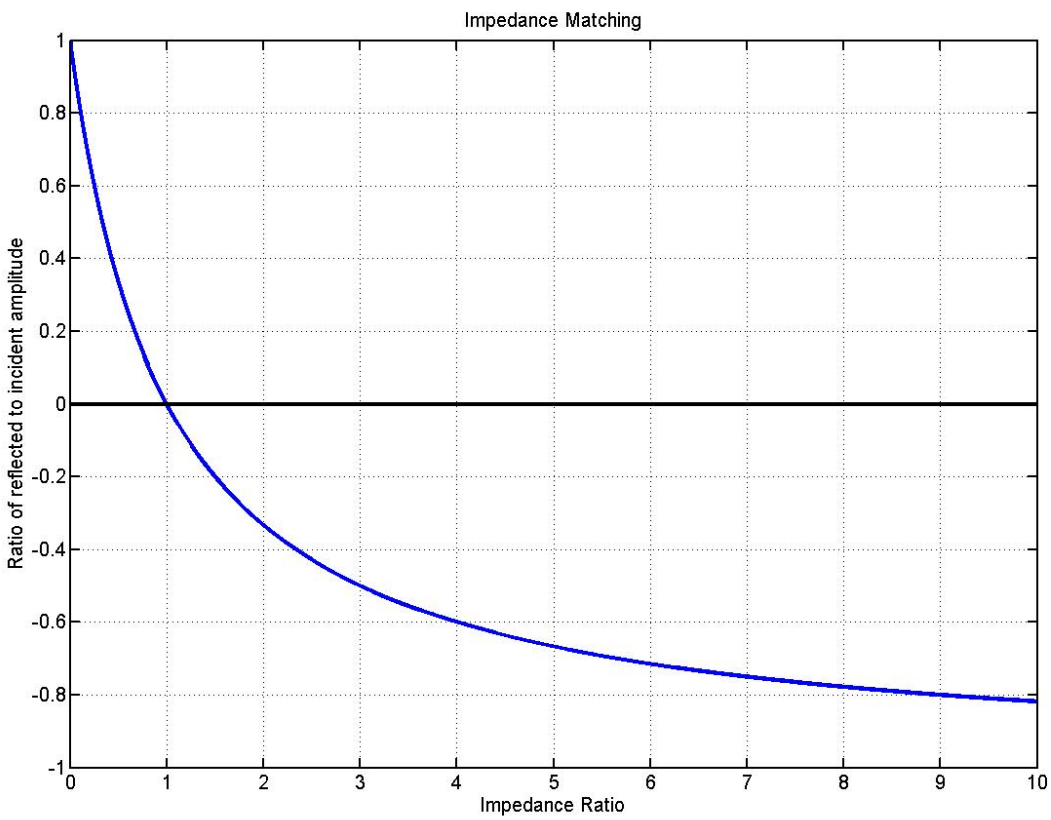
Name:____________________________

Impedance Matching Friday 5

We have been saying that if the impedance is matched between two media, then there will be no reflection of waves as energy passes from one medium to another. If a wave pulse passes from medium number 1 with impedance Z_1 and strikes a medium number 2 that has impedance Z_2, the ratio of the amplitude of a reflected wave pulse A_r to the incident wave pulse A_i is given by:

$$\frac{A_r}{A_i} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = \frac{1 - Z_1/Z_2}{1 + Z_1/Z_2}$$



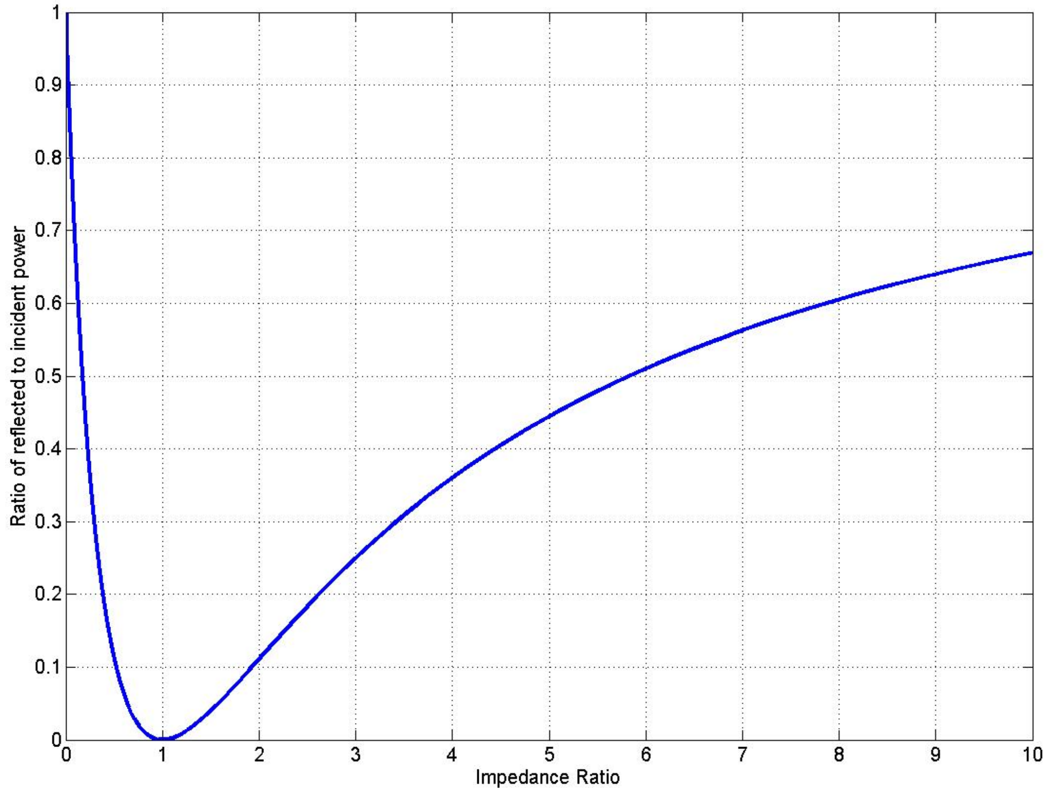
Plot of the ratio A_r/A_i given the impedance ratio Z_2/Z_1

Fig. 1

Notice that if a wave reflects from a low impedance (Z_2 less than Z_1) then the reflected pulse has the same sign and nearly the same amplitude as the incident pulse. If it reflects from a high impedance (Z_2 greater than Z_1) then the sign is inverted. This is what we saw with reflections on the wave machine in lecture.

If the impedance ratio is 1, this means that the impedances are the same and then the amplitude of the reflected wave is zero. This corresponds to "matched impedances".

The energy in the wave is always a positive number and it is proportional to the square of the amplitude. If we look at the power (energy flow) in the reflected wave we see that it goes to zero when the impedances are matched. This is shown in the next figure. The numbers in this graph are simply the square of the numbers in the first graph.



Fraction of power that is reflected as a function of the impedance ratio.
Fig.2

Sound in a tube
An important example of sound propagation is sound traveling inside of a tube. Because sound propagation in the tube is confined and guided by the tube, the situation is one-dimensional. In open air you use the specific impedance $Z_s=\rho S$ (the product of the mass density ρ and the speed of sound S). Inside of a tube, the impedance is different and has different units $Z=Z_s/A$ where A is the area of the tube ($A=\pi r^2$ where r is the tube radius). When sound that is inside of a tube comes to the end of a tube, there is a miss-match of the impedance and the situation is complex. It turns out that the impedance of the end of the pipe depends on the ratio of the radius of the tube r to the sound wavelength λ. This is because sound diffracts out and around the end of the tube, and diffraction depends on the size of the wavelength compared to the size of the opening. Figure 3 shows both the impedance ratio Z_2/Z_1 and the ratio of the reflected power to the incident power for sound as a function of the ratio between the tube radius and the wavelength. You can also think of the horizontal axis as a frequency axis, once you specify the radius of the tube. If

you know the speed of sound *S*, the tube radius *r* and the frequency *f*, then you can determine the ratio of tube radius to wavelength using:

$$\frac{r}{\lambda} = \frac{r}{S} \cdot f$$

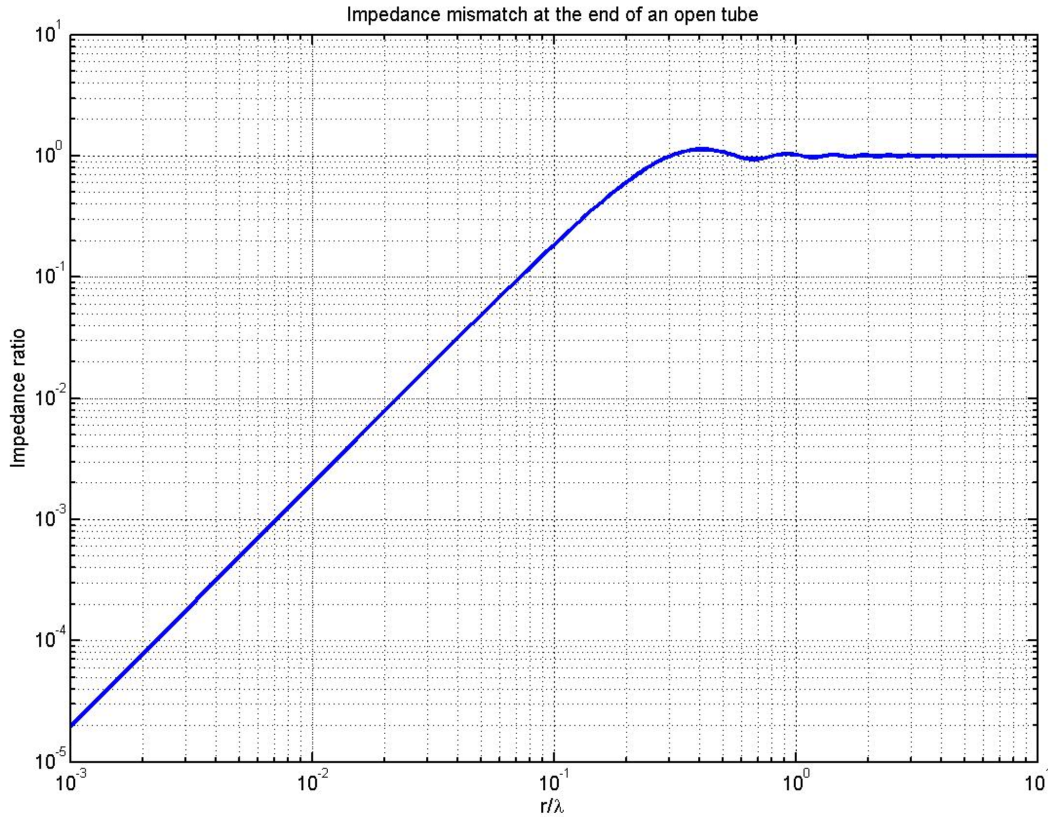


Figure 3. Impedance mismatch at the end of an open tube.

Examples:

1) Sound traveling through concrete hits a layer of steel. How much of the sound will be reflected? The speed of sound in steel is about 5900 m/s and the mass density of steel is about 7800 kg/m^3. This gives an acoustic specific impedance of about 46,000,000 Kg/m^2/s. The speed of sound in concrete is about 3100 m/s the mass density is 2600 kg/m^3. Thus the impedance of concrete is about 8,000,000 kg/m^2/s.

The impedance ratio is

$$\boldsymbol{\frac{Z_2}{Z_1} = \frac{46,000,000}{8,000,000} = 5.75}$$

So from figure 2 we see that about .5 or half of the energy is reflected.

2) What is the amplitude of the reflected wave compared to the amplitude of the incident wave for the situation in question 1?

This we read off of figure (or from the formula) that the ratio is about –0.7. So the wave reflects with a change of sign (This is a shift of phase of π or 180 degrees).

3) The tube of a flute has an inside diameter of 1.9 cm. What is the fraction of power reflected when concert A (440 Hz) hits the end of the tube? Take the speed of sound to be 340m/s.

The wavelength is λ=(340m/s)/(440 Hz)=.77m.

The ratio of the tube radius (*r*=1.9/2=0.95 cm=.0095m) to the wavelength is r/λ=(.0095m)/(.77m)=0.0123 This number can be written as 1.23x 10^{-2} Looking at the graph in figure 3 we see that the impedance ratio is between 10^{-2} and 10^{-3} (about .003). Now, using the graph in figure 2 you can see that the reflection is very high (Using the formula we see that it is .9881 which means that only about 1.2% of the sound power percent escapes out of the tube.)